Fractionally Quantized Topological Nonlinear Thouless Pumping of Solitons

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Abstract: We theoretically predict and experimentally observe that nonlinearity can act to fractionally quantize the motion of solitons in photonic Thouless pumps. Such fractionalization is unique to the nonlinear regime.

Thouless pumps are 1+1 dimensional reductions of Chern insulators for which one wavevector dimension is substituted with a periodic modulation [1]. In analogy to the quantized conductance of Chern insulators, Thouless pumps show quantized transport assuming adiabaticity and filled bands. As 1+1 dimensional reductions, Thouless pumps do not need to break time-reversal symmetry (or its equivalent) and have therefore been implemented and observed in various experimental platforms [2], perhaps most successfully in photonic waveguide systems, due to their flexible design. Additionally, at high optical power, the nonlinear response of the underlying host material mediates interactions between photons described via nonlinearities in the mean-field limit. Understanding topological protection in the presence of nonlinearities is especially important for envisioning complex photonic topological devices.

We show quantized soliton pumping in arrays in evanescently-coupled single-mode waveguides, for which the dynamics of high-power, monochromatic light is described by the discrete nonlinear Schrödinger equation (a.k.a. Gross-Pitaevskii equation):

$$i\frac{\partial}{\partial z}\Psi_n(z) = \sum_m H^{(\text{lin})}_{n,m}(z)\Psi_m(z) - g|\Psi_n(z)|^2\Psi_n(z). \tag{1}$$

Here z denotes the propagation distance, Ψ_n the electric field amplitude on site n, and $g > 0$ the strength of a focusing Kerr nonlinearity. The tight-binding Hamiltonian $H^{(\text{lin})}_{n,m}(z)$ describes a Thouless pump, in particular the off-diagonal Aubry-André-Harper model (AAH) [3, 4]. A schematic diagram of our photonic implementation of an AAH model with five sites per unit cell and equal on-site potential is shown in Fig. 1a-c. A periodic modulation of the hopping strength is achieved by modulating the distances between the waveguides.

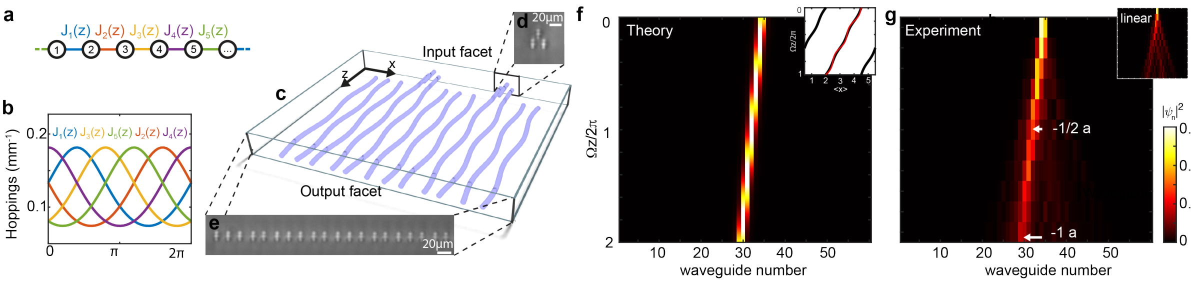

Fig. 1. Quantized fractional Thouless pumping. **a.** a Schematic of a five-site AAH-model. **b.** Modulation of the hopping over one period to create a Thouless pump. **c.** Schematic implementation of the model shown in **a-b** in evanescently-coupled waveguides. **d.** and **e.** White-light micrograph of the input and output facet. **f.** Calculated adiabatic soliton evolution over two pumping periods. The inset shows the center of mass position of the soliton (red) and the multi-band Wannier function of the two lowest bands (black) over one period. **g.** Experimentally measured intensity of the fractionally pumped soliton showing a displacement of -1/2 unit cells (a denotes lattice constant) after one period and -1 unit cell after two periods. The inset shows the significant diffraction for linear propagation. Adapted from [5].

The Kerr nonlinearity in Eq. (1) gives rise to the formation of discrete spatial solitons by balancing diffraction; these solitons are localized eigenstates that preserve their spatial shape during propagation for a z-independent Hamiltonian. (This is to be distinguished from temporal solitons, which preserve their temporal shape.) Figure 1f shows the adiabatic z-evolution over two periods for a fractionally pumped soliton. During propagation the wavefunction remains localized, and we have shown numerically that – in the adiabatic limit – the wavefunction resembles the shape of an instantaneous soliton at each point during the pump cycle [6]. This nonlinear adiabaticity for solitons can be understood as the analog of the adiabatic theorem for linear eigenstates. It has been shown theoretically and experimentally that low-power solitons are pumped by the Chern number of the band from which they bifurcate [6–8], and the solitons come back to the same wavefunction after each pump cycle. Here, however, in the regime of higher input power ($gP/J^{\max}$=2.15; with P=$\sum_n |\Psi_n|^2$), the soliton comes back to itself only after two periods and is displaced by -1/2 unit cells after one period and -1 unit cell after two periods. While we only show one example here, soliton pumping forms quantized plateaux over a range of input powers. We explain the mechanism behind this fractional quantization of soliton motion below.

Quantization of soliton pumping can be understood following a simple argument [6]: If a soliton exists in one unit cell, it also exists in all other unit cells due to translation invariance. As Thouless pumps are periodic, the solitons at the beginning and end of each period are identical. Therefore, if the soliton ends up in the same soliton eigenstates after one period, it has to be either the initial soliton or translated by an integer number of unit cells. Similarly, if it ends up in the same soliton only after multiple periods, its displacement is quantized after multiple periods. By expanding the wavefunction in the basis of instantaneous Wannier states and neglecting inter-band couplings, it has been shown that a low-power soliton tracks the position of the single-band Wannier state of the band from which it bifurcates [7, 8]; it therefore shows integer displacement. With increasing power, the single-band assumption is no longer justified. Instead, multiple bands have to be considered. Here, we observe numerically that the soliton follows the maximally localized multi-band Wannier function (of the two lowest bands), as shown in the inset of Fig. 1f. While the combined displacement of the multi-band Wannier functions is an integer and given by the sum of the Chern numbers, the individual displacement can be fractional. For the case shown here, the displacement is -1/2 unit cells after each period.

We observe fractionally quantized nonlinear Thouless pumping of solitons over two periods in arrays of evanescently-coupled waveguides fabricated using femtosecond direct laser-writing. To reach the necessary degree of nonlinearity, we use 2-ps-long, high-power laser pulses (wavelength 1030 nm), that are pre-chirped to minimize spectral broadening due to self-phase modulation such that we can approximate the hopping strength to be wavelength-independent [9]. To efficiently excite the fractionally pumped soliton, we use a triple-coupler that converts a single-site excitation into an effective two-site excitation before the Thouless pumping starts (see Fig. 1c,f). We map out the propagation of the wavefunction as a function of z by repeatedly cutting the sample and imaging the output facet of the waveguide array as a function of input power. For linear propagation ($g = 0$), we measure significant diffraction as expected (see inset in Fig. 1g) and no quantization. At high input power ($gP/J^{\max}$=2.15), the intensity remains mostly localized during propagation as the nonlinearity balances the diffraction, forming a spatial soliton (see Figure 1g). After one period, the soliton is localized on a single site (and displaced by -1/2 unit cells). It only returns to its initial shape after two periods with a displacement of -1 unit cell, as dictated by the sum of the Chern numbers of the two lowest bands (+2-3=-1), which determines the winding of the multi-band Wannier states and is in agreement with the theoretical simulations.

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