Orthogonal STBC-MIMO Index Coded PSK Modulation for Prioritized Receivers

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***Abstract*—Multiple input multiple output (MIMO) scheme which employs Alamouti code with index coded PSK modulation over Rayleigh fading channel has been studied. In this work, for the noisy index coding problem we generalize the MIMO scheme for any space time block codes (STBCs) obtained from orthogonal designs in which the central server transmits symbols over $N_t \times N_r$ MIMO Rician fading channel. We show that, for a chosen index code and 2^N-PSK signal set at very high SNR, error performance of a receiver is decided by minimum inter-set distance seen by the receiver and for achieving its best ML decoding performance, we must choose the mappings that maximizes minimum inter-set distance. Further for improving the performance of high priority receivers for a chosen index code of length N we are transmitting N bits using a rotated M-PSK ($M = 2^N$) constellation.**

***Index Terms*—Noisy index coding, MIMO, Space time block codes (STBCs).**

I. Introduction

A. Background

The index coding problem (ICP) [1], [2], was studied as the noiseless form of broadcasting with side information. ICP was initially implemented to reduce the number of transmissions for satellite communication. Several significant engineering problems, such as topological interference management [3], content delivery [4], and device-to-device (D2D) communication [5], have utilized the ICP concept. It includes a central server that intends to send a set of messages over a broadcast channel to a set of receivers that are already familiar with a subset of the messages as side information and request another subset of messages. Consider the case of a central server with set of m messages denoted by $\mathcal{X} = \{x_1, x_2, \ldots, x_m\}$ where $x_i \in F_2$ which it broadcasts as coded messages to a set of n receivers denoted by $\mathcal{R} = \{R_1, R_2, \ldots, R_n\}$. Each receiver $R_i \in \mathcal{R}$ knows a priori a proper subset K_i of the messages, wants a subset W_i of the messages, where $W_i \cap K_i = \emptyset$ and is identified by the pair (W_i, K_i).

***Definition* 1.** An index code for ICP ($\mathcal{X}$,$\mathcal{R}$) over binary field $F_2 = \{0, 1\}$ consists of

1) An encoding function for the sender, $E : \{0, 1\}^m \to \{0, 1\}^N$, and

2) Set of decoding function corresponding to each receiver R_i, $D_i : \{0, 1\}^{N+|K_i|} \to \{0, 1\}^{|W_i|}$ such that $D_i(\mathrm{E}(\mathcal{X}), K_i) = W_i$, for each $i \in \{1,2,\ldots,n\}$.

The purpose of a noiseless ICP, as defined in [6], is to identify an index code that minimizes the length N, which is equal to the number of binary transmissions made by the server to satisfy the demands of all the receivers. The index code with the shortest length is known as an optimal index code. In [6], the classification of noiseless ICPs is discussed.

The ICP over a noisy broadcast channel is the most realizable. In [7], ICP over noisy channels with a central server supporting binary transmissions was studied. M-ary modulation schemes have been considered for the transmission of coded messages because, as demonstrated in [8], M-ary modulation can accomplish a significant reduction in bandwidth compared to binary transmissions.

In [9], the case of noisy index coding over Additive White Gaussian Noise (AWGN) broadcast channels, with prioritized receivers, was investigated for a given index code and predefined receiver priority order. The notion of prioritized receivers was introduced to make noisy ICP more practical. For example, a content delivery network such as (Hotstar or YouTube) might decide to provide superior performance to premium receivers who pay more than other receivers. A decision rule for the Maximum Likelihood (ML) decoder was derived for transmission of symbols over the AWGN broadcast channel, which provides the optimal error performance for any receiver R_i. In [9], an algorithm for determining the optimal set of mappings for a given ICP with prioritized receivers was proposed. It was demonstrated that mapping based on maximizing minimum inter-set distance (***Definition* 3**) was more pertinent than mapping based on maximizing minimum Euclidean distance. To further enhance the error performance of receivers with the highest priority, coded messages were broadcasted using a rotated PSK constellation, as in [10], in which half of the signal points were rotated while the other half remained fixed.

In [11] noisy index coding over SISO Rayleigh fading broadcast channel has been studied. In this paper we extend the MIMO scheme based on Alamouti code in [9] to any STBCs obtained from orthogonal designs [12] in order to attain diversity gain in addition to coding gain.

B. Our Contributions

We consider a noisy ICP ($\mathcal{X}$,$\mathcal{R}$) with m messages and n receivers over MIMO Rician fading channel. Even though Rayleigh fading is an outstanding approximation in a large number of practical situations. It can be demonstrated that the Rician model has the Line-of-Sight (LOS) component. In terms of probability of error, it is demonstrated that the Rician fading exhibits less error probability for a Receiver R_i than the Rayleigh fading due to the presence of a LOS component. The main contributions of this work are listed below:

- We extend the proposed MIMO scheme based on Alamouti code [9] to any STBCs obtained from orthogonal designs, for the noisy ICP, to achieve maximum diversity gain.
- We derive the ML decision rule that provides the optimal performance for any receiver R_i in a MIMO scheme, given a particular index code and mapping.
- We show that, for a chosen index code and 2^N-PSK signal set, with transmission of symbols over MIMO Rician fading channel, to achieve best ML decoding performance of a receiver at high SNR, the mappings must maximize the minimum inter-set distance.
- Like in [10] we map the broadcast vectors (***Definition* 2**) to a rotated 2^N-PSK constellation to enhance the probability of error performance over MIMO Rician fading channel for high priority receivers.

