Meson screening mass at finite chemical potential

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August 8, 2022

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Screening correlators

- Hadronic correlators are useful for calculating the hadronic energy eigenvalues using the spectral decomposition of the excitations.
- At high temperatures, the temporal span is constrained as $N_\tau = 1/(aT)$ making the long distance behaviour for temporal correlator difficult to study.
- Instead, we calculate correlator propagating in spatial direction called screening correlator and extract screening mass from it.
- The inverse of a screening mass is the screening length, i.e., the spatial distance beyond which the effects of a test hadron are effectively screened.
- Staggered screening correlator ansatz

$$C(n_z) = \sum_i A_i^{no} \cosh\left[M_i^{no}(N_\sigma/2 - n_z)\right] - (-1)^{n_z} A_i^{osc} \cosh\left[M_i^{osc}(N_\sigma/2 - n_z)\right]$$

Screening correlators: Finite density

- We have a pretty good estimate of these screening masses at zero density (HotQCD collaboration, Phys.Rev.D 100 (2019) 9, 094510), but not at finite density because of the complex determinant problem making its simulation intractable.
- We try to overcome this issue by Taylor expanding the required observable in terms of μ/T around $\mu = 0$ and obtain the coefficients of the expansion.
- We use this to get method to obtain screening mass of pseudoscalar channel at finite μ.
- Isoscalar chemical potential : $\mu_S = \mu_u = \mu_d$.

Free theory

- Free theory correlator equation (Journal of High Energy Physics 2007(03): 022.)

$$C(z,\mu_S) = \frac{3T^2}{2z}e^{-2\pi Tz}\left(\left(1+\frac{1}{2\pi Tz}\right)\cos(2\mu_S z) + \frac{\mu_S}{\pi T}\sin(2\mu_S z)\right) + \mathcal{O}(e^{-4\pi Tz})$$

- For $\mu_S = 0$, we get

$$C(z) = \frac{3T^2}{2z}e^{-2\pi Tz}\left(1+\frac{1}{2\pi Tz}\right)$$

$$C''(z) = \tfrac{\partial^2 C}{\partial \mu_S^2} = -6T^2 e^{-2\pi zT}\left(z\left(1+\frac{1}{2\pi zT}\right) - \frac{1}{\pi T}\right)$$

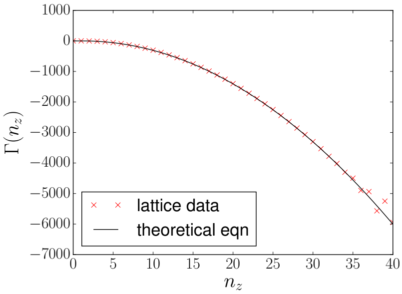
$$C''''(z) = \tfrac{\partial^4 C}{\partial \mu_S^4} = 12z^2T^2 e^{-2\pi zT}\left(2z\left(1+\frac{1}{2\pi zT}\right) - \frac{4}{\pi T}\right)$$

- Also, we obtain

$$\Gamma_{Free}(z) \equiv \tfrac{C''(z)}{C(z)} = -4z\left(z - \frac{1}{\pi T\left(1+\frac{1}{2\pi zT}\right)}\right)$$

$$\Sigma_{Free}(z) \equiv \tfrac{C''''(z)}{C(z)} = 16z^3\left(z - \frac{2}{\pi T\left(1+\frac{1}{2\pi zT}\right)}\right)$$

Free theory



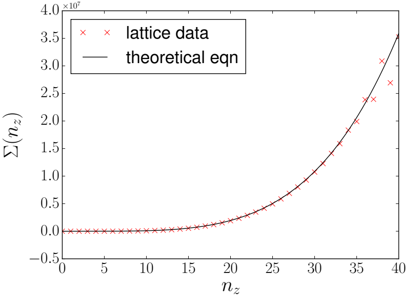


Figure: Free theory plots for (left) $\Gamma(n_z)$ and (right) $\Sigma(n_z)$ for $80^3 \times 8$ lattice along with theoretical curve.

Finite temperature

- Rewriting the free theory correlator

$$C(z,\mu_S) = \frac{3T^2}{2z}\text{Re}\left[e^{-2\pi Tz + i\,2\mu_S z}\left(\left(1+\frac{1}{2\pi Tz}\right) - i\frac{\mu_S}{\pi T}\right)\right]$$

it is easy to see $M_{scr}^{free} = 2\pi T + i\,2\mu_S$ and $A^{free} = \frac{3T^2}{2z}\left(\left(1+\frac{1}{2\pi Tz}\right) - i\frac{\mu_S}{\pi T}\right)$.

