# Index Coded - NOMA in Vehicular Ad Hoc Networks

Sreelakshmi Pazhoor, *Student Member, IEEE*, Jesy Pachat, *Member, IEEE*,
Anjana Ambika Mahesh, *Student Member, IEEE*, Deepthi P. P., *Member, IEEE*,
and B. Sundar Rajan, *Fellow, IEEE*

***Abstract*—The demand for multimedia services is growing day by day in vehicular ad hoc networks (VANETs), resulting in high spectral usage and network congestion. Non-orthogonal multiple access (NOMA) is a promising wireless communication technique to solve the problems related to spectral efficiency effectively. The index coding (IC) is a powerful method to improve spectral utilization, where a sender aims to satisfy the needs of multiple receivers with a minimum number of transmissions. By combining these two approaches, in this work, we propose a novel technique called index coded NOMA (IC-NOMA), where we apply NOMA techniques on index coded data to reduce the number of transmissions further. This work shows that the IC-NOMA system demands a specific design for index codes to reap the advantages of NOMA. We have done the feasibility analysis of the proposed method in a general scenario and proposed an index code design to integrate IC over NOMA for the best efficiency. Through detailed analytical studies it is validated that the proposed transmission system provides improved spectral efficiency and power saving compared to conventional IC systems.**

***Index Terms*—Caching, index coding, Non-orthogonal multiple access, vehicular communications.**

## I. Introduction

VEHICULAR Adhoc Networks (VANETs) have become a developing platform in modern intelligent transport systems. The prime goal of VANETs is to provide road safety and infotainment services to the users. The VANET scenario where a set of users demand some popular multimedia content is called the popular content dissemination scenario [1]. The popular content may include navigational information such as road safety, real-time traffic data, road accident information etc. When multiple vehicles request common data, they can collaboratively download the data from a roadside unit (RSU) with reduced download completion time [2]. But in a general scenario, users will be interested in some common information for navigation as well as multimedia data of their choice for entertainment. Hence this work assumes that vehicles have some common demands together with some distinct individual demands. All the vehicles are assumed to be equipped with long-term evolution-vehicle (LTE-V) based communication [3] devices. LTE-V is a promising integrated solution for supporting V2X communications with low latency and high reliability. The vehicles can download data packets as they pass through the range of RSU. The vehicles may not be able to download the complete set of packets demanded by them when they pass the RSU, as this depends on the speed of the vehicle and the range of RSU. In [4] authors have discussed the scenario where the vehicles can share their acquired data and satisfy their demands through cooperative communication. In this work, we assume that there is no cooperative communication among the vehicles. Vehicles are unwilling to collaborate due to privacy and security issues or the lack of incentive.

The exchange of data needs to be less time-consuming, and any mistake can result in casualties. The concepts of index coding (IC) techniques can be exploited for overcoming bandwidth limitations and for the fast distribution of data [5]. Index coding is a source coding variant first proposed by Birk and Kol. The participating users have some prior details, called side information [6]. Based on each user's side information, the source would send coded blocks of data that would jointly allow users to extract the requested information in a minimum number of transmissions. The side information may be a linear combination of message packets, called coded side information [7], [8]. The primary goal of an IC problem is to minimize the number of transmissions required and improve bandwidth efficiency. The index coding technique can be introduced to VANETs, which lets RSUs to serve the users with side information by reduced number of transmissions. The number of transmissions reduces, and thus the network load, which improves the overall transmission efficiency [5].

V2X communicating units process and share huge amounts of data which can result in increased latency and irregular connectivity. 5G networking technologies built into vehicular networks satisfy the increasing communication needs. One of the most critical factors in improving network specifications is the implementation of an effective multiple access technique. Non-orthogonal multiple access (NOMA) techniques [9] are found to have increased spectral efficiency and decreased latency compared with orthogonal multiple access (OMA) techniques. By accommodating far more users than OMA, NOMA meets the demand for massive connectivity. NOMA supports different

Manuscript received 10 March 2022; revised 3 June 2022; accepted 5 June 2022. Date of publication 14 June 2022; date of current version 19 September 2022. This work was supported by the Science and Engineering Research Board (SERB) of Department of Science and Technology (DST), Government of India, through J.C. Bose National Fellowship to B. Sundar Rajan. A part of this paper has been accepted in IEEE 95th Vehicular Technology Conference: VTC 2022-Spring in Helsinki, Finland. The review of this article was coordinated by Dr. Trung Q. Duong. *(Corresponding author: B. Sundar Rajan.)*

Sreelakshmi Pazhoor, Jesy Pachat, and Deepthi P. P. are with the Electronics and Communication Department, National Institute of Technology, Calicut 673601, India (e-mail: sreelakshmi.pazhoor@gmail.com; jesypachat@gmail.com; deepthi@nitc.ac.in).

Anjana Ambika Mahesh and B. Sundar Rajan are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India (e-mail: anjanamahesh@iisc.ac.in; bsrajan@iisc.ac.in).

Digital Object Identifier 10.1109/TVT.2022.3182891

users using the same resources. NOMA introduces controllable interference to superpose different user signals at the expense of a tolerable increase in the complexity of the receivers. The channel gain of users vary depending on their distance from the transmitter, among other factors. Users with a substantial difference in channel gain need a single NOMA transmission with superposition coding (SC) [10], [11]. The far users are assigned higher powers due to their weak channel conditions than near users with good channel conditions. The signals of far users are superposed linearly with signals of near users for transmission. A decoding process called successive interference cancellation (SIC) takes place at the receiver based on the distance from the sender [12]. The far-user must decode the data from the superposed signal by treating the interference from any other device signal as noise. The others must decode and deduct the highest power data from the received signal to decode the desired signal. Then all users are served concurrently by SC at transmitter and SIC at the receiver using available bandwidth. NOMA-based studies are evolving day in and day out. Allocating one frequency channel to multiple users at the same time makes NOMA a promising multiple access system with improved spectral efficiency. Cache-assisted NOMA (CA-NOMA) exploits cached and previously decoded data for better cancellation of interference called cache-enabled interference cancellation [13].

The work in [14] and [15] discusses a mode-selection scheme for two user NOMA system where the transmission mode is switched between NOMA and IC based on the cached data of the paired users. There, the authors propose to use index coding if both the users have data of each other in their cache. If the cached information is not available, the transmission scheme switches to NOMA. Thus the work in [14] and [15] doesn't apply NOMA on index coded data. Applying NOMA principles on index-coded data is a non-trivial task because the index coding scheme is broadcast in nature while in NOMA, the far users cannot retrieve the near user data. When power domain NOMA is applied on index-coded data for improved bandwidth efficiency, an important aspect to be considered is that the far users cannot get the index coded data transmitted at the power level of the near user. This may demand extra transmissions of some index-coded data to facilitate decoding for the far user. Therefore, in order to ensure an overall improvement in bandwidth efficiency and transmitted power, there is a need to develop a design strategy where index code for near users and far users are designed separately by exploiting NOMA transmission.

### *A. Contributions*

In this work, we propose a transmission scheme called index coded-NOMA (IC-NOMA) where the index coded packets are superposed using power domain NOMA principles to serve the users with a minimum number of transmissions. To the best of our knowledge, this is the first work that discusses a transmission scheme which applies NOMA principles on index-coded data.

In an index coding solution, the users decode their demands from the received index coded packets using the side information available with them. The decodability in index coding assumes that all the index coded packets are available to all the users. In the proposed IC-NOMA scheme, the index coded packets of far users are superposition coded in the power domain with index coded packets of near users. With the successive interference cancellation decoding of NOMA, the far users cannot decode the index coded data corresponding to near user transmitted at the low power level. This may demand additional transmission of index coded data for decodability at the far user. On the other side, the index coded packets for far users will be made available to the near users as coded side information through power domain superposition without consuming extra bandwidth. Therefore, in order to ensure an overall improvement in bandwidth efficiency and transmitted power, the index code design of far users and near users need to be carried out separately. Grouping of the users as far user and near user subgroups also need to be done carefully to reap the advantages of NOMA transmission scheme. This work proposes an algorithm to divide the users receiving broadcast transmissions into far and near groups and design suitable index codes for each group separately to fit the NOMA concepts. The index code design of the far group takes care of the fact that near user data is not needed for decoding. Near user index code design makes use of the additional index coded side information to bring in further reduction in the number of transmissions.

We compare the performance of the proposed transmission algorithm with conventional IC analytically. It is seen that the proposed algorithm offers improved bandwidth efficiency along with power efficiency.

We have also presented a practical scenario in VANET where the proposed system could be applied. The VANET scenario, where the vehicles are moving in a multilane track with almost equal speed, is considered. Under this framework, we combine NOMA principles and IC techniques to develop a transmission strategy with superior spectral efficiency.

The main contributions of this paper are summarized below:

- We propose IC-NOMA as a transmission scheme with superior spectral efficiency.
- It is shown that IC-NOMA needs a unique index code design to get bandwidth improvement. An algorithm to develop index code designs for IC-NOMA is presented in this work.
- A practical VANET scenario is discussed where the proposed IC-NOMA scheme can be applied to bring improved efficiency.
- Through detailed analytical studies, it is shown that the proposed transmission scheme provides improved spectral efficiency and power saving compared to conventional IC systems.

## II. PRELIMINARIES AND BACKGROUND

In this section, we discuss some basics of NOMA and IC. These promising wireless techniques have become a research hotspot in vehicular communications.

