



A tilt instability in the cosmological principle

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Abstract We show that the Friedmann–Lemaître–Robertson–Walker (FLRW) framework has an instability towards the growth of fluid flow anisotropies, even if the Universe is accelerating. This flow (tilt) instability in the matter sector is invisible to Cosmic No-Hair Theorem-like arguments, which typically only flag shear anisotropies in the metric. We illustrate our claims in the setting of “dipole cosmology”, the maximally Copernican generalization of FLRW that can accommodate a flow. Simple models are sufficient to show that the cosmic flow need not track the shear, even in the presence of a positive cosmological constant. We also emphasize that the growth of the tilt hair is fairly generic if the effective equation of state $w \rightarrow -1$ at late times (as it does in standard cosmology), irrespective of the precise model of dark energy. The generality of our theoretical result puts various recent observational claims about late time anisotropies and cosmic dipoles in a new light.

1 Introduction and motivation

Copernicus put forward the viewpoint that we are not privileged observers in the Universe [1]. Copernican viewpoint has influenced physics and in particular cosmology ever since [2–4]. After Hubble’s discovery in 1929 [5], the Copernican principle has been dubbed the “cosmological principle”. It states that Universe is homogeneous and isotropic on constant cosmic time slices and is formulated within the FLRW cosmology. The current concordance flat Λ Cold Dark Matter (Λ CDM) model, sometimes also called the standard model of cosmology, is a specific model within the FLRW framework.

Flat Λ CDM model culminates the cosmology today and it rests upon cosmological data at various different redshifts. However, steadily improving precision in observations has created tensions within the model [6–8]. Many models, almost all within the FLRW framework, have been proposed to resolve the tensions, but none of them seem fully satisfactory [9, 10]. This has promoted the idea that the current cosmological tensions may be symptoms of a much deeper issue, a violation of the cosmological principle [11–13]. This viewpoint resonates with the growing concern that the cosmological principle may not pass all observational tests, see [14] for a recent review.

The simplest (ie., the most Copernican) setting that can accommodate a cosmic flow is the “dipole cosmology” paradigm of [15] that generalizes the FLRW framework. This is a tilted axially symmetric Bianchi V/VII_h cosmology that falls into the broader rubric of tilted homogeneous models [4, 16, 17]. The “tilt” refers to a flow in the fluid that is *not* orthogonal to the homogeneous time slices. See [18–28] for earlier analysis within tilted cosmology setting. As such, dipole cosmology is a minimal setting within the tilted models for formulating a cosmic dipole. It has 2 functions of the time coordinate in the background metric – an overall Hubble expansion rate and the metric anisotropy parametrized by shear. It also has 3 functions in the cosmic fluid sector – energy density, pressure and the tilt. Part of our goal in this letter is to emphasize that this is an extremely simple and tractable framework that can accommodate a dipole flow, that generalizes the Friedmann equations while still being ODEs. The simplicity of this set up as a model-building paradigm, especially with non-trivial equations of state including mixtures [29]. See [30, 31] for two fluid models within a generic titled cosmology setup.

A second motivation, and the main point of this Letter, is that this class of models allow us to investigate certain stability properties of the (conventional) FLRW framework that is often not emphasized. Standard lore dictates that cos-

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mic acceleration washes away the anisotropies. Statements of this kind are usually called Cosmic No-Hair Theorems [32], and they usually refer to very fast (exponential) falloff of the shear anisotropy in the metric. Dipole cosmology enables us to see that even while the shear anisotropies die down, the tilt anisotropies of an accelerating Universe need not. We will illustrate this here in the simple setting of fluids with constant equations of state together with a positive cosmological constant Λ . This is an extremely simple scenario and is implicit in the older work of [25]. Our goal is to point out that this is the simplest instantiation of the more general scenario noted in [15] where large classes of models with the effective equation of state of the Universe goes to -1 at late times, were shown to have growing tilt anisotropies at late times.