- From this, we propose following ansatz for finite temperature correlator

$$\begin{aligned} C(z;\mu_S) &= \text{Re}\left[\left(A_R(\mu_S) - iA_I(\mu_S)\right)e^{-z(M_R(\mu_S)+iM_I(\mu_S))}\right] \\ &= e^{-zM_R(\mu_S)}\left\{A_R(\mu_S)\cos\left(zM_I(\mu_S)\right) + A_I(\mu_S)\sin\left(zM_I(\mu_S)\right)\right\} \end{aligned}$$

with $M_{scr}(\mu_S) = M_R(\mu_S) + iM_I(\mu_S)$ and the amplitude is $A(\mu_S) = A_R(\mu_S) - iA_I(\mu_S)$.

Finite temperature

- For the Staggered Dirac operator $\mathcal{M}(\mu)$ at finite chemical potential μ, the γ_5-hermiticity property of the Dirac operator gets modified to

$$\gamma_5 \, \mathcal{M}^\dagger(\mu)_{(n,m)} = \mathcal{M}(-\mu)_{(m,n)} \, \gamma_5$$

- Because of the modified γ_5-hermiticity, the correlator $C(z, \mu_i, \mu_j) = \left\langle \mathrm{Tr} \left[P(\mu_i)_{n,0} \Gamma P(\mu_j)_{0,n} \Gamma^\dagger \right] \right\rangle$ has following conjugation property

$$C(z; -\mu_S) = C^*(z; \mu_S)$$

- This conjugation property implies the real and imaginary parts of the screening mass and amplitude to be even and odd functions of μ_S respectively.

- Taylor-expanding the correlators in powers of μ_S to $\mathcal{O}(\mu_S^4)$ and collecting terms with μ_S^2 and μ_S^4, we get

$$\Gamma(z) = \left.\frac{C''}{C}\right|_{\mu_S=0} = \frac{A_R''}{A_R} + z\left[2\frac{A_I'}{A_R}M_I' - M_R''\right] - z^2(M_I')^2$$
$$= \alpha_2 z^2 + \alpha_1 z + \alpha_0$$

$$\Sigma(z) = \left.\frac{C''''}{C}\right|_{\mu_S=0} = \frac{A_R''''}{A_R} + z(4\frac{A_I'}{A_R}M_I''' + 4\frac{A_I'''}{A_R}M_I' - M_R'''' - 6M_R''\frac{A_R''}{A_R})$$
$$+z^2(3M_R''^2 - 12\frac{A_I'}{A_R}M_I'M_R'' - 4M_I'M_I''' - 6M_I'^2\frac{A_R''}{A_R})$$
$$+z^3(6M_R''M_I'^2 - 4\frac{A_I'}{A_R}M_I'^3) + z^4(M_I'^4)$$
$$= \beta_4 z^4 + \beta_3 z^3 + \beta_2 z^2 + \beta_1 z + \beta_0$$

- Expression for Γ for both free theory and finite temperature is quadratic in z with highest power coefficient $\alpha_2 = M_I'^2$.

- Expression for Σ for both free theory and finite temperature is quartic in z with highest power coefficient $\beta_4 = M_I'^4$.

- We obtain M_R'' as

$$M_R'' = \frac{1}{4}\left(2\alpha_1 - \frac{\beta_3}{\alpha_2}\right)$$

N_σ	β	T[GeV]	m_l	m_s	configurations
64	9.670	2.90	0.0001399	0.002798	6000
64	9.360	2.24	0.00018455	0.003691	6000

Table: The list of configurations used for the finite temperature screening mass analysis. All the configurations used here have $N_\tau = 8$ using 2+1 HISQ action, with LCP as defined in (Phys. Rev. D90 (2014), p. 094503).

Finite temperature: Temperature effect

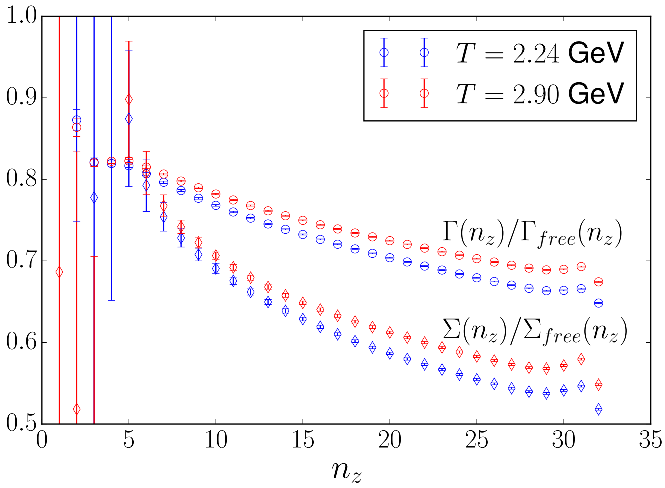


Figure: Plot show the temperature dependence behaviour of $\Gamma(n_z)$ and $\Sigma(n_z)$ for two temperatures $T = 2.24$ GeV and $T = 2.90$ GeV.