C. Organisation and Notation

The paper is organized as follows. In section II we have discussed the preliminaries of noisy index coding [9]. In section III we consider MIMO scheme which employs orthogonal STBCs with index coded PSK modulation. In section IV, we discuss the simulation results to encapsulate our work. Finally, section V concludes the paper.

The mathematical notation defined in paper are as follows: The set $\{1, 2, \ldots, m\}$ is denoted by $[m]$. A vector is denoted as $\mathbf{x} = \begin{pmatrix} x_1 & x_2 & \ldots & x_m \end{pmatrix} \in F_2^m$ and a matrix is denoted by an upper-case bold letter, as in $\mathbf{L}$.

II. PRELIMINARIES

The summary of noisy index coded PSK modulation is described in this section. The notions of Effective broadcast vector sets and Inter-set distance [9] will be briefly defined.

A. Review of [9]

In a noisy ICP, a central server with m messages transmit messages to n receivers across a SISO AWGN broadcast channel. Considering each receiver wants a single unique message. The pair (W_i, K_i), want set and known set, uniquely identify each receiver R_i. Let the set of indices $b_i = \{j : x_j \in K_i\}$ be the set corresponding to the known set. Each receiver requests the message $x_{z(i)}$, where z: $[n] \to [m]$ is a map from the set of receivers to the set of messages.

***Definition* 2. Effective broadcast vector set:** Let $a_i \in F_2^{|K_i|}$ be the side of information realization of receiver R_i. The effective broadcast vector set is the subset of F_2^N that must be taken into account by each receiver R_i for decoding because each receiver R_i knows certain messages from its known set. It is denoted as $C_{\mathbf{L}}(a_i)$ for encoding matrix $\mathbf{L}$,

$$C_{\mathbf{L}}(a_i) = \{\mathbf{y} \in F_2^N : \mathbf{y} = \mathbf{x}\mathbf{L}, x_{b_i} = a_i, x_l \in F_2, l \in [m] \setminus b_i\}.$$

The effective broadcast vector set $C_{\mathbf{L}}(a_i)$ can further be separated into two subsets, namely zero effective broadcast vector set $C_{\mathbf{L}0}(a_i)$ and one effective broadcast vector set $C_{\mathbf{L}1}(a_i)$, based on the i^{th} receiver desired message bit.

$$C_{\mathbf{L}0}(a_i) = \{\mathbf{y} \in F_2^N : \mathbf{y} = \mathbf{x}\mathbf{L}, x_{b_i} = a_i, x_{z(i)} = 0, \\ x_l \in F_2, l \in [m] \setminus (b_i \cup z(i))\}.$$

$$C_{\mathbf{L}1}(a_i) = C_{\mathbf{L}}(a_i) \setminus C_{\mathbf{L}0}(a_i).$$

The complex signal points in 2^N-PSK constellation corresponding to $C_{\mathbf{L}}(a_i)$, for receiver R_i is termed as **Effective broadcast signal set** ($S_{\mathbf{L}}(a_i)$).

***Definition* 3. Inter-set Distance:** The minimum euclidean distance between the signal points corresponding to $C_{\mathbf{L}0}(a_i)$ and signal points corresponding to $C_{\mathbf{L}1}(a_i)$ is termed as inter-set distance seen by receiver R_i corresponding to the side information realization a_i, denoted as,

$$d_{IS}(S_{\mathbf{L}}(a_i)) \triangleq min\{|s_a - s_b| : s_a \in S_{\mathbf{L}0}(a_i), s_b \in S_{\mathbf{L}1}(a_i)\}$$

and **Minimum inter-set distance** for receiver R_i is minimum of inter-set distance among all the $S_{\mathbf{L}}(a_i)$,

$$d_{IS,min}^{(i)} \triangleq \min_{a_i \in F_2^{|K_i|}} d_{IS}(S_{\mathbf{L}}(a_i)).$$

III. MIMO INDEX CODED PSK MODULATION

In this section, we look into MIMO index-coded PSK modulation incorporating orthogonal STBCs [12] across a Rician fading channel and develop a decision rule for ML decoding that provides the optimal probability of error performance for any receiver R_i. Subsequently, we determine an upper bound on pairwise error probability.

A. STBCs from Orthogonal design

A Generalized Linear Complex Orthogonal Design (GLCOD) in l complex variables c_1,c_2,...,c_l is a $P \times Q$ matrix $\mathbf{C}(c_1, c_2, \ldots, c_l)$ such that the following:

- the entries of $\mathbf{C}$ are complex linear combinations of 0, $\pm c_i$, $\forall\ i \in [l]$ and their conjugates;
- $\mathbf{C}^H\mathbf{C} = \mathbf{D}$, where $\mathbf{D}$ is diagnol matrix whose entries are linear combination of $\mid c_i \mid^2$, $\forall\ i \in [l]$ [12].