### *A. Non-Orthogonal Multiple Access (NOMA)*

NOMA can be widely categorized into two, namely power domain NOMA and code domain NOMA. In power domain NOMA, different user signals are superposed on the power

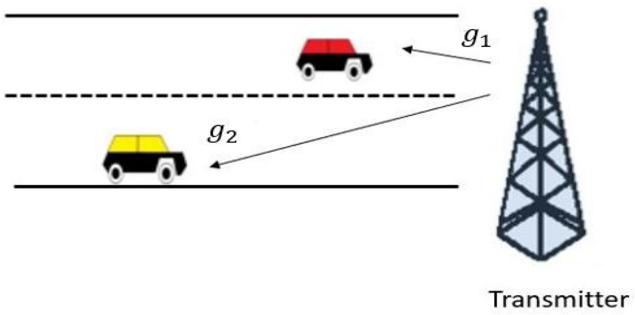

Fig. 1. NOMA scenario in VANET with $M = 2$.

domain depending on their channel conditions. The users served simultaneously by a single superposed transmission form a cluster [16].

Assuming $M$ users per cluster and each user denoted as $V_i$ with $i \in [M]$, let $x_i$ be the data requested by user $V_i$. Without loss of generality, let user $V_M$ be the farthest user and $V_1$ be the nearest user, and the channel gains be $g_1 > g_2 > g_3 \ldots\ldots > g_M$. The signal $s_i$ be the encoded form of $x_i$, $s_i = enc(x_i)$. Let the additive white Gaussian noise with zero mean and variance $\sigma_i^2$ at $V_i$ be represented as $n_i$.

Considering the channel gain of each user, the signals requested by them are linearly superposed in power domain at the transmitter as

$$S = \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i, \tag{1}$$

where $P$ is the transmission power, $\alpha_i P$ is the amount of power allocated to the signal corresponding to user $V_i$ with $\alpha_1 < \alpha_2 < \alpha_3 \ldots\ldots < \alpha_M$ and $\sum_{i=1}^{M} \alpha_i = 1$. Then the received signal at $V_i$ is

$$Z_i = \sqrt{g_i} S + n_i. \tag{2}$$

For user de-multiplexing, SIC is carried out at near users [17], based on the power with which the base station transmits the signals. The far user decodes the desired information, considering interference due to others as noise. All other users perform SIC, where the highest power data is decoded and progressively cancelled to decode the desired information.

Fig. 1 shows a conventional NOMA scenario with $M = 2$. In this case, the transmitted signal is $S = \sqrt{\alpha_1 P} s_1 + \sqrt{\alpha_2 P} s_2$ with $\alpha_2 = 1 - \alpha_1$. Let $\alpha < 0.5$ be the power allocation factor and $\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$. The transmitted signal is given by $S = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha) P} s_2$. Users $V_1$ and $V_2$ receives $Z_1 = \sqrt{g_1} S + n_1$ and $Z_2 = \sqrt{g_2} S + n_2$ respectively. Far user $V_2$ decodes $x_2$ by considering interference due to $x_1$ as noise, $V_1$ performs SIC of $x_2$ to decode $x_1$. After SIC the signal at $V_1$ can be represented as $Z_1' = Z_1 - \sqrt{g_1}\sqrt{(1-\alpha)P} s_2'$ with $s_2'$ as the signal intended for $V_2$ decoded at $V_1$.

## B. Index Coding

Consider a single sender holding a set of messages and serving a set of users. The user can demand a subset of these messages while caching a different subset of these messages. The data dissemination problem under such a scenario can be addressed through index coding approach to reduce the number of transmissions from the sender. The set of data packets demanded by each user is known as the want set, whereas the set of data packets cached are known as side information or known set. Each user can decode its data demands from the broadcast index coded transmissions by making use of the side information available in its cache.

TABLE I
TABLE REPRESENTING THE KNOWN SET AND WANT SET DISTRIBUTION OF $N = 4$ USERS

| Receiver | Known set | Want set |
|---|---|---|
| $V_1$ | $x_1$ | $\{x_2\}$ |
| $V_2$ | $x_2$ | $\{x_1, x_3, x_4\}$ |
| $V_3$ | $x_3$ | $\{x_1, x_4\}$ |
| $V_4$ | $x_4$ | $\{x_3\}$ |

Consider a single sender with $n$ messages $\mathcal{X} = \{x_1, x_2, \ldots x_n\}$, $x_i \in \mathbb{F}_q$, where $\mathbb{F}_q$ denotes the finite field with $q$ elements for a prime power $q$. Let there be $N$ receivers each with some prior knowledge of these messages over a broadcast channel [18]. A receiver denoted as $V_i, i \in [N]$ needs a subset of the messages denoted as $\mathcal{W}_i$, with $\mathcal{W}_i \subseteq \mathcal{X}$, known as the want set. Also, each receiver is holding a subset of these messages termed as known set or side information and denoted as $\mathcal{K}_i$, with $\mathcal{K}_i \subseteq \mathcal{X}$. Let the known set and want set of index coding problem is denoted as $\mathcal{K} = \{\mathcal{K}_i : \quad i \in [N]\}$ and $\mathcal{W} = \{\mathcal{W}_i : \quad i \in [N]\}$. The index coding problem with $n$ messages, $N$ receivers, known set $\mathcal{K}$ and want set $\mathcal{W}$ be denoted as $\mathcal{I}(n, N, \mathcal{K}, \mathcal{W})$.

*Definition 1:* For instance $\mathcal{I}(n, N, \mathcal{K}, \mathcal{W})$ of an index coding problem with input vector $\mathbf{x} \in \mathbb{F}_q^n$, then the corresponding index code [6], $\mathbf{y} \in \mathbb{F}_q^l$ of length $l$, consists of

1) An encoding function $\mathcal{F} : \mathbb{F}_q^n \to \mathbb{F}_q^l$.
2) And corresponding decoding functions for each receiver $\mathcal{G}_i : \mathbb{F}_q^l \times \mathbb{F}_q^{|\mathcal{K}_i|} \to \mathbb{F}_q^{|\mathcal{W}_i|}$ for $i \in [N]$.

If $l$ is minimum then the index code designed is optimal. For a linear index code $\mathbf{y}$, the encoding function is a linear transformation described as, $\mathbf{y} = \mathbf{L}\mathbf{x}$, where $\mathbf{L} \in \mathbb{F}_q^{l \times n}$ is known as the encoding matrix and is defined as $\mathbf{L} = \mathcal{F}(n, N, \mathcal{K}, \mathcal{W})$.

The coding and decoding in IC can be explained with an example.

*Example 1:* Let Table I represents the known set and want set of $N = 4$ receivers. The encoding matrix $\mathbf{L}$ for an optimal linear index code is given by

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

The corresponding index coded transmissions are $y_1 = x_1 + x_2$, $y_2 = x_2 + x_3$ and $y_3 = x_3 + x_4$. The index code length is given by $l = 3$. The decoding process of different users is described in Table II.

In [7], [8] authors give a generalization of index coding problem, where the elements of known set are linear combinations of the messages. Such index coding problems are known as

TABLE II
TABLE SHOWING DECODING PROCESS AT RECEIVERS

| Receiver | Known set | Want set | Decoding Process |
|---|---|---|---|
| $V_1$ | $x_1$ | $\{x_2\}$ | $y_1 + x_1 = x_2$ |
| $V_2$ | $x_2$ | $\{x_1, x_3, x_4\}$ | $y_1 + x_2 = x_1$ , $y_2 + x_2 = x_3$, $y_2 + y_3 + x_2 = x_4$ |
| $V_3$ | $x_3$ | $\{x_1, x_4\}$ | $y_1 + y_2 + x_3 = x_1$, $y_3 + x_3 = x_4$ |
| $V_4$ | $x_4$ | $\{x_3\}$ | $y_3 + x_3 = x_4$ |

index coding with coded side information (ICCSI). The coded side information at receiver $V_i$ is represented using the vector, $\mathbf{a}_i \in \mathbb{F}_q^{d_i}$ such that $\mathbf{a}_i = \mathbf{B}_i \mathbf{x}^T$, where $\mathbf{B}_i \in \mathbb{F}_q^{d_i \times n}$ is known as the side-information generator matrix. Under this set up let $\mathbf{L}_c$ be the encoding matrix used at the sender such that the demands of all users are satisfied in minimum number of transmissions. So the index coding vector for ICCSI problem is given as $\mathbf{y}_c = \mathbf{L}_c \mathbf{x}$. Here $\mathbf{L}_c = \mathcal{F}(n, N, \mathcal{K}_c, \mathcal{W})$, where $\mathcal{K}_c = \{\mathbf{a}_i : i \in [N]\}$ is the coded side information.

## III. SYSTEM MODEL AND MOTIVATING EXAMPLE

In this section, we discuss the system model for the proposed IC-NOMA scheme and a method to design index codes for IC-NOMA to achieve improved bandwidth efficiency.

### A. System Model

This work considers a downlink VANET scenario in which vehicles travel on a one-way multi-lane road. The vehicles travel at speed such that the relative velocity between them is negligible. The system model of this work considers LTE-V or Cellular-V2X (C-V2X) [19] standard. As per the technical specifications published by 3GPP in 2016, this standard enable both direct and network-based communications. Thus, the vehicles can exchange information with infrastructures such as roadside units (RSU) or base stations (BS). The RSU and vehicles in its range can communicate directly over 5.9GHz unlicensed frequency, whilst the vehicles can interact with the base station using the C-V2X licensed spectrum.