Let us list a few of our main results. (1) Even when the shear dies off at late times, the tilt or bulk flow can be relatively large or can even grow in very large classes of models. (2) Even with a very small initial value for tilt and almost FLRW background, we can get a sizable bulk flow, signalling a tilt instability in the FLRW cosmology. (3) These features can arise even in an accelerating Universe (say, one with a positive cosmological constant). (4) Together with the cases considered in [15,29], the results of this paper show that these observations, while not absolutely generic, are quite easily realized (and generic indeed, in some regions of parameter/initial-condition space). The punchline is that at late times, even if we are dealing with metrics which are homogeneous and almost isotropic, we still could have substantial bulk flows and cosmic dipoles. It can be of significance for late time observational cosmology and cosmic tensions, even though to make a precise statement, we will need more detailed model building within the dipole cosmology paradigm [29].

Let us emphasize a simple but potentially confusing point. The notion of tilt (i.e., a non-zero flow of the cosmic fluid on constant time slices), is not an observer-dependent concept. A standard observer living on those flows, would see a metric with non-zero time-space off-diagonal terms. Likewise, if the cosmic fluid has different components, each component can have its own tilt, see [29]. The relative tilt of these fluids is a physical observable. A detailed discussion of dipole cosmography will be presented elsewhere [33].

Our analysis is done using the dipole cosmology equations (presented below) that generalize the FLRW Friedmann equations. They can be viewed as keeping track of the non-linear evolution of tilt perturbations around the FLRW system. Interpreted this way, our result is a demonstration that FLRW cosmology is not stable against homogeneous, anisotropic tilt perturbations even when the Universe is accelerating. In this work we focus on the main results; more detailed analyses and discussions may be found in [15,29,33–35].

2 Dipole cosmology, the basic setup

We start with a review of the dipole cosmology set up [15,16]. The most general cosmological metric in 4 dimensions has 6 functions of space and time. Assuming a cosmic (comoving) time coordinate t , and spatial homogeneity on constant time slices, we remain with 3 functions of t . In our dipole cosmology besides homogeneity we also assume axisymmetry which removes one more function and hence the metric involves only 2 functions of t . The metric in the dipole cosmology setting takes the form

$$ds^2 = -dt^2 + a^2(t) \left[e^{4b(t)} dz^2 + e^{-2b(t)-2A_0z} \times (dx^2 + dy^2) \right], \quad (1)$$

where $a(t)$ is the over all scale factor, $b(t)$ parameterizes the anisotropy and $A_0 \neq 0$ is a constant of dimension of inverse length. While it may be set to 1 by a choice of units, we keep it for later convenience. See [4,15–18,20,25,26,29,34] for earlier discussions and analysis in the titled/dipole cosmology setting. We may define the Hubble expansion rate $H(t)$ and the cosmic shear $\sigma(t)$ as usual

$$H := \frac{\dot{a}}{a}, \quad \sigma := 3\dot{b} \quad (2)$$

where *dot* denotes derivative w.r.t. t . When $\sigma = 0$ the metric reduces to an open FLRW universe.

In this metric the z direction is chosen to be along the cosmic flow (the tilt). To see this, we note that the in (t, z, x, y) frame the energy momentum tensor of a perfect fluid with energy density ρ and pressure p is

$$\begin{aligned} T^a_b &= T_{\text{iso}}^a_b + T_{\text{tilt}}^a_b \\ T_{\text{iso}}^a_b &= \text{diag}(-\rho, p, p, p) \\ T_{\text{tilt}}^a_b &= (\rho + p) \sinh \beta \\ &\times \begin{pmatrix} -\sinh \beta & ae^{2b} \cosh \beta & 0 & 0 \\ -\cosh \beta / (ae^{2b}) & \sinh \beta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (3)$$

where $T_{\text{iso}}^a_b$ is the energy momentum tensor of a usual isotropic perfect fluid. The off-diagonal terms in T^a_b are a manifestation of the non-zero bulk flow along z direction; β is the rapidity of the fluid as measured by the observer in the comoving frame of the metric and parameterizes the tilt, the bulk flow. The important point in (3) is that when $\rho + p = 0$, i.e. for a cosmological constant, β drops out and we recover the usual FLRW setup. In other words, a cosmological constant can be viewed as having any value of the tilt.