The symbol transmission rate of $\mathbf{C}(c_1, c_2, ..., c_l)$ is defined as l/P. When the only entries of $\mathbf{C}$ are $\{0, \pm c_1, \pm c_2, \ldots, \pm c_l\}$ and their conjugates, $\mathbf{C}$ is referred to as a Complex Orthogonal design (COD) [12].

In [13] Alamouti proposed the first STBC from 2×2 orthogonal design with rate 1 for 2 transmit antennas. Orthogonal codes with rate 3/4 were introduced for three and four transmit antennas [14], [15].

$$\mathbf{C}_2(c_1,c_2)=\begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \tag{1}$$

$$\mathbf{C}_3(c_1,c_2,c_3)=\begin{bmatrix} c_1 & c_2 & c_3 \\ -c_2^* & c_1^* & 0 \\ -c_3^* & 0 & c_1^* \\ 0 & -c_3^* & c_2^* \end{bmatrix} \tag{2}$$

$$\mathbf{C}_4(c_1,c_2,c_3)=\begin{bmatrix} c_1 & c_2 & c_3 & 0 \\ -c_2^* & c_1^* & 0 & c_3 \\ -c_3^* & 0 & c_1^* & -c_2 \\ 0 & -c_3^* & c_2^* & c_1 \end{bmatrix} \tag{3}$$

In [16] an obvious and straightforward realizations of orthogonal designs of square size with rate $(l+1)/2^l$, where $Q=2^l$ is the number of transmit antennas, was given as follows. Let $\mathbf{C}_1=c_1\mathbf{I}_1$ ($\mathbf{I}_n$ is $n\times n$ Identity matrix) then $\mathbf{C}_{2^l}(c_1,c_2,...,c_{l+1})$ can be constructed for l = 1,2,3,... as follows

$$\mathbf{C}_{2^l}(c_1,c_2,...,c_{l+1})= \begin{bmatrix} \mathbf{C}_{2^{l-1}}(c_1,c_2,...,c_l) & c_{l+1}\mathbf{I}_{2^{l-1}} \\ -c_{l+1}^*\mathbf{I}_{2^{l-1}} & \mathbf{C}_{2^{l-1}}^H(c_1,c_2,...,c_l) \end{bmatrix} \tag{4}$$

By omitting some columns from (4), orthogonal designs for a certain number of transmit antennas that are not powers of 2 can be derived.

B. Proposed MIMO Scheme

We propose a MIMO scheme for any STBCs obtained from orthogonal designs. Consider the scenario in which the central server has N_t transmit antennas and each receiver has N_r receive antennas. At each symbol time, m messages are present with the server, $\mathbf{x}=\begin{pmatrix} x_1 & x_2 & \dots & x_m \end{pmatrix}\in F_2^m$ which it broadcast as coded messages to n receivers, where each receiver knows a priori a proper subset of messages and wants single message.

We consider a orthogonal STBC $\mathbf{C}(c_1,c_2,...,c_l)$ of size $P\times N_t$. For square orthogonal STBC $P=N_t$. Let $\mathbf{x}_i$ be the message vector at symbol time i, corresponding broadcast vector be $\mathbf{y}_i$ and corresponding 2^N-PSK signal point is $c_i=\mathcal{M}(\mathbf{x}_i\mathbf{L})$ $\forall i\in[l]$ ($\mathcal{M}$ be the map from broadcast vectors to 2^N-PSK constellation points). After mapping of broadcast vectors to 2^N-PSK signal points, the central server incorporates a orthogonal STBC $\mathbf{C}(c_1,c_2,...,c_l)$ of size $P\times N_t$.

Consider a receiver R_i, we can write the received signal matrix as

$$\mathbf{Y}=\mathbf{C}(c_1,c_2,...,c_l)\mathbf{H}+\mathbf{W} \tag{5}$$

where $\mathbf{Y}\in\mathbb{C}^{P\times N_r}$ ($\mathbb{C}$ denotes the complex field) is the received signal matrix, $\mathbf{C}(c_1,c_2,...,c_l)\in\mathbb{C}^{P\times N_t}$ is the STBC matrix, $\mathbf{H}\in\mathbb{C}^{N_t\times N_r}$ denotes the channel matrix which contains independent and identically distributed (i.i.d) complex valued gaussian channel gains h_{uv} having non zero mean and variance unity between the u^{th}, $1\le u\le N_t$ transmit antenna and v^{th}, $1\le v\le N_r$ receive antenna.