Finite temperature: Excited states effect

Considering the contribution of the first excited states in our expression, we get

$$C(z) = A_0 \exp^{-M_0 z} + A_1 \exp^{-M_1 z} = A_0 \exp^{-M_0 z}\left[1 + \frac{A_1}{A_0}\exp^{-(\Delta M)z}\right]$$

$$\Gamma(z) \simeq \frac{\left(\alpha_2 z^2 + \alpha_1 z + \alpha_0\right)}{1 + \frac{A_1}{A_0}\exp^{-(\Delta M)z}}$$

$$\Sigma(z) \simeq \frac{\left(\beta_4 z^4 + \beta_3 z^3 + \beta_2 z^2 + \beta_1 z + \beta_0\right)}{1 + \frac{A_1}{A_0}\exp^{-(\Delta M)z}}$$

where

$$\alpha_i = \alpha_{0i} + \frac{A_1}{A_0}\alpha_{1i}\exp^{-\Delta M z} \simeq \alpha_{0i}$$

$$\beta_i = \beta_{0i} + \frac{A_1}{A_0}\beta_{1i}\exp^{-\Delta M z} \simeq \beta_{0i}$$

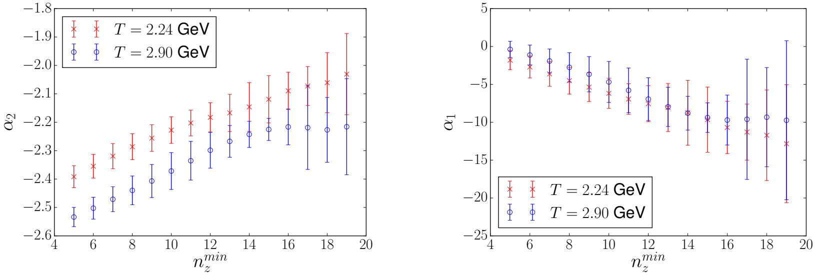
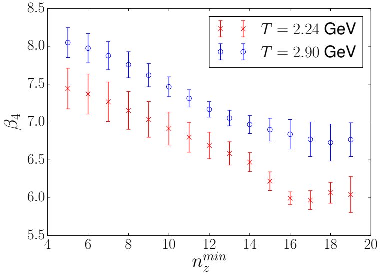


Figure: Fit coefficients obtained for $\Gamma(z)$ to the fitting ansatz $\Gamma(z) = \frac{(\alpha_2 z^2 + \alpha_1 z + \alpha_0)}{1 + \frac{A_1}{A_0} \exp^{-(\Delta M)z}}$ keeping $n_z^{max} = 25$.

Finite temperature: Fit coefficients



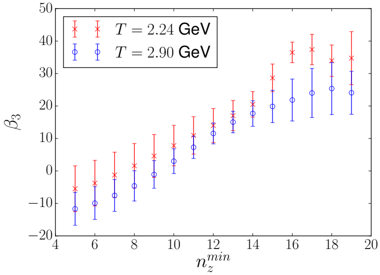


Figure: Fit coefficients obtained for $\Sigma(z)$ to the fitting ansatz $\Sigma(z) = \frac{(\beta_4 z^4 + \beta_3 z^3 + \beta_2 z^2)}{1 + \frac{A_1}{A_0} \exp^{-(\Delta M)z}}$ keeping $n_z^{max} = 25$.

Finite temperature: Screening mass

Temp T	α_2	α_1	β_4	β_3	TM_R''	M_I'
2.24 GeV	-2.06(11)	-11.7(5.6)	6.04(24)	34.7(8.2)	-0.22(39)	1.43(3)
2.90 GeV	-2.23(11)	-9.3(6.5)	6.73(24)	25.3(8.0)	-0.24(43)	1.49(4)
Free theory	-4	10.2	16	-81.5	0	2

Table: Values for the polynomial fit parameters with the TM_R'' and M_I' for two temperatures and free theory. The analysis is done on lattices with $N_\tau = 8$ and $N_\sigma = 64$.

Conclusions

- We verified the free theory expression for screening correlator derived analytically at finite isoscalar chemical potential by looking at its derivatives on lattice.
- We derived a new procedure for calculating the screening masses at small finite chemical potential using symmetry arguments.
- We calculated TM_R'' at two temperatures which at present analysis is zero within error but leaning to have a negative value. To get further accurate values, we need to go to larger lattices.
- We also looked M_I' values at two temperatures which seem to approach the correct free theory limit.