In a VANET scenario, a set of vehicles could be interested in some common information while each individual vehicle can have some distinct demand for some entertainment data. Therefore the demand set of vehicles can have some packets demanded in common. The vehicles report their demands to the server through RSU or BS. The server will send the entire set of demanded packets to the next RSU through BS in the direction of vehicle movement. RSU delivers the whole set of demanded packets to the vehicles when they enter its range. Given the range of RSU, speed of vehicles and the total number of messages demanded; it is unable to satisfy the user demands completely by RSU. Each vehicle receives some of the packets as their side information. This work assumes that each vehicle has a cache where data packets delivered by RSU nodes are stored as side information to reduce the cost associated with network usage. As the vehicles are not entering the RSU range simultaneously, their side information could be different when they leave the range of RSU. A set of vehicles moving in one direction within a range of BS forms a cluster. The vehicles will send the index set of their side information through a wireless backchannel to BS. The known set and want set of vehicles in the cluster determines index code design for the cluster. In order to apply NOMA principles on the index coded data the vehicles in a cluster need to be further divided as far group and near group. In IC-NOMA, the index code design for each group will be done separately. The BS broadcast IC-NOMA transmissions to the vehicles in its range. Hence the scenario under consideration consists of different phases as follows:

1) Reporting Phase: The vehicles can report their demands to server through RSU or BS. The server on receiving user demands send the total demand set to the next RSU and base station (BS) in the forward direction of the vehicles through the respective wireline channels.
2) R2V Phase: The RSU disseminate data packets to users in its transmission range, and each user receives subset of the packets delivered by RSU as their side information.
3) V2N Phase: In this phase vehicles communicate only to the BS. The set of vehicles entering the range of base station forms cluster. The vehicles communicate the index set of their side information to the BS through a wireless back channel. The BS updates the demand set of vehicles in the cluster by removing the messages already received by them in R2V phase. The base station performs IC-NOMA transmissions for users entering the communication range of BS.

### B. IC-NOMA in VANET

In this section, we introduce the concepts of the proposed transmission strategy with superior spectral efficiency called IC-NOMA, which is an integration of NOMA principles and index coding techniques. Unlike traditional NOMA, where message packets are superposition coded, IC-NOMA superposes index coded packets in the power domain.

Consider a cluster of $N$ vehicles with $\mathcal{D}_i, i \in [N]$ as the initial set of demands which the vehicle $V_i$ communicates to the server via RSU or BS. The server sends the whole set of demanded packets denoted as, $\mathcal{X} = \bigcup_{i \in [N]} \mathcal{D}_i$ to the RSU through the BS in the direction of vehicle movement. During R2V phase, $V_i$ will receive their side information packets denoted as $\mathcal{K}_i$. For the IC problem under consideration, the demands of each user is modified at BS as $\mathcal{W}_i = \mathcal{D}_i \backslash \{\mathcal{D}_i \cap \mathcal{K}_i\}$ to remove the demanded messages that are received as side information during R2V phase.

The index coded symbols designed for users in cluster is determined by known set and want set of users in the cluster. Let $g_i, i \in [N]$ denote the channel gain between the BS and the receiving vehicle $V_i$. As mentioned earlier, the users in a cluster are further divided to two groups for applying NOMA principles over index coded data. Let $N_f$ be the number of users in far group

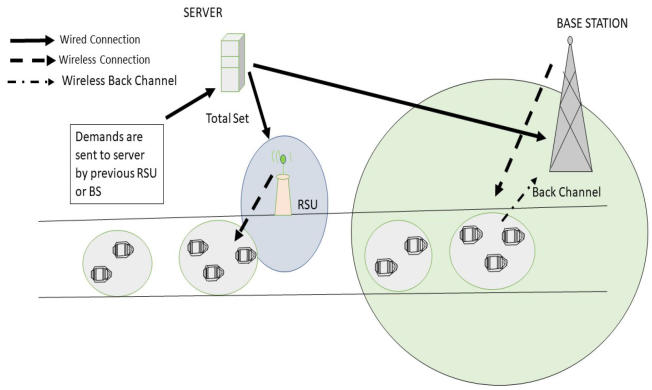

Fig. 2. VANET scenario with $N = 5, N_f = 2$.

TABLE III
EXAMPLE 2 OF IC-NOMA PROBLEM

| vehicle | Known set $\mathcal{K}_i$ | Want set $\mathcal{W}_i$ |
|---|---|---|
| $V_1$ | $\{x_2\}$ | $\{x_1\}$ |
| $V_2$ | $\{x_1\}$ | $\{x_2\}$ |
| $V_3$ | | $\{x_3\}$ |

and $N_n = N - N_f$ be the number of users in near group. The channel gain of users are denoted as $g_1 \geq g_2 \ldots\ldots \geq g_{N_n} > g_{N_n+1} \geq g_{N_n+2} \ldots\ldots \geq g_N$. Fig. 2 represents VANET scenario under consideration with $N = 5$, $N_f = 2$ and $N_n = 3$.

The minimum number of transmissions needed for the IC-NOMA scenario is represented by $l^{IC-NOMA}$ and that for conventional IC is represented by $l^{IC}$. Later we prove that $l^{IC-NOMA} \leq l^{IC}$.

The $j^{th}$ IC-NOMA signal received by user $V_i$ can be represented as

$$Z_i^j = \sqrt{g_i} S_j + n_i, \quad i \in [N],\ j \in [l^{IC-NOMA}], \tag{3}$$

where $S_j$ with $j \in [l^{IC-NOMA}]$ represents the $j^{th}$ transmitted signal, $g_i$ for $i \in [N]$ represents the channel gain between the user $V_i$ and the BS and $n_i$ represents the additive white Gaussian noise with zero mean and variance $\sigma_i^2$ at user $V_i$.

### *C. Motivating Example*

In this section we show the proposed transmission strategy called IC-NOMA through an example.

*Example 2:* Let $N = 3$, Table III displays known set and corresponding want set of each vehicle. The linear solution for the given IC problem requires at least two transmissions. One such optimal linear index code is given by the encoding matrix $\mathbf{L}$ as

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The corresponding index coded transmissions are $y_1 = x_1 + x_2$ and $y_2 = x_3$.

In this example, Let $V_1, V_2$ be the near users and $V_3$ be the far user so that $N_f = 1$ and $N_n = 2$. The IC-NOMA transmission combines the concepts of NOMA with IC to transmit the signals $s_1 = enc(y_1)$ and $s_2 = enc(y_2)$.

Hence the IC-NOMA transmitted signal in this case is $S = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2$ with $\alpha < 0.5$. Here data requested by two near users $V_1, V_2$ are paired by index coding under the same power level. The far user $V_3$ gets $x_3$ transmitted at highest power, near users $V_1$ and $V_2$ decode $y_2 = x_3$ and perform SIC to get the index coded packet $y_1 = x_1 + x_2$. They exploit the side information to decode the desired data. Eventually, the designed scheme is equivalent to a two-user NOMA system with two power levels, though there are three users. Hence the number of users served simultaneously by a single transmission increases.

The total number of required transmissions in IC-NOMA for Example 3 is $l^{IC-NOMA} = 1$.

### *D. Index Code Design for IC-NOMA*

In a traditional index coding problem, optimal linear index code can be designed based on want set and known set of each user. But some IC solutions cannot be combined with NOMA principles efficiently even when the index code is optimal. Hence it is required to see how index code design needs be modified to reap additional power or bandwidth saving when combined with NOMA. In NOMA decoding, near user apply SIC for decoding and can retrieve the information intended for both the near and far user (i.e., the data sent at both the power levels). So in IC-NOMA, index coded data sent to the far user can be used as an additional coded side information to meet the demands of the near user. In contrast, the demands of the far users should be satisfied with the index coded packets transmitted at the higher power, because the index coded packets sent at the lower power levels are considered as interfering noise and are discarded. Therefore, the signals at the highest power levels should provide a complete IC solution to meet the demands of far users. Hence an index code design that considers the information redistribution through NOMA decoding need to be developed for achieving improved transmission efficiency.

Let $x = [x_1\ x_2 \ldots x_n]^T$ be the set of messages to be communicated. Considering only the want set and known set of far users denoted as $\mathcal{W}_f = \{\mathcal{W}_i : \quad i \in \mathcal{I}_f\}$ and $\mathcal{K}_f = \{\mathcal{K}_i : \quad i \in \mathcal{I}_f\}$ with $\mathcal{I}_f$ as the index set of far users, a linear IC problem is formulated. A linear index code specified by the encoding matrix $\mathbf{L}_f = \mathcal{F}(n, |\mathcal{I}_f|, \mathcal{K}_f, \mathcal{W}_f)$ is given by

$$\mathbf{y}_f = \mathbf{L}_f \mathbf{x}, \tag{4}$$

where $\mathbf{L}_f \in \mathbb{F}_q^{l_f \times n}$, the length of index code designed for far users is denoted as $l_f$. Let $\mathcal{Y}_f = \{y_{f_i} : \quad i \in [l_f]\}$ be the set of index coded packets designed for far users. The near users decode the data of far users to perform SIC; hence an index code is designed for near users by exploiting the respective side information along with the index code of far users as coded side information.

Consider $\mathcal{I}_n$ as the index set of near users. The side information set at each near user can be modified as $\mathcal{K}_i' = \{\mathcal{K}_i \cup \{\mathcal{Y}_f\} : i \in \mathcal{I}_n\}$ and $\mathcal{K}_n' = \{\mathcal{K}_i' : \quad i \in \mathcal{I}_n\}$. The total want set of near users are represented as $\mathcal{W}_n' = \{\mathcal{W}_i \backslash \mathcal{W}_i^c : \quad i \in \mathcal{I}_n\}$, where $\mathcal{W}_i^c$ is the demand set of near user $V_i$ satisfied through coded side information of far users. Hence the index coding problem for near users is an ICCSI problem with encoding matrix $\mathbf{L}_n^c =$

TABLE IV
USERS WITH THEIR DEMANDS (BEFORE R2V PHASE)

| User | Demands ($\mathcal{D}_i$) |
|---|---|
| $V_1$ | $\{x_1, x_4, x_5, x_6\}$ |
| $V_2$ | $\{x_1, x_5, x_6\}$ |
| $V_3$ | $\{x_1, x_2, x_6\}$ |
| $V_4$ | $\{x_1, x_3, x_4, x_5, x_7\}$ |
| $V_5$ | $\{x_3, x_6\}$ |

TABLE V
EXAMPLE 3 OF IC-NOMA PROBLEM

| User | Known set | want set $\mathcal{W}_i$ (After R2V Phase) |
|---|---|---|
| $V_1$ | $\{x_1, x_2, x_3\}$ | $\{x_4, x_5, x_6\}$ |
| $V_2$ | $\{x_2, x_3, x_4\}$ | $\{x_1, x_5, x_6\}$ |
| $V_3$ | $\{x_3, x_4, x_5\}$ | $\{x_1, x_2, x_6\}$ |
| $V_4$ | $\{x_5, x_6, x_7\}$ | $\{x_1, x_3, x_4\}$ |
| $V_5$ | $\{x_6, x_7\}$ | $\{x_3\}$ |

$\mathcal{F}(n, |\mathcal{I}_n|, \mathcal{K}'_n, \mathcal{W}'_n)$. The index coded vector is obtained as,

$$\mathbf{y}^c_n = \mathbf{L}^c_n \mathbf{x}, \tag{5}$$

where $\mathbf{L}^c_n \in \mathbb{F}_q^{l_n \times n}$, and $l_n$ is the length of index code designed for near users.