Using phasor notation, the complex channel gain h_{uv} is represented as $h_{uv}=a_{uv}\cdot e^{j\phi_{uv}}$, where a_{uv} is the amplitude of fading process and a_{uv}, $\forall u\in[N_t],\forall v\in[N_r]$ is Rician distributed with PDF,

$$p(a_{uv})=2a_{uv}(1+K)\exp(-(K+a_{uv}^2(1+K)))I_0(2a_{uv}\sqrt{K(1+K)}) \tag{6}$$

where K is the Rician parameter defined as the ratio of line-of-sight path to remaining multipaths and $I_0(.)$ is the zero-order modified Bessel function of the first kind.

$\mathbf{W}\in\mathbb{C}^{P\times N_r}$ denotes the noise matrix which contains complex-valued AWGN with zero mean and variance N_0. The channel is assumed to be quasi static and perfect channel state information is available at the receiver.

C. Maximum-Likelihood Decoder

We establish a decision rule corresponding to the ML decoder for a receiver R_i. To arrive at the ML decision rule, we adopt a methodology akin to that of [9].

Take into account the decoding of the message vector $\mathbf{x}_l$, ($c_l=\mathcal{M}(\mathbf{x}_l\mathbf{L})$). Since STBCs from orthogonal designs can be decoded symbol by symbol, the equation for decoding each symbol can be expressed as $d=\sum_{u=1}^{N_t}\sum_{v=1}^{N_r}a_{uv}^2c_l+w'$, where d is the decision variable and w' is distributed as $\mathcal{CN}(0,N_0\sum_{u=1}^{N_t}\sum_{v=1}^{N_r}a_{uv}^2)$.

Consider receiver R_i wants $x_{z(i)}$. The conditional probability density of d given that $\mathcal{M}(\mathbf{x}_l\mathbf{L})$ is transmitted and fading amplitudes $a_{uv},\forall u\in[N_t],\forall v\in[N_r]$ are perfectly known at the receiver is

$$p(d|\mathcal{M}(\mathbf{x}_l\mathbf{L}),a_{uv},u,v)=\frac{\exp\left(-\frac{|d-\sum_{u=1}^{N_t}\sum_{v=1}^{N_r}a_{uv}^2\mathcal{M}(\mathbf{x}_l\mathbf{L})|^2}{N_0\sum_{u=1}^{N_t}\sum_{v=1}^{N_r}a_{uv}^2}\right)}{(\pi N_0\sum_{u=1}^{N_t}\sum_{v=1}^{N_r}a_{uv}^2)}. \tag{7}$$

Following an approach akin to one used in [9] we can write decision rule (ignoring ties) as,

$$\mathbf{P}(x_{z(i)}=0|x_{b_i}=a_i,d,a_{uv},u,v)\lessgtr_0^1 \mathbf{P}(x_{z(i)}=1|x_{b_i}=a_i,d,a_{uv},u,v) \tag{8}$$

Using Bayes rule in (8), we obtain decision rule in terms of likelihood functions as

$$p(d|x_{z(i)}=0,x_{b_i}=a_i,a_{uv},u,v)\mathbf{P}(x_{z(i)}=0)\lessgtr_0^1 p(d|x_{z(i)}=1,x_{b_i}=a_i,a_{uv},u,v)\mathbf{P}(x_{z(i)}=1) \tag{9}$$

$S_{\mathbf{L}0}(a_i)$ and $S_{\mathbf{L}1}(a_i)$ is the set of all signal points corresponding to broadcast vector with $x_{z(i)}=0$ and $x_{z(i)}=1$ with $x_{b_i}=a_i$. Therefore,

$$p(d|x_{z(i)}=0,x_{b_i}=a_i,a_{uv},u,v)=p(d|S_{\mathbf{L}0}(a_i),a_{uv},u,v) \tag{10}$$

$$p(d|x_{z(i)}=1,x_{b_i}=a_i,a_{uv},u,v)=p(d|S_{\mathbf{L}1}(a_i),a_{uv},u,v) \tag{11}$$

Assuming that all messages take values 0 and 1 with equal probability, from (9), (10) and (11) we obtain decision rule as

$$\sum_{k:s_k\in S_{\mathbf{L}0}(a_i)} p(d|s_k, a_{uv}, u, v) \lessgtr_0^1 \sum_{k:s_k\in S_{\mathbf{L}1}(a_i)} p(d|s_k, a_{uv}, u, v) \tag{12}$$

From (7) and (12), we obtain ML decision rule as,

$$\sum_{k:s_k\in S_{\mathbf{L}0}(a_i)} \left(\exp\left(-\frac{|\, d-\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2 s_k)\,|^2}{N_0\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2}\right)\right) \lessgtr_0^1 \sum_{k:s_k\in S_{\mathbf{L}1}(a_i)} \left(\exp\left(-\frac{|\, d-\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2 s_k)\,|^2}{N_0\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2}\right)\right) \tag{13}$$

From (13), we can infer that euclidean distance of scaled signal points (by $\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2$) in $S_{\mathbf{L}0}(a_i)$ to d, with respect to scaled signal points in $S_{\mathbf{L}1}(a_i)$ is the basis for ML decision rule.