3 Evolution equations of dipole cosmology

In our dipole cosmology we have 5 functions of t , $\rho(t)$, $p(t)$, $a(t)$, $b(t)$, $\beta(t)$ which are to be specified by Einstein’s field equations

$$R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}, \tag{4}$$

or the continuity equations $\nabla^a T_{ab} = 0$, yielding

$$H^2 - \frac{1}{9}\sigma^2 - \frac{A_0^2}{a^2}e^{-4b} = \frac{\rho}{3} + \frac{1}{3}(\rho + p) \sinh^2 \beta \tag{5a}$$

$$\dot{\sigma} + \sigma \left(3H - \frac{2A_0}{a} \tanh \beta e^{-2b} \right) = 0 \tag{5b}$$

$$\dot{\rho} + 3H(\rho + p) = -(\rho + p) \tanh \beta \left(\dot{\beta} - \frac{2A_0}{a} e^{-2b} \right) \tag{5c}$$

$$\dot{p} + H(\rho + p) = -(\rho + p) \left(\frac{2}{3}\sigma + \dot{\beta} \coth \beta \right). \tag{5d}$$

Moreover, we note that the above equations imply

$$\sigma = \frac{1}{4A_0} a e^{2b} (\rho + p) \sinh 2\beta \tag{6}$$

Note also that for $p = -\rho$, β drops out of equations and the solution to the above equations for $\rho > 0$ is a de Sitter geometry in an open Universe slicing.

Among the above 4 equations, (5a) and (5c) are generalization of the 2 Friedmann equations of the usual FLRW cosmology, while (5b) (or (6)) and (5d) are new and govern dynamics of the shear and the tilt. To solve the above equations one needs to supplement them with another equation, which may be taken to be the equation of state (EoS).

4 Examples of dipole cosmology models

In usual cosmological model building, one typically works with non-interacting multi-component fluids each with a constant equation of state w_i and energy density and pressure ρ_i , $p_i = w_i \rho_i$. For discussing general dynamical behavior of the system, it is handy to consider an effective cosmic fluid with $\rho = \sum_i \rho_i$ and $p = \sum_i w_i \rho_i$ and an effective EoS $w_{eff} = p/\rho$. This effective EoS is in general time dependent. In [15] we considered scenarios where the $w_{eff}(t)$ mentioned above had the feature that $w_{eff}(t) \rightarrow -1$ at late times, so that the late Universe accelerates. Many examples of this type were considered and a general criterion was provided for when such models will lead to increasing tilt at late times. In particular, if the approach of $w_{eff}(t)$ to -1 at late times is exponential and sufficiently fast, it was noted that the tilt would grow. It should be emphasized that this is *not* a particularly stringent demand: indeed, it was pointed out that

the effective (time-dependent) equation of state of any cosmology with a cosmological constant component, like the standard flat Λ CDM cosmology satisfies the demand. The phenomenon of tilt growth is fairly generic in the space of physically interesting models.

In this paper, we will discuss a simple class of accelerating models that also exhibit late time tilt growth. These are models with a fluid with constant equation of state w and a cosmological constant. This is a setting that has been studied previously, e.g. see [25], and our goal is to show that these old results are a specific realization of our broader claims in [15]. Our secondary goal is to present the relevant equations in a form that is a transparent generalization of the Freedman equations. We will see that in these dipole w - Λ models, tilt can increase at late times if w is stiff enough. This is consistent with our claim that tilt growth can be a fairly generic phenomenon in accelerating Universes.

4.1 Dipole w - Λ models

As an illustrative example we study models involving a fluid of constant EoS w and a cosmological constant Λ . It follows from our earlier discussion that a Λ can be incorporated into (5) by defining

$$p = w\tilde{\rho} - \Lambda, \quad \rho = \tilde{\rho} + \Lambda, \quad -1 < w \leq 1. \tag{7}$$

For a generic w , (5) implies

$$\begin{aligned} \frac{\ddot{a}}{a} &= \dot{H} + H^2 \\ &= \frac{\Lambda}{3} - \frac{\tilde{\rho}}{6}(1 + 3w) - \frac{2}{9}\sigma^2 - \frac{\tilde{\rho}}{3}(1 + w) \sinh^2 \beta \end{aligned} \tag{8a}$$

$$\begin{aligned} &\dot{\beta} (\coth \beta - w \tanh \beta) \\ &= (3w - 1)H - \frac{2}{3}\sigma - \frac{2wA_0}{a(t)} e^{-2b} \tanh \beta \end{aligned} \tag{8b}$$

$$\tilde{\rho}^{\frac{w}{1+w}} a e^{2b} \sinh \beta = C = const. \tag{8c}$$

$$\sigma = \frac{C(1 + w)}{2A_0} \tilde{\rho}^{\frac{1}{1+w}} \cosh \beta \tag{8d}$$