The side information of each user, along with the distance from the BS, significantly influence the design of IC-NOMA. This is illustrated through an example.

*Example 3:* Consider the VANET scenario shown in Fig. 2 with $N = 5$. Table IV displays the want set of each vehicle.

The server sends the total demand set $\mathcal{X} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ to RSU through BS in the direction of vehicle movement. The vehicles on entering the RSU range receives data packets as side information as shown by Table V . Since the demands of some of the users will be satisfied by side information during R2V phase, the want set is modified at BS as $\mathcal{W}_i = \mathcal{D}_i \backslash \{\mathcal{D}_i \cap \mathcal{K}_i\}$, Table V shows the modified want set.

Here it is assumed that spacing between vehicles $V_1$ and $V_2$ is same as that between $V_2$ and $V_3$ as well as $V_4$ and $V_5$ while spacing between $V_3$ and $V_4$ is longer. Since the overlap in side information between consecutive vehicles depends on spacing between them, the overlap in side information between $V_3$ and $V_4$ is lesser. Also since $V_5$ enters last, it could download lesser number of packets compared to other vehicles.

The encoding matrix representing one of the possible linear solutions for the given index coding problem is

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

The index coded transmissions designed are $y_1 = x_1 + x_4, y_2 = x_2 + x_5$, $y_3 = x_3 + x_6$, $y_4 = x_4 + x_7$. The index code length is given by $l^{IC} = 4$.

When combining the designed index codes with NOMA principles, the far users should be able to decode the desired data without getting the data of near users. We cannot use the above index code design in the NOMA scenario, as the far users need index coded packets of near users for decoding the desired data. Hence in an IC-NOMA scenario, the index code design need to be modified for combining with NOMA.

TABLE VI
EXAMPLE 3 OF IC-NOMA PROBLEM FOR FAR USERS

| vehicle | Known set | want set |
|---|---|---|
| $V_4$ | $\{x_5, x_6, x_7\}$ | $\{x_1, x_3, x_4\}$ |
| $V_5$ | $\{x_6, x_7\}$ | $\{x_3\}$ |

TABLE VII
EXAMPLE 3 OF IC-NOMA PROBLEM FOR NEAR USERS WITH MODIFIED WANT SET AND KNOWN SET

| vehicle | Known set | want set |
|---|---|---|
| $V_1$ | $\{x_1, x_2, x_3, x_1 + x_7, x_3 + x_6, x_4 + x_7\}$ | $\{x_5\}$ |
| $V_2$ | $\{x_2, x_3, x_4, x_1 + x_7, x_3 + x_6, x_4 + x_7\}$ | $\{x_5\}$ |
| $V_3$ | $\{x_3, x_4, x_5, x_1 + x_7, x_3 + x_6, x_4 + x_7\}$ | $\{x_2\}$ |

TABLE VIII
THE WANT SET DISTRIBUTION FOR $N = 5$ WITH THE KNOWN SET SAME AS GIVEN IN TABLE V

| User | Want set |
|---|---|
| $V_i$ | $\mathcal{W}_i$ |
| $V_1$ | $\{x_4, x_5, x_7\}$ |
| $V_2$ | $\{x_5, x_6\}$ |
| $V_3$ | $\{x_1, x_6\}$ |
| $V_4$ | $\{x_1, x_2, x_3\}$ |
| $V_5$ | $\{x_1\}$ |

In this example, as described earlier spacing between $V_3$ and $V_4$ is higher than others. Therefore $V_1$, $V_2$ and $V_3$ are grouped as near users while $V_4$ and $V_5$ are grouped as far users. Thus $N_f = 2$ and $N_n = 3$ in this case.

In IC-NOMA, to develop index codes that can bring in improved transmission efficiency when combined with NOMA scheme, we consider the index coding problem as two different index coding problems: One as a conventional IC problem to design index codes for far users; and the other as an ICCSI for near users.

First the encoding matrix $\mathbf{L}_f$ is designed to develop index codes for far users as per (4) based on Table VI for far users. Hence the encoding matrix $\mathbf{L}_f$ corresponding to far users is designed as

$$\mathbf{L}_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

The far user index code is given by $y_{f_1} = x_1 + x_7$, $y_{f_2} = x_3 + x_6$ and $y_{f_3} = x_4 + x_7$. The index code length for far users is $l_f = 3$.

Here some of the near user demands are satisfied by the index code of far user. Hence the want set along with the known set of near users are modified as shown in Table VII.

As per (5) index coded solutions can be developed for near users based on Table VII. Using $y_{f_1}$, $y_{f_2}$ and $y_{f_3}$ as coded side information along with the respective side information, we can design an encoding matrix $\mathbf{L}^c_n$ for near users.

$$\mathbf{L}^c_n = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

$y^c_{n_1} = x_2 + x_5$ are the index code designed for near users $V_1, V_2$ and $V_3$ with length as $l_n = 1$.

The index codes designed are combined using NOMA principles in IC-NOMA. Hence the transmitted signals in IC-NOMA is $S_1 = \sqrt{\alpha P} s_{n_1} + \sqrt{(1-\alpha)P} s_{f_1}$, $S_2 = \sqrt{P} s_{f_2}, S_3 = \sqrt{P} s_{f_3}$, where $\alpha < 0.5$ and $s_{n_1} = enc(y^c_{n_1})$, $s_{f_1} = enc(y_{f_1})$, $s_{f_2} = enc(y_{f_2})$ and $s_{f_3} = enc(y_{f_3})$. Hence the number of required transmissions, in this case, is $l^{IC-NOMA} = 3$.

The far users receiving $S_1$ decode $s_{f_1}$ considering interference due to $s_{n_1}$ as noise, and from $S_2$ and $S_3$ they decode $s_{f_2}$ and $s_{f_3}$. The near users receiving $S_1$ get the far user data $s_{f_1}$ perform SIC to get $s_{n_1}$, and from $S_2$ and $S_3$ they decode $s_{f_2}$ and $s_{f_3}$. Hence the far users get $\{s_{f_1}, s_{f_2}, s_{f_3}\}$, whereas the near users get $\{s_{f_1}, s_{f_2}, s_{f_3}, s_{n_1}\}$.

In IC-NOMA, the index coded packets of far users are combined with index coded packets of near users suitably at two power levels as in a conventional NOMA using SC. The near users retrieve the index codes designed for them through SIC, followed by the decoding of desired data. Hence in Example IV, the total number of required transmissions in IC-NOMA to serve $N = 5$ users is $l^{IC-NOMA} = 3$, whereas traditional index coding requires at least 4 transmissions.

## IV. DESIGN OF IC-NOMA

The spectral efficiency offered by the IC-NOMA system can be achieved only through the innovative design of the index codes so as to reap the advantages of NOMA through SC. In order to develop an index code design for NOMA, the proposed system groups the users into two as far users and near users, by considering the channel gain of each user from the BS. The proposed algorithm is specifically developed for situations where the users can be divided into two groups based on their respective channel gains but can be extended to a more general scenario with more number of groups. The channel gain difference between any two users in the same group is considered as very small while the difference between channel gains of two users in two different groups could be considerably greater.

We search for the maximum and minimum channel gains $g_{max}$ and $g_{min}$ from a group of $N$ users. Let the index set of far users be denoted as $\mathcal{I}_f$ and the index set of near users as $\mathcal{I}_n$, where $\mathcal{I}_f = \{i : \quad |g_{max} - g_i| > |g_{min} - g_i|\}$ and $\mathcal{I}_n = \{i : \quad |g_{max} - g_i| < |g_{min} - g_i|\}$, where $i \in \ [N]$.

Consider the scenario where there are n messages. Let $x = [x_1 x_2 \ldots x_n]^T$ be the input vector. After grouping far and near users separately, we design an index code of length $l_f$ for far users based on the elements in known set $\mathcal{K}_f = \{\mathcal{K}_i : \quad i \in \mathcal{I}_f\}$ and elements in want set $\mathcal{W}_f = \{\mathcal{W}_i : \quad i \in \mathcal{I}_f\}$. Let $\mathbf{y}_f$ denote the index code designed for far users with encoding matrix as $\mathbf{L}_f$. Then $\mathbf{y}_f = \mathbf{L}_f \mathbf{x}$.

After designing index codes for far users, we design an index code $\mathbf{y}^c_n$ of length $l_n$ for the near users. Since every near user decodes the data of far users to perform SIC according to NOMA principles, the known set of each near user is updated based on the set of coded side information denoted by $\mathcal{Y}_f$ as $\mathcal{K}'_i = \{\mathcal{K}_i \cup \mathcal{Y}_f : \quad i \in \mathcal{I}_n\}$. The index codes for near users are designed based on $\mathcal{K}'_n = \{\mathcal{K}'_i : \quad i \in \mathcal{I}_n\}$ and $\mathcal{W}'_n = \{\mathcal{W}_i \backslash \mathcal{W}^c_i : \quad i \in \mathcal{I}_n\}$, where $\mathcal{W}^c_i$ represents the set of demands of near user $V_i$ satisfied by the index code of far users. Then $\mathbf{y}^c_n = \mathbf{L}^c_n \mathbf{x}$.