D. Upper Bound on Pairwise Error Probability

In this subsection, for symbol transmission over $N_t \times N_r$ MIMO Rician fading channel, we derive an upper bound on the pairwise error probability.

Let's consider receiver R_i, demanding a message $x_{z(i)} = 1$. After scaling by $(\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2)$, the decoder will locate a signal point that is close to d. If signal point lies in $S_{\mathbf{L}1}(a_i)$ then it is error free. If signal point lies in $S_{\mathbf{L}0}(a_i)$ then decoder makes an error.

Assuming that fading amplitudes $a_{uv}, \forall u \in [N_t], \forall v \in [N_r]$ are perfectly known at the receiver R_i , ML detection requires minimization of the metric,

$$m(s_k|d, a_{uv}, u, v) = \left|d - \sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2 s_k\right|^2 \tag{14}$$

where $s_k \in S_{\mathbf{L}1}(a_i)$ is transmitted. The decoder makes an error if and only if it decodes to $s_{k'} \in S_{\mathbf{L}0}(a_i)$,

$$\begin{aligned}&\mathbf{P}\{s_k \to s_{k'}|a_{uv}, u, v\}\\ &= \mathbf{P}\Big(\Big|d - \sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2 s_{k'}\Big|^2 \le \Big|d - \sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2 s_k\Big|^2\Big)\end{aligned} \tag{15}$$

$$= \mathbf{P}\Big(\Big|\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2(s_k - s_{k'})\Big|^2 + 2Re\big(\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2(s_k - s_{k'})w'^*\big) \le 0\Big)$$

Now, Let

$$X = Re\Big(\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2(s_k - s_{k'})w'^*\Big)$$

where X is distributed as Gaussian with zero mean and variance

$$\sigma_X^2 = \frac{N_0 \cdot \sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2}{2}\Big(\Big|\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2(s_k - s_{k'})\Big|^2\Big).$$

Let $A = \frac{1}{2}\Big(\Big|\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2(s_k - s_{k'})\Big|^2\Big)$, we have

$$\mathbf{P}\{s_k \to s_{k'}|a_{uv}, u, v\} = \mathbf{P}(A + X \le 0) = Q\left(\frac{A}{\sigma_X}\right) \tag{16}$$

$Q(x)$ is a Gaussian tail function. The Gaussian tail function can be upper bounded by an exponential function as,

$$Q(x) \le \frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$$

The conditional pairwise error probability becomes,

$$\begin{aligned}&\mathbf{P}\{s_k \to s_{k'}|a_{uv}, u, v\} \le\\ &\qquad \frac{1}{2}\exp\left(-\frac{\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2|(s_k - s_{k'})|^2}{4N_0}\right)\end{aligned} \tag{17}$$

In order to find an upper bound on pairwise error probability, we average (17) with respect to identical and independent Rician distributions of a_{uv} with parameter K to arrive at

$$\begin{aligned}&\mathbf{P}\{s_k \to s_{k'}\} \le\\ &E_{a_{uv},u,v}\left[\frac{1}{2}\exp\left(-\frac{\sum_{u=1}^{N_t}\sum_{v=1}^{N_r} a_{uv}^2|(s_k - s_{k'})|^2}{4N_0}\right)\right]\end{aligned}$$

Now, Let

$$\frac{|(s_k - s_{k'})|^2}{4N_0} = m^2$$

$$\mathbf{P}\{s_k \to s_{k'}\} \le \frac{1}{2}\prod_{u=1}^{N_t}\prod_{v=1}^{N_r} E_{a_{uv}}\left[\exp\left(-a_{uv}^2 m^2\right)\right] \tag{18}$$

$$= \frac{1}{2}\left[\left(\frac{1+K}{m^2+1+K}\right)\exp\left(-\frac{m^2 K}{m^2+1+K}\right)\right]^{N_t N_r}$$

$$= \frac{1}{2}\left[\left(\frac{1+K}{\frac{|(s_k-s_{k'})|^2}{4N_0}+1+K}\right)\exp\left(-\frac{K\left(\frac{|(s_k-s_{k'})|^2}{4N_0}\right)}{\frac{|(s_k-s_{k'})|^2}{4N_0}+1+K}\right)\right]^{N_t N_r}$$

Assuming normalized signal power, we have $SNR = \frac{1}{N_0}$. So, at high SNR we obtain the upper bound on pairwise error probability as,

$$\mathbf{P}\{s_k \to s_{k'}\} \le \frac{1}{2}\left[\frac{4(1+K)}{SNR|(s_k - s_{k'})|^2}\exp(-K)\right]^{N_t N_r}. \tag{19}$$

As a result, we may deduce from (19) that the suggested MIMO scheme provides the maximum diversity gain of $N_t N_r$. It is significant to remember that, in contrast to traditional MIMO schemes, the inter-set distance and Rician parameter K dictate the coding gain of the noisy ICP with MIMO schemes for any orthogonal STBCs, with transmission of symbols across Rician fading channel. Therefore, we must select mappings that maximizes the minimum inter-set distance if we want a receiver to achieve its highest coding gain, at high SNR. Further, we use rotated PSK constellations to enhance the ML decoding performance of high priority receivers. Let $s_0, s_1, ..., s_{2^N-1}$ be the 2^N signal points. Only the alternate signal points $s_0, s_2, ..., s_{2^{N-1}-1}$ are rotated by an angle θ, where θ varies from $0 < \theta < 2\pi/M$ ($M = 2^N$), keeping the other half of the points stationary, in order to increase minimum inter-set distance for highest priority receivers. Moreover, among the mappings with same minimum inter-set distance we select the one with less multiplicity of signal pairs.