In our analysis we consider non-negative cosmological constant $\Lambda \geq 0$ and take $\beta > 0$. From these equations we learn:

1. As in the FLRW case, for $w \leq -1/3$ we get an accelerated expansion for any $\Lambda \geq 0$.
2. As Universe expands $a(t)$ grows and $\rho(t)$ drops.
3. The shear $\sigma(t)$ goes to zero (exponentially fast for accelerated expansion) and the universe isotropizes rapidly.
4. Since $-1 < w \leq 1$, and for $\beta > 0$, $\coth \beta > 1$ and $\tanh \beta < 1$, the coefficient of $\dot{\beta}$ term is always positive. While the last term in (8b) does not have a definite sign, it becomes insignificant at late times due to the expansion.

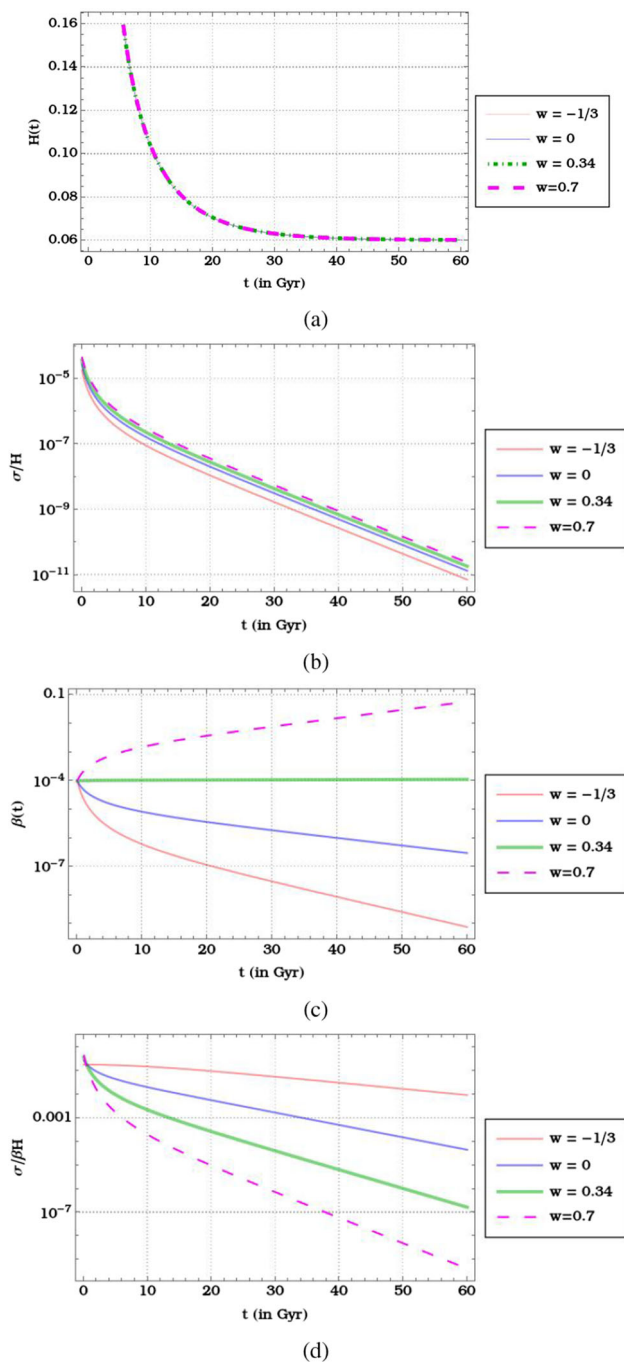


Fig. 1 Evolution of overall Hubble parameter $H(t)$, σ/H , tilt β and the dimensionless ratio $\sigma/(\beta H)$ for dipole w - Λ model. Initial values are $a_{in} = 1$, $b_{in} = 0$, $\rho_{in} = 0.6$, $\beta_{in} = 10^{-4}$ and $\Lambda = 0.0109$. In these plots we have adopted the units in which $A_0 = 1$. We have plotted 4 representative values of w ; β grows for $w > 1/3$ at late times. While all the 4 cases have essentially the same $H(t)$ they differ in evolution of β

Therefore, for $w > 1/3$ the sign of $\dot{\beta}$ is positive and the tilt can grow. This is in accord with the results of [25].