**Algorithm 1:** Grouping of users and updating near users.

```
Require: $g_i, \mathcal{K}_i$ and $\mathcal{W}_i \ \forall i \in [N]$, $x = [x_1\ x_2 \ldots x_n]^T$, $N$.
$g_{max} = max_{i \in [N]}(g_i)$.
$g_{min} = min_{i \in [N]}(g_i)$.
$\mathcal{I}_f = [N]$. $\mathcal{I}_n = [N]$.
for i=1 to $N$ do
    if $|g_{max} - g_i| > |g_{min} - g_i|$ then
        $\mathcal{I}_n = \mathcal{I}_n \backslash \{i\}$.
    else
        $\mathcal{I}_f = \mathcal{I}_f \backslash \{i\}$.
    end if
end for
$N_f = |\mathcal{I}_f|$, $N_n = |\mathcal{I}_n|$.
$\mathcal{K}_f = \{\mathcal{K}_i : \quad i \in \mathcal{I}_f\}$.
$\mathcal{W}_f = \{\mathcal{W}_i : \quad i \in \mathcal{I}_f\}$.
Design encoding matrix $\mathbf{L}_f = \mathcal{F}(n, N_f, \mathcal{K}_f, \mathcal{W}_f)$.
$\mathbf{y}_f = \mathbf{L}_f \mathbf{x}$.
Let $\mathcal{Y}_f$ denote the set of index coded packets designed
  for far users.
$\mathcal{Y}_f = \{y_{f_i} : \quad i \in [l_f]\}$.
$\mathcal{K}'_i = \{\mathcal{K}_i \cup \mathcal{Y}^f : \quad i \in \mathcal{I}_n\}$.
$\mathcal{K}'_n = \{\mathcal{K}'_i : \quad i \in \mathcal{I}_n\}$.
Let $\mathcal{W}^c_i$ denotes the set of demands of near user $V_i$
  satisfied by coded side information of far users.
$\mathcal{W}'_n = \{\mathcal{W}_i \backslash \mathcal{W}^c_i : \quad i \in \mathcal{I}_n\}$.
Design encoding matrix $\mathbf{L}^c_n = \mathcal{F}(n, N_n, \mathcal{K}_n,' \mathcal{W}'_n)$.
$\mathbf{y}^c_n = \mathbf{L}^c_n \mathbf{x}$.
return $\mathbf{y}_f, \mathbf{y}^c_n$.
```

Hence the conventional IC problem is converted to two IC problems in IC-NOMA. One is a conventional index coding problem for far users, and other is an ICCSI problem for near users. The index code design for IC-NOMA is given by Algorithm 1.

IC-NOMA includes a set of NOMA messages superposing the index coded packets and a set of index coded messages, if needed to satisfy the user needs. Each NOMA transmission of IC-NOMA superposes two index coded packets in the power domain and is similar to a conventional NOMA with two power levels. The transmitted set of NOMA messages and the set of index code messages in the IC-NOMA system are denoted respectively as

$$S^{IC-NOMA}_k = \begin{cases} S^{NOMA}_k & k \in [l^{NOMA}] \\ \& & \\ S^{IC}_k & k = \{l^{NOMA}+1, \ldots l^{IC-NOMA}\}, \end{cases} \tag{6}$$

$$S^{NOMA}_k = \sqrt{\alpha P} s_{n_k} + \sqrt{(1-\alpha)P} s_{f_k}, \qquad k \in [l^{NOMA}]$$

$$S_k^{IC} = \begin{cases} \sqrt{P} s_{f_k}, \quad k = \{l^{NOMA}+1, \ldots l^{IC-NOMA}\}, \\ \qquad\qquad \text{if } l_f > l_n \\ \text{or} \\ \sqrt{P} s_{n_k}, k = \{l^{NOMA}+1, \ldots l^{IC-NOMA}\}, \\ \qquad\qquad \text{if } l_f < l_n, \end{cases} \tag{7}$$

where $s_{f_i} = enc(y_{f_i})$, $i \in [l_f]$ and $s_{n_i} = enc(y_{ni}^c)$, $i \in [l_n]$. The required number of NOMA transmissions in IC-NOMA system is $l^{NOMA} = min(l_f, l_n)$. If $l^{IC-NOMA}$ represents the total number of transmissions needed in IC-NOMA system as NOMA and IC, then $l^{IC-NOMA} = max(l_f, l_n)$.

Hence the IC-NOMA system consists of three different cases as follows:

$Case\ \ I$ : When $l_f = l_n = l_{NOMA}$, demands of both the far and near users are satisfied by $l_{NOMA}$ number of NOMA transmissions.

$Case\ \ II$ : When $l_f > l_n$, a total of $l_f$ transmissions will be required by IC-NOMA system, with $l_n$ number of NOMA transmissions and $l_f - l_n$ number of index coded transmissions. Far user receives NOMA transmissions and index coded transmissions to decode the desired data. Near user decodes far user data from NOMA and index coded transmissions (as coded side information) and perform SIC to decode the desired data.

$Case\ \ III$ : When $l_f < l_n$, a total of $l_n$ transmissions will be required by IC-NOMA system, with $l_f$ number of NOMA transmissions and $l_n - l_f$ number of index coded transmissions. Far users requires only NOMA transmissions to decode the desired data. Near users requires NOMA and index coded transmissions to decode the desired data.

*Claim 1:* For a given index coding problem, IC-NOMA serves the user demands with less than or equal number of transmissions compared to conventional IC.

*Proof:* For a given index coding problem with $N$ number of users, let $l^{IC}$ be the optimal length of a conventional index code. Considering the same problem as two sub-problems in the IC-NOMA scenario, let $l_f$ be the length of index code designed for far users with $N_f < N$ and $l_n$ be the length of index codes designed for near users with $N_n < N$. Since the conventional index coding problem is split as two sub-problems in IC-NOMA, $l_f \leq l^{IC}$ and $l_n \leq l^{IC}$, and as given above $l^{IC-NOMA} = max(l_f, l_n)$. Hence $l^{IC-NOMA} \leq l^{IC}$. Thus IC-NOMA serves the user demands with less than or equal number of transmissions compared to conventional IC. □

## V. RATE ANALYSIS

In this section, we quantify the improvement in achievable information rate of the proposed IC-NOMA system over conventional IC. The achievable rate at each user for a given transmission depends on their respective channel gains. Let $g_n = min(g_i), i \in \mathcal{I}_n$ denote the channel gain of user with worst channel in near group and $g_f = min(g_i), i \in \mathcal{I}_f$ denote the channel gain of user with worst channel in far group. All users in a cluster receive the index coded packets broadcasted by the BS. Hence, the IC-NOMA transmissions should have sufficient power so that even the users with the worst channel conditions in both the near group and the far group should meet their rate requirements. The rate analysis is performed at each receiver by assuming additive white Gaussian noise with zero mean and unit variance. All the rates in subsequent discussions are expressed in bits per channel use.

### *A. Conventional IC Scenario*

Let P be the power required per transmission in a conventional IC scenario. Let the number of index coded transmissions needed to satisfy user demands be $l^{IC}$. Thus the transmitted signal is

$$S_i^{IC} = \sqrt{P} enc(y^i) \quad i = \{1, 2, \ldots . l^{IC}\}. \tag{8}$$

The minimum achievable rates that could be achieved by all the users in the near and far group in index coding problem is denoted as $R_f^{IC}$ and $R_n^{IC}$ and are given by

$$\begin{aligned} R_f^{IC} &= log_2\left(1 + min(g_i)P\right), \quad i \in \mathcal{I}_f, \\ &= log_2\left(1 + g_f P\right), \end{aligned} \tag{9}$$

$$\begin{aligned} R_n^{IC} &= log_2\left(1 + min(g_i)P\right), \quad i \in \mathcal{I}_n, \\ &= log_2(1 + g_n P), \end{aligned} \tag{10}$$

where $\mathcal{I}_f$, $\mathcal{I}_f$ denotes the index set of far users and near users respectively. Since $g_n > g_f$, we have $R_f^{IC} < R_n^{IC}$. Thus the rate that can be supported in index coded transmission is limited by that of the far user as,

$$R^{IC} = R_f^{IC}. \tag{11}$$

### *B. IC-NOMA Scenario*

In proposed IC-NOMA scheme, the data transmitted could be either simple index coded data or the index coded data combined with NOMA, as seen earlier. In proposed IC-NOMA scheme, the achievable information rate for NOMA transmissions is indicated as $R^{IN-NOMA}$. The achievable information rate for index coded transmissions of IC-NOMA system in Case II and Case III are represented as $R^{IN-IC^{(i)}}$ with $i = 2$ and $i = 3$ respectively.

*1) CASE I ($l_f = l_n$):* In this case IC-NOMA system will have NOMA transmissions only to satisfy user demands. For comparison, the power per transmission is considered to be the same as that of conventional index coded transmission i.e., $P$. Then,

$$\begin{aligned} S_k^{IC-NOMA} &= S_k^{NOMA} \\ &= \sqrt{\alpha P} s_{n_k} + \sqrt{(1-\alpha)P} s_{f_k} \quad k \in [l_{NOMA}]. \end{aligned} \tag{12}$$

Let $R_{f_i}^{IN-NOMA}$ denotes the information rate that could be achieved by far user $V_i$ from NOMA transmission. Then,

$$R_{f_i}^{IN-NOMA} = log_2\left(1 + \frac{(1-\alpha)P g_i}{\alpha P g_i + 1}\right). \tag{13}$$

Since we have, $g_f = min(g_i)$, $i \in \mathcal{I}_f$, the minimum information rate that could be achieved by the users in far group

$R_f^{IN-NOMA}$ is given by

$$R_f^{IN-NOMA} = log_2\left(1+\frac{(1-\alpha)Pg_f}{\alpha P g_f+1}\right). \tag{14}$$

Near user performs SIC of far user data. Let $R_n^{IN-NOMA}$ denote the minimum achievable rate in the near group. Then,

$$R_n^{IN-NOMA} = log_2(1+\alpha P g_n). \tag{15}$$

The minimum achievable rate for the near group and far group of the proposed IC-NOMA system is determined by the lowest channel gain of the users in each group.

The sum rate of transmission channel for this case denoted as $R^{IN-NOMA}$ for a single NOMA transmission of IC-NOMA system is

$$\begin{aligned} R^{IN-NOMA} &= R_f^{IN-NOMA} + R_n^{IN-NOMA} \\ &= log_2\left\{\frac{1+Pg_f}{1+\alpha P g_f}(1+\alpha P g_n)\right\}. \end{aligned} \tag{16}$$

Consider (11) and (16) to analyze the the improvement in the rate that can be supported by the proposed system.

$$R^{IN-NOMA} - R^{IC} = log_2\left(\frac{1+\alpha P g_n}{1+\alpha P g_f}\right). \tag{17}$$

Since $g_n > g_f$,

$$R^{IN-NOMA} > R^{IC}. \tag{18}$$

Hence the proposed system can support a higher information rate compared to conventional IC.

*2) CASE II ($l_f > l_n$):* In this case, there will be a total of $l_f$ number of transmissions in IC-NOMA, with $l_n$ NOMA transmissions and $l_f - l_n$ index coded transmissions. The IC-NOMA transmissions are given by

$$S_k^{IC-NOMA} = \begin{cases} S_k^{NOMA} = \sqrt{\alpha P} s_{n_k} + \sqrt{(1-\alpha)P} s_{f_k}, \\ \qquad\qquad\qquad\qquad k \in [l_n] \\ \text{and} \\ S_k^{IC} = \sqrt{P} s_{f_k}, \quad k = \{l_n+1, \ldots l_f\}. \end{cases} \tag{19}$$

The far user receives NOMA transmissions and IC transmissions to decode the desired data. Near user decodes far user data from NOMA and IC transmissions (as coded side information) to perform SIC to decode the desired.

The information rate for NOMA transmission of IC-NOMA will be the same as (16). The index coded transmissions in this case are index coded packets designed for far users. The information rate that can be supported by IC-NOMA under this scenario is decided by the information rate supported by the IC transmission of IC-NOMA, which is nothing but the achievable data rate at far user. Therefore achievable rate of IC-NOMA for IC transmission for Case II is calculated as

$$R^{IN-IC^{(2)}} = log_2(1+Pg_f) = R^{IC}. \tag{20}$$

The improved information rate that can be supported by NOMA transmissions is given in (17). From (20), it can be seen that, the achievable rate of index coded transmission is same as that of conventional IC.

*3) CASE III ($l_f < l_n$):* Under this scenario, there will be a total of $l_n$ number of transmissions in IC-NOMA, with $l_f$ NOMA transmissions and $l_n - l_f$ IC transmissions. The IC-NOMA transmissions are given by

$$S_k^{IC-NOMA} = \begin{cases} S_k^{NOMA} = \sqrt{\alpha P} s_{n_k} + \sqrt{(1-\alpha)P} s_{f_k}, \\ \qquad\qquad\qquad\qquad k \in [l_f] \\ \text{and} \\ S_k^{IC} = \sqrt{P} s_{n_k}, \quad k = \{l_f+1, \ldots l_n\}. \end{cases} \tag{21}$$

Far users require only NOMA transmissions to decode the desired data. Near users require NOMA and IC transmissions to decode the desired data. Near users decode far user data from NOMA transmissions (as coded side information) to perform SIC to decode desired. The achievable rate for NOMA transmission of IC-NOMA is given by (16).

The achievable rate for IC transmission of IC-NOMA system under Case III is that of the near user data rate and is given as,

$$R^{IN-IC^{(3)}} = log_2(1+Pg_n). \tag{22}$$

The improvement in achievable rate for index coded transmission of IC-NOMA compared to conventional IC can be obtained from (11) and (22) as,

$$R^{IN-IC^{(3)}} - R^{IC} = log_2\left(\frac{1+Pg_n}{1+Pg_f}\right). \tag{23}$$

The improvement in achievable information rate for NOMA transmissions is given in (17). It can be seen that there is an improvement in the achievable information rate for both IC and NOMA transmissions of IC-NOMA system in this case.

Thus it can be seen that the achievable rate for proposed IC-NOMA is greater than conventional IC under Case I and Case III for all transmissions. For Case II the achievable rate for NOMA transmission is greater than conventional IC while the rate for IC transmission is same as in conventional IC.

## VI. Transmission Power Analysis

In this section, we demonstrate how the proposed IC-NOMA system outperforms traditional IC in terms of power consumption. The power per transmission required for the IC-NOMA system to provide an achievable information rate at least as good as that of conventional IC is evaluated.

Let $P^{IC}$ be the power per transmission for conventional IC, $P_a$ be the power per NOMA transmission for IC-NOMA and $P_b^{(i)}$ for $i = 2$ and $i = 3$ be the power per index coded transmission of IC-NOMA system for Case II and Case III.

The total power consumption is computed for IC and IC-NOMA scheme, to quantify the power saving of the proposed system.

### *A. Case I ($l_f = l_n = l^{NOMA}$)*

Considering $P^{IC}$ as the power per transmission for conventional IC and $P_a$ as the power per NOMA transmission for

IC-NOMA, (11) (16) are modified respectively as

$$R^{IC} = log_2(1 + g_f P^{IC}) \,, \tag{24}$$

$$R^{IN\text{-}NOMA} = log_2 \left\{ \frac{1 + P_a g_f}{1 + \alpha P_a g_f} (1 + \alpha P_a g_n) \right\}. \tag{25}$$

To find the minimum power requirement for IC-NOMA transmission that makes $R^{IN\text{-}NOMA}$ at least as good as $R^{IC}$; consider (24) and (25).

$$log_2(1 + g_f P^{IC}) = log_2 \left\{ \frac{1 + P_a g_f}{1 + \alpha P_a g_f} (1 + \alpha P_a g_n) \right\}. \tag{26}$$

Let $P^{IC} = P_a + \zeta$ where $-p \leq \zeta \leq +p, \quad p = |P^{IC} - P_a|$. Then,

$$\zeta = \frac{(1 + P_a g_f)(\alpha P_a (g_n - g_f))}{g_f (1 + \alpha P_a g_f)}. \tag{27}$$

From (27), we have $\zeta > 0 \Rightarrow P^{IC} > P_a$ i.e., power per transmission for IC is greater than that of IC-NOMA. Then the power per NOMA transmission of IC-NOMA system is

$$P_a = P^{IC} - \zeta. \tag{28}$$

Hence we have power saving per transmission for the IC-NOMA system; now we have to find the total power saving across the system considering the required number of transmissions.

Let $l^{IC}$ be the required number of typical IC transmissions needed to satisfy the user demands. The total power saving denoted as $P^{(1)}_{Saving}$ for Case I is given by

$$\begin{aligned} P^{(1)}_{Saving} &= l^{IC} P^{IC} - l^{NOMA} P_a \\ &= (l^{IC} - l^{NOMA}) P_a + \zeta l^{IC}. \end{aligned} \tag{29}$$

### B. Case II ($l_f > l_n$)

In this scenario, we need to calculate the power per transmission of the IC-NOMA scheme to ensure that the achievable information rate for NOMA and IC transmissions is at least as good as conventional IC.

From (28) we know that the NOMA transmissions of IC-NOMA demands less power per transmission to support an information rate at least as good as conventional IC. Let $P_b^{(2)}$ be the power per IC transmission for Case II of IC-NOMA system. Then (20) is modified as

$$R^{IN\text{-}IC^{(2)}} = log_2(1 + P_b^{(2)} g_f). \tag{30}$$

From (24) and (30), it can be seen that the power per transmission required will be the same for both IC transmission of IC-NOMA and conventional IC to have equal achievable rate. Then,

$$P_b^{(2)} = P^{IC}. \tag{31}$$

Hence in the IC-NOMA system with $l_f > l_n$, we have non-zero power saving for all the $l_n$ number of NOMA transmissions and zero power savings for $l_f - l_n$ number of IC transmissions; when compared to conventional IC. In this case the total power saving denoted as $P^{(2)}_{Saving}$ for Case II is given by

$$\begin{aligned} P^{(2)}_{Saving} &= l^{IC} P^{IC} - (l_n P_a + (l_f - l_n) P^{IC}) \\ &= (l^{IC} - l_f) P^{IC} + \zeta l_n. \end{aligned} \tag{32}$$

### C. Case III ($l_f < l_n$)

The NOMA transmission of the IC-NOMA system saves power compared to conventional IC to achieve equal information rate as given by (28).

Considering $P_b^{(3)}$ as the power per IC transmission for Case III of IC-NOMA system, (22) is modified as

$$R^{IN\text{-}IC^{(3)}} = log_2(1 + P_b^{(3)} g_n). \tag{33}$$

To find the minimum power per transmission required to make the information rate for the IC transmission of IC-NOMA system at least as good as that of conventional IC, consider (24) and (33).

$$log_2(1 + P^{IC} g_f) = log_2(1 + P_b^{(3)} g_n). \tag{34}$$

Let $P^{IC} = P_b^{(3)} + \zeta_1$, where $-p_1 < \zeta_1 < p_1$, $p_1 = |P^{IC} - P_b^{(3)}|$. Then,

$$\zeta_1 = \frac{(g_n - g_f) P_b^{(3)}}{g_f}. \tag{35}$$

From (35), $\zeta_1 > 0 \Rightarrow P^{IC} > P_b^{(3)}$, i.e., power per transmission for conventional IC is greater than that of IC transmission of IC-NOMA. The power per transmission required for IC transmissions of IC-NOMA system to provide an information rate at least as good as IC is

$$P_b^{(3)} = P^{IC} - \zeta_1. \tag{36}$$

Thus for $l_f < l_n$, we have power savings in the IC-NOMA system for NOMA and IC transmissions.

To find the total power saving across the system, consider the required number of transmissions

$$\begin{aligned} P^{(3)}_{Saving} &= l^{IC} P^{IC} - (l_f (P^{IC} - \zeta) + (l_n - l_f)(P^{IC} - \zeta_1)) \\ &= (l^{IC} - l_n) P^{IC} + l_f \zeta + (l_n - l_f) \zeta_1. \end{aligned} \tag{37}$$

From (29), (32), and (37), it can be seen that for providing same information rate, the IC-NOMA system will have positive power savings when compared to conventional IC.

## VII. QUALITY OF SERVICE (QOS) REQUIREMENT ANALYSIS

This section compares the performance of the proposed IC-NOMA system with that of the conventional IC in terms of transmission power for meeting the rate required to satisfy quality of service demands of the user. The following discussions assume R as the rate requirement at each individual user to meet QoS. The minimum transmission power required to meet the QoS requirement under both schemes are analyzed here.

Let $P^{IC}$ be the power per transmission required for conventional IC to meet the QoS requirement. In IC-NOMA to meet the QoS requirement, consider the power per NOMA transmission as $P_c$ and power per IC transmission for Case II and Case III as $P_d^{(2)}$ and $P_d^{(3)}$ respectively.

### A. Conventional IC Scenario

Let $P^{IC}$ be the power per transmission for conventional IC. The achievable rates at near and far users due to IC transmission is as follows:

$$R_f^{IC} = log(1 + P^{IC} g_f), \tag{38}$$

$$R_n^{IC} = log(1 + P^{IC} g_n). \tag{39}$$

The power per transmission $P^{IC}$ required to meet QoS requirement R is given by:

$$P^{IC} = max\left(\frac{2^R - 1}{g_n}, \frac{2^R - 1}{g_f}\right) \tag{40}$$

$$= \frac{2^R - 1}{g_f}. \tag{41}$$

Let $l^{IC}$ be the required number of transmissions in conventional IC scenario. The total power requirement for each transmission to meet the QoS requirement is

$$P_{total}^{IC} = \frac{2^R - 1}{g_f}\, l^{IC} = P^{IC}\, l^{IC}. \tag{42}$$

### B. IC-NOMA Scenario

*1) Case I ($l_f = l_n = l^{NOMA}$):* The far and near users satisfy their demands using $l^{NOMA}$ number of NOMA transmission only. The achievable rates at near and far users are given respectively as

$$R_n^{IN-NOMA} = log(1 + \alpha P_c g_n), \tag{43}$$

$$R_f^{IN-NOMA} = log\left(1 + \frac{(1-\alpha) P_c g_f}{\alpha P_c g_f + 1}\right). \tag{44}$$

Let $P_{c_n}$ and $P_{c_f}$ be the minimum transmission powers required to meet the QoS requirement $R$ at near and far users, respectively. Then,

$$P_{c_n} = \frac{2^R - 1}{\alpha g_n} = \frac{P^{IC} g_f}{\alpha g_n}, \tag{45}$$

$$P_{c_f} = \frac{2^R - 1}{g_f(1 - \alpha - \alpha(2^R - 1))}$$

$$= \frac{P^{IC}}{1 - \alpha - g_f \alpha P^{IC}}. \tag{46}$$

Hence the transmission power $P_c$ required to meet QoS requirement R for both far and near users is

$$P_c = max\left(\frac{P^{IC} g_f}{\alpha g_n}, \frac{P^{IC}}{1 - \alpha - g_f \alpha P^{IC}}\right). \tag{47}$$

In this case the total power requirement of IC-NOMA system with $l^{NOMA}$ number of NOMA transmissions is given by

$$P_{total}^{IC-NOMA^{(1)}} = P_c\, l^{NOMA}. \tag{48}$$

The power per NOMA transmission $P_c$ will be always greater than $P^{IC}$ due to superposition coding. Now whether IC-NOMA can provide power saving compared to IC depends upon how much $l^{NOMA}$ is smaller than $l^{IC}$. It can be seen that for the case $l^{NOMA} = l^{IC}$, IC-NOMA requires more power than conventional IC.

*2) CASE II ($l_f > l_n$):* The far and near users receive NOMA and index coded transmissions to meet their demands. There will a total of $l_f$ number of transmissions in IC-NOMA system, with $l_n$ number of NOMA transmissions and $l_f - l_n$ number of IC transmissions. The transmission power required for IC transmission of IC-NOMA to meet QoS is given as

$$P_d^{(2)} = P^{IC} = \frac{2^R - 1}{g_f}. \tag{49}$$

In this case, the total power requirement of the IC-NOMA system to meet the QoS requirement at each transmission is given by

$$P_{total}^{IC-NOMA^{(2)}} = P_c\, l_n + P_d^{(2)}\,(l_f - l_n)$$

$$= P_d^{(2)}\, l_f + (P_c - P_d^{(2)})\, l_n. \tag{50}$$

Here also power saving of IC-NOMA depends on the values of $l_f$ and $l_n$. There could be power saving when $l_f$ is less than $l^{IC}$. When $l_f = l^{IC}$ and $l_n = 0$, the power required by IC-NOMA system will be same as that required by conventional IC. But when $l_f = l^{IC}$ and $l_n \neq 0$, the total power required by IC-NOMA system will be more than conventional IC.

*3) CASE III ($l_f < l_n$):* This scenario of the IC-NOMA system consists of both NOMA and index coded transmissions. The far users satisfy their respective demands from NOMA transmissions, whereas near users require both NOMA and index coded transmissions to satisfy the respective demands. In this case, IC-NOMA system will have a total of $l_n$ transmissions with $l_f$ NOMA transmissions and $l_n - l_f$ number of index coded transmissions.

The transmission power required for IC transmission of IC-NOMA to meet QoS is given as

$$P_d^{(3)} = \frac{2^R - 1}{g_n} = P^{IC}\, \frac{g_f}{g_n}. \tag{51}$$

Since $\frac{g_f}{g_n} < 1$, the power required per IC transmission of IC-NOMA system is less than that required by conventional IC to meet the QoS. The total power consumption is calculated as

$$P_{total}^{IC-NOMA^{(3)}} = P_c\, l_f + P_d^{(3)}\,(l_n - l_f)$$

$$= (P_c - P_d^{(3)})\, l_f + P_d^{(3)}\, l_n. \tag{52}$$

In this case also, the power saving of IC-NOMA compared to IC depends upon the individual values of $l_f$ and $l_n$. But in this case there could be power saving even when $l_n = l^{IC}$, since $P_d^{(3)} < P^{IC}$.

## VIII. SELECTION OF INDEX CODE FOR THE FAR USERS

The discussions on transmit power requirement for the QoS in Section VII, throws light on to the fact that the proposed algorithm for IC-NOMA can be modified further to improve the power efficiency. From the power efficiency calculations, the following two conclusions can be drawn:

- For the cases $l^{IC} = l_f = l_n$ and $l^{IC} = l_f > l_n \neq 0$, the IC-NOMA system demands more power than conventional IC. In this case, it is beneficial to proceed with conventional IC.

TABLE IX
THE SET OF POSSIBLE INDEX CODE DESIGNS FOR IC-NOMA SYSTEM BY CONSIDERING WANT SET DISTRIBUTION AS IN TABLE VIII AND KNOWN SET AS IN TABLE V

| | Index codes designed. | |
|---|---|---|
| $l_f$ & $l_n$ | $\mathcal{Y}_f$ | $\mathcal{Y}_n^c$ |
| $l_f = 3$, $l_n = 3$ | $\{x_1, x_2 + x_7, x_3\}$ | $\{x_2 + x_5, x_3 + x_6, x_4 + x_1\}$ |
| $l_f = 3$, $l_n = 2$ | $\{x_1 + x_6, x_2 + x_7, x_3 + x_6\}$ | $\{x_2 + x_5, x_4\}$ |
| $l_f = 3$, $l_n = 1$ | $\{x_1 + x_7, x_2 + x_5, x_3 + x_6\}$ | $\{x_4 + x_1\}$ |

- For all other cases, the power saving of IC-NOMA is directly depending upon the individual values of $l_f$ and $l_n$.

Moreover, for a given $l_f$ the scheme with minimum value for $l_n$ can minimize the overall power consumption. Considering this fact, it is required to arrive at an index code design that minimizes $l_n$ for a given $l_f$. For a given $l_f$ there can be many possibilities for far user index code design. Since actual design of far user index code decides the coded side information of the near user, the length $l_n$ of the near user index code depends heavily on the actual design of index code for far user. This can be made clear by considering Example 4.

*Example 4:* Consider an IC problem with $N = 5$, let the known set distribution be the same as in Table V and the want set distribution be as in Table VIII. Table IX shows the set of possible index codes designed for scenario under consideration. It can be seen that the optimal length of conventional index code for this problem is $l^{IC} = 4$.

Table IX shows the variation in length of near user index code according to the change index code design for far users. From Table IX it can be seen that the length of index code for near user $l_n$ varies from 3 to 1 for the same value of $l_f$.

Hence there is a need to design index code for far users by considering the index coding problem of near users, such that the solution to the far user index coding problem minimizes the index code length of the near users. Thus the proposed algorithm for IC-NOMA is modified by considering the above insights. The modified algorithm is presented as Algorithm 2.

Consider $l_f$ as the optimal index code length corresponding to the index coding problem of far users. Let $\mathcal{L}_f$ be the set of all possible optimal index codes designed for far users. The index code $\mathbf{L}_f^j \in \mathcal{L}_f$ with $j \in [|\mathcal{L}_f|]$ that minimize $|\mathbf{y}_n^{cj}|$ can give the best performance in terms of power efficiency. Algorithm 2 gives the modified design of index code for IC-NOMA.