Special Case: For Rayleigh fading, parameter K is zero as channel only has multipath components. Then the inequality (19) can be expressed as,

$$\mathbf{P}\{s_k \to s_{k'}\} \le \frac{1}{2}\left[\frac{4}{SNR|(s_k - s_{k'})|^2}\right]^{N_t N_r}. \tag{20}$$

Example 1 - Let $n = m = 7$, $W_i = x_i, \forall i \in [7]$ and $K_i, \forall i \in [7]$ are the side information available at the receivers $R_i, \forall i \in [7]$ where $K_1 = \{x_2, x_3, x_4, x_5, x_6\}, K_2 = \{x_1, x_3, x_4, x_7\}, K_3 = \{x_1, x_2, x_4\}, K_4 = \{x_1, x_2, x_3\}, K_5 = \{x_4, x_6\}, K_6 = \{x_5\}$ and $K_7 = \{\phi\}$. The decreasing order of priority among the receivers is $\{R_1, R_2, R_3, R_4, R_5, R_6, R_7\}$. The index code considered for the problem is of length $N = 3$ and given by:

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

where $\mathbf{L}$ is the encoding matrix. The index coded bits are given by $\mathbf{y} = \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}$ where $y_1 = x_1 + x_2 + x_3 + x_4$, $y_2 = x_5 + x_6$, $y_3 = x_7$.

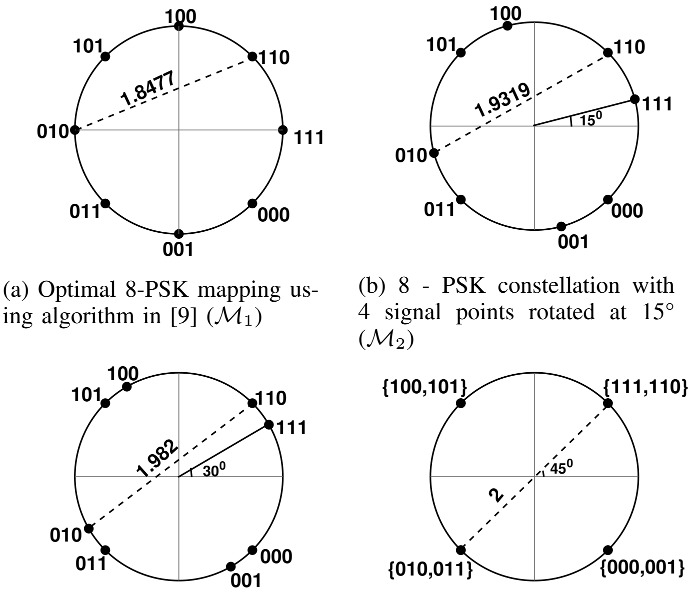
After maximizing the minimum inter-set distance for this ICP, 64 optimal 8-PSK mappings were found. After rotating four alternate constellation points, the number of optimal mappings decreased to 32. In addition, when contemplating mappings with a smaller multiplicity of pairs, the number of optimal mappings is reduced to 16. One such mapping is shown in Fig. 1(a).

Table I lists the effective broadcast vector set ($C_{\mathbf{L}}(a_i)$), 0-effective broadcast vector set ($C_{\mathbf{L}0}$) and 1-effective broadcast vector set ($C_{\mathbf{L}1}$) as seen by receiver R_1. We consider the highest priority receiver R_1 with side information as (00101). The effective broadcast vector set of R_1 is $C_{\mathbf{L}} = \{(110), (111), (010), (011)\}$. The 0-effective broadcast vector set is $C_{\mathbf{L}0} = \{(110), (111)\}$ and the 1-effective broadcast vector set is $C_{\mathbf{L}1} = \{(010), (011)\}$.