5. Cosmological constant Λ does not explicitly appear in (8b) and therefore it is plausible that β growth only depends on w ($> 1/3$) and not the value of Λ ; β growth can happen in accelerating/decelerating cosmologies. This is indeed what we have verified numerically. In fact, we find that β growth is faster in cases with Λ , due to faster growth of H .

The above is also confirmed by numerical evolution of the equations for $\Lambda = 0$ and $\Lambda > 0$ cases. We plot the latter in Fig. 1.

5 Discussion and outlook

We have formulated and analysed dipole cosmology, which is constructed to accommodate a cosmic bulk flow in a minimal generalization of the FLRW framework. Our theoretical analysis is based upon Einstein field equations while allowing for a tilt in the energy momentum tensor of the cosmic fluid. It reveals a few important facts. The tilt can remain large and even grow in time, while geometry isotropises (cosmic shear dying off fast), illustrating that FLRW cosmology is unstable against tilt perturbations. This claim is true even in an accelerating Universe, in particular one containing a positive cosmological constant. It is in fact quite generic in Universes where $w_{eff}(t) \rightarrow -1$ at late times. This statement is our most significant claim.

We also uncovered another interesting feature in some classes of dipole cosmology models [15]: while β goes to zero at very late times, there could be intermediate stages where the tilt can grow. While these results are not in the context of a realistic model, they exhibit the theoretical possibility of instability in FLRW setting due to dipole deformations/perturbations. Given the various claims about late time flows and dipole anisotropies in the recent literature, we feel that this observation is worthy of broader notice.

Our findings may seem at odds with the common lore, which is based on intuitions gained from Wald's celebrated cosmic no-hair theorem [32] and the fact that inflation leaves us in a Universe in which shear is suppressed. The cosmic evolution afterward does not provide sources for the shear and it remains small in the course of cosmic history. Our analysis emphasizes that tilt and shear are two separate concepts, and our dynamical equations (5) reveal that $\sigma \ll 1$ does not imply $\beta \ll 1$ and in fact β can be or become of order 1. These results are compatible with the usual lore, but they highlight the important distinction between having an (almost) isotropic and homogeneous metric and having a homogeneous and isotropic cosmology, which is usually taken to be synonymous. In particular, an observer living in

a flowing fluid of galaxies will see a homogeneous Universe with a dipole anisotropy even if the shear anisotropies in the metric are small [33]. Viewed in this light, the CMB observations which are usually taken as observational confirmation of the usual FLRW cosmologies (modulo CMB anomalies [14, 36]), primarily indicate isotropy of the background metric over which the light propagates and does not exclude bulk flows and cosmic dipoles [33].

In the w - Λ example we considered here, tilt-growth and the associated instability happens for $w > 1/3$. On the other hand, thermal corrections to the Stefan–Boltzmann law of non-relativistic particles yields $w < 1/3$. There are two comments to be made here. (1) In our analysis above we confined ourselves to the $\beta > 0$ cases. One may analyze the $\beta < 0$ cases. All the main results itemized above are still true for $\beta < 0$ cases, with the important addition that tilt growth happens for $w \geq 1/3$. This includes the physically important and interesting case of radiation. See [29] for a detailed analysis. A crucial result is that the relative flow between matter and radiation can increase at late times. This instability may be interpreted as a non-kinematical dipole component in the CMB [14, 36]. (2) Cosmic fluids may have a field theory description. For example, as is customary in inflationary model building, consider a scalar field with kinetic energy T and potential U , then $w_{eff} = (T - U)/(T + U)$ [37]. For small enough potentials we typically deal with w_{eff} close to 1. So, the instabilities we discussed here can be relevant for general cosmological model building.

Our analysis prompts several theoretical and observational questions, the most important one being the instability of homogeneous and isotropic cosmologies to *homogeneous* tilt perturbations. Even if one starts with an isotropic Universe after inflation, a small tilt perturbation in the course of cosmic evolution can yield a sizable bulk flow or cosmic dipole. Therefore theoretically, FLRW setup needs to be revisited as the cosmological framework, and it is natural to expect non-kinematical dipoles/tilts in various different cosmological observations.

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