*Remark 1:* By selecting the optimal index code for far users that minimizes the index code length of near users as in Algorithm 2, the above mentioned cases $l^{IC} = l_f = l_n$ and $l^{IC} = l_f > l_n \neq 0$, will be changed to $l^{IC} = l_f > l_n = 0$, which is same as conventional IC.

**Algorithm 2:** Selection of index code for far users.

```
Require: $g_i, \mathcal{K}_i$ and $\mathcal{W}_i\ \forall i \in [N]$, $x = [x_1\ x_2 \ldots x_n]^T$, $N$.
$g_{max} = max_{i \in [N]}(g_i)$.
$g_{min} = min_{i \in [N]}(g_i)$.
$\mathcal{I}_f = [N]$.
$\mathcal{I}_n = [N]$.
for i=1 to $N$ do
    if $|g_{max} - g_i| > |g_{min} - g_i|$
        $\mathcal{I}_n = \mathcal{I}_n \backslash \{i\}$.
    else
        $\mathcal{I}_f = \mathcal{I}_f \backslash \{i\}$.
    end if
end for
$N_f = |\mathcal{I}_f|$, $N_n = |\mathcal{I}_n|$.
$\mathcal{K}_f = \{\mathcal{K}_i : \quad i \in \mathcal{I}_f\}$.
Let $l_f$ be optimal index code length considering IC problem for the far users.
$\mathcal{L}_f = \{\mathbf{L}_f^j : \quad \mathbf{L}_f^j = \mathcal{F}(n, |\mathcal{I}_f|, \mathcal{K}_f, \mathcal{W}_f)$; such that length of the code is $l_f\}$.
for $j = 1 : |\mathcal{L}_f|$ do
    $\mathbf{y}_f^j = \mathbf{L}_f^j \mathbf{x}$;  $\mathcal{Y}_f^j$ be the set corresponding to the vector $\mathbf{y}_f^j$.
    $\mathcal{K}_i^j = \{\mathcal{K}_i \cup \mathcal{Y}_f^j : \quad i \in \mathcal{I}_n\}$.
    $\mathcal{K}_n^j = \{\mathcal{K}_i^j : \quad i \in \mathcal{I}_n\}$.
    Let the set of demands of near user $V_i$ satisfied by coded side information of far users is denoted as $\mathcal{W}_i^{cj}$.
    $\mathcal{W}_n^j = \{\mathcal{W}_i \backslash \mathcal{W}_i^{cj} : \quad i \in \mathcal{I}_n\}$.
    Design encoding matrix as
    $\mathbf{L}_n^{cj} = \mathcal{F}(n, |\mathcal{I}_n|, \mathcal{K}_n^j, \mathcal{W}_n^j)$.
    $\mathbf{y}_n^{cj} = \mathbf{L}_n^{cj} \mathbf{x}$.
end for
Choose $\{\mathbf{L}_f^{j^*}, \mathbf{L}_n^{cj^*}\}$ pair for which the length of $\mathbf{y}_n^{cj^*} = \mathbf{L}_n^{cj^*} \mathbf{x}$ is minimum.
return $\mathbf{y}_f^{j^*}, \mathbf{y}_n^{cj^*}$.
```

TABLE X
THREE DIFFERENT CASES OF IC-NOMA SYSTEM BY CHOOSING DIFFERENT DEMAND SET AND CHOOSING THE SAME KNOWN SET

| User | Known set | Demand set $\mathcal{D}_i$ | | |
|---|---|---|---|---|
| $V_i$ | $\mathcal{K}_i$ | $l_f = l_n$ | $l_f > l_n$ | $l_f < l_n$ |
| $V_1$ | $\{x_1, x_2, x_3\}$ | $\{x_1, x_2, x_4, x_7\}$ | $\{x_4, x_7\}$ | $\{x_4, x_5, x_6\}$ |
| $V_2$ | $\{x_2, x_3, x_4\}$ | $\{x_2, x_3, x_5\}$ | $\{x_2, x_3, x_4, x_5\}$ | $\{x_1, x_3\}$ |
| $V_3$ | $\{x_3, x_4, x_5\}$ | $\{x_2, x_3, x_4, x_5, x_6\}$ | $\{x_1, x_2, x_6\}$ | $\{x_1, x_2, x_3, x_5, x_7\}$ |
| $V_4$ | $\{x_5, x_6, x_7\}$ | $\{x_1, x_5, x_6, x_7\}$ | $\{x_2, x_3, x_7\}$ | $\{x_5, x_6\}$ |
| $V_5$ | $\{x_6, x_7\}$ | $\{x_3, x_6\}$ | $\{x_1, x_6\}$ | $\{x_3, x_6, x_7\}$ |
| $V_6$ | $\{x_7\}$ | $\{x_1, x_7\}$ | $\{x_1\}$ | $\{x_3, x_7\}$ |

## IX. ILLUSTRATION OF RATE, TRANSMISSION POWER AND QOS ANALYSIS

In this section, we attempt to quantify the improvement in performance gain of the proposed IC-NOMA system in terms of achievable information rate and power saving analytically with example. The example considered for analytical studies is the VANET scenario with $N = 6$ having same known set with different demand set conditions as given in Table X. It can be seen that for all the three want set distributions considered in the example, the length of the optimal linear index code for conventional IC is same given as $l^{IC} = 4$.

Table XI shows the index code design for three want set conditions developed through Algorithm 2 of the proposed IC-NOMA scheme.

TABLE XI
INDEX CODES DESIGNED FOR THREE DIFFERENT CASES OF IC-NOMA SYSTEM

| Case | $\mathcal{Y}_f$ | $\mathcal{Y}_n^c$ |
|---|---|---|
| $l_f = l_n$ | $\{x_1 + x_7, x_3 + x_6\}$ | $\{x_2 + x_4, x_4 + x_5\}$ |
| $l_f > l_n$ | $\{x_1 + x_7, x_2 + x_5, x_6 + x_3\}$ | $\{x_4 + x_1\}$ |
| $l_f < l_n$ | $\{x_3 + x_7\}$ | $\{x_1 + x_4, x_5 + x_2, x_6 + x_3\}$ |

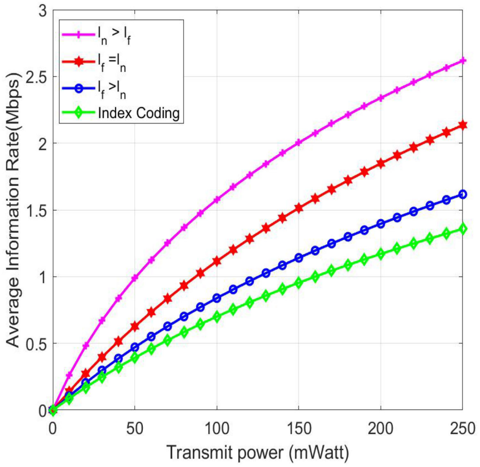

Fig. 3. Comparison of proposed system with conventional IC to show the improvement in average information rate for three different cases as given in Table X.

### A. Rate Analysis

Fig. 3 compares the average achievable rate of the proposed scheme as discussed in Section V considering a typical index coding scenario. In each IC-NOMA scenario, a total of $l^{IC-NOMA}$ transmission will be there with $l^{NOMA}$ NOMA transmissions and $l^{IC-NOMA} - l^{NOMA}$ IC transmissions. The average information rate, $R_{avg}$ for each case of IC-NOMA system is calculated as

$$R_{avg} = \frac{l^{NOMA} R^{IN-NOMA} + (l^{IC-NOMA} - l^{NOMA}) R^{IN\text{-}IC^{(i)}}}{l^{IC-NOMA}}.$$

We consider $R^{IN\text{-}IC^{(i)}}$ for $i = 2$ and $i = 3$ as the information rate of index coded transmission of IC-NOMA system for Case II and Case III respectively. The system performance varies with the lengths of index codes designed for far and near users. The plot shows that in IC-NOMA, the average information rate is high for $l_f < l_n$ when compared to the other other two cases. This is because, the information rate for the IC part of the IC-NOMA system for $l_f < l_n$ will be determined by the channel gains of near users.

For the case $l_f > l_n$, both the far and near users need the NOMA part and IC part to satisfy their demands. The information rate for the IC part of the IC-NOMA system for $l_f > l_n$ will be determined by the channel gains of far users. Hence the information rate, in this case, will be less than that of the case $l_f < l_n$.

In IC-NOMA, for $l_f = l_n$, the far and near users satisfy their respective demands by NOMA transmissions only. Each NOMA transmission, in this case, would have a higher achievable rate than IC transmissions in the $l_f > l_n$ scenario. Hence IC-NOMA system with $l_f = l_n$ will outperform the scenario $l_f > l_n$ in terms of achievable information rate.

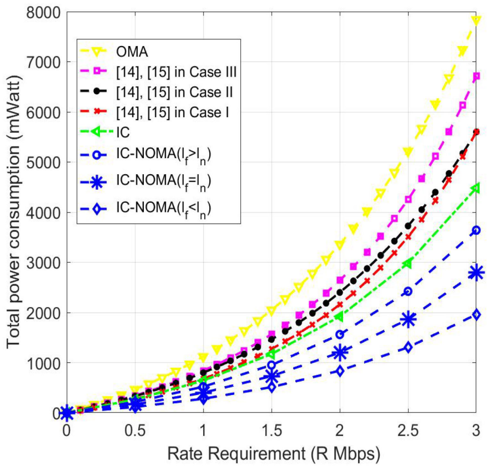

Fig. 4. The comparison of total power consumption for the proposed IC-NOMA system with conventional IC, OMA and transmission scheme in [14], [15] to meet the QoS requirement (R) at individual user.

### B. QoS Requirement