For the optimal 8-PSK mapping depicted in Fig. 1(a), the minimum inter-set distance for highest priority receiver R_1 is 1.8477, and by rotating four alternate constellation points with 30°, the minimum inter-set distance increased to 1.982. The minimum inter-set distance for receiver R_1 keeps on increasing

TABLE I: Effective broadcast vector sets and its partitions seen by R_1 for ICP in Example 1.

a_1	$C_{\mathbf{L}}$	$C_{\mathbf{L}0}$	$C_{\mathbf{L}1}$
(00000)	(000),(001),(100),(101)	(000),(001)	(100),(101)
(00001)	(010),(011),(110),(111)	(010),(011)	(110),(111)
(00010)	(010),(011),(110),(111)	(010),(011)	(110),(111)
(00011)	(000),(001),(100),(101)	(000),(001)	(100),(101)
(00100)	(100),(101),(000),(001)	(100),(101)	(000),(001)
(00101)	(110),(111),(010),(011)	(110),(111)	(010),(011)
(00110)	(110),(111),(010),(011)	(110),(111)	(010),(011)
(00111)	(100),(101),(000),(001)	(100),(101)	(000),(001)



(a) Optimal 8-PSK mapping using algorithm in [9] ($\mathcal{M}_1$)

(b) 8 - PSK constellation with 4 signal points rotated at 15° ($\mathcal{M}_2$)

(c) 8 - PSK constellation with 4 signal points rotated at 30° ($\mathcal{M}_3$)

(d) 8 - PSK constellation with 4 signal points rotated at 45° ($\mathcal{M}_4$)

Fig. 1: 8-PSK mappings with different angles of rotation for ICP in Example 1.

as the angle of rotation is increased from 0° to 45° as shown in Fig. 1(b), Fig. 1(c) and Fig. 1(d). At 45° rotation, the minimum inter-set distance for receiver R_1 is 2 which is the maximum possible inter-set distance for PSK constellation, whereas for receiver R_7 the minimum inter-set distance is 0.

IV. SIMULATION RESULTS

Consider the noisy ICP in Example 1. We investigate the performance of the receivers with 2×2 MIMO Rician fading channel using Alamouti code (1) and compared it with the SISO scheme for mapping $\mathcal{M}_1$ displayed in Fig. 1(a). It is significant to note that the observed diversity gain for the

2×2 MIMO scheme with Alamouti code is 4, compared SISO scheme's diversity gain of 1. Fig. 2 shows the simulation results.

Now, using a 4×1 MIMO Rician fading channel and 4×4 orthogonal STBC in (3), we investigate the performance of receivers for ICP in Example 1 at various degree of rotation. We compare the optimal mapping $\mathcal{M}_1$ with mappings that were obtained by rotating half of the signal points at 15° ($\mathcal{M}_2$), 30° ($\mathcal{M}_3$). According to Fig. 3(a), the probability of error performance for the highest priority receiver R_1 improves. The probability of error performance for receiver R_2 in this case stays the same because the minimum inter-set distance seen by receiver R_2 is same for mappings $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$. As the angle of rotation increases, the minimum inter-set distance seen by receivers R_3, R_4, R_5, and R_6 increases, improving their performance, however for receiver R_7, performance declines as the minimum inter-set distance seen by this receiver decreases. Fig. 3(b) displays the performance of receivers at an angle of rotation of 45° ($\mathcal{M}_4$). Highest priority receiver R_1 performs best with an angle of rotation of 45°, whereas for receiver R_7, the minimum inter-set distance is 0 because the signal points in both the $S_{\mathbf{L}0}(a_i)$ and $S_{\mathbf{L}1}(a_i)$ are mapped to the same constellation point. So, it is not possible to decode for receiver R_7. As a result, we only take into account scenarios in which $0 \leq \theta < 45$.

When we emphasis on the performance of all the receivers for cases $0 \leq \theta < 45$, it is evident that the 4×1 MIMO scheme with 4×4 orthogonal STBC achieves a diversity gain of 4.

From Table II we see that for the highest priority receiver R_1, when the probability of error is 10^{-3}, energy per bit to noise power spectral density ratio $(Eb/No, dB)$ required for receiver R_1 decreases with increase in angle of rotation.

Simulation results comparing the performance of receivers for ICP in Example 1 using 4×1 MIMO scheme and 4×4 orthogonal STBC in (3) over Rayleigh and Rician fading channel with mapping $\mathcal{M}_1$ has been shown in Fig. 4. As can be seen, the presence of the LOS component makes all receivers perform better over the Rician fading channel than Rayleigh

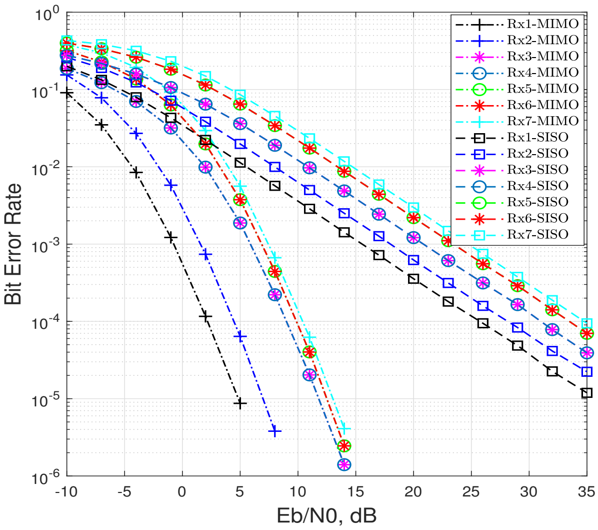
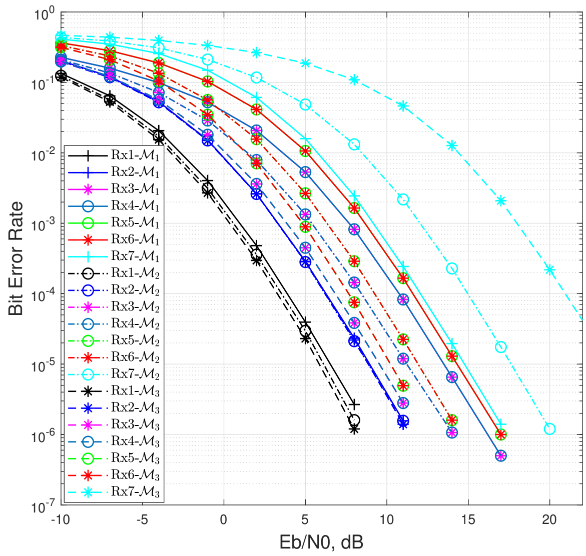
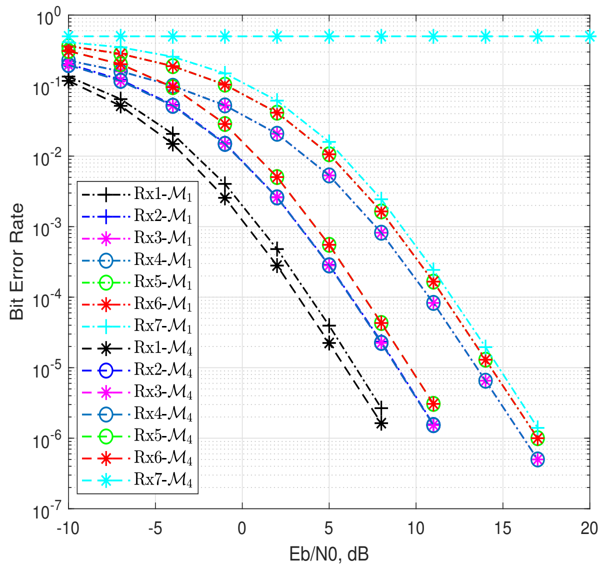


Fig. 2: Simulation results comparing the performance of receivers between 2×2 MIMO scheme and SISO scheme for ICP in Example 1.



(a)



(b)

Fig. 3: Simulation results comparing the performance of receivers over 4×1 MIMO Rician fading channel for ICP in Example 1 with different angles of rotation (a) Mapping $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$. (b) Mapping $\mathcal{M}_1$ and $\mathcal{M}_4$.

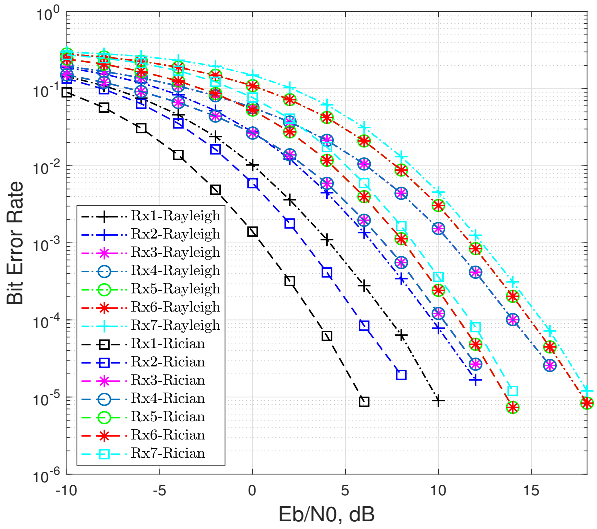


Fig. 4: Simulation results comparing the performance of receivers for 4×1 MIMO scheme over Rayleigh and Rician fading channel for ICP in Example 1.

fading channel.

The simulation results supports our assertion that for noisy ICP over $N_t \times N_r$ MIMO Rician fading channel incorporating orthogonal STBCs provides diversity gain of $N_t N_r$ and metric minimum inter-set distance dictates the coding gain.

TABLE II: $(Eb/No, dB)$ for all receivers in Example 1 for different angles of rotation.

Angle of Rotation	R_1	R_2	R_3	R_4	R_5	R_6	R_7
0°	0.97	3.30	7.68	7.68	8.65	8.65	9.16
15°	0.58	3.30	5.39	5.39	6.32	6.32	12.05
30°	0.35	3.30	3.85	3.85	4.83	4.83	17.98

V. Conclusion

Noisy ICP over $N_t \times N_r$ MIMO Rician fading channel is considered. We suggest a MIMO scheme for any orthogonal STBCs in order to acquire diversity gain of $N_t N_r$ in addition to coding gain. We have demonstrated that mapping the broadcast vectors to the constellation points is crucial, and that the metric minimum inter-set distance dictates the receivers ML decoding performance. Moreover, we can increase the minimum inter-set distance between the 0-effective broadcast signal sets and 1-effective broadcast signal sets by employing rotated 2^N-PSK constellation for high priority receivers, although performance for some low priority receivers might deteriorate.

VI. Acknowledgment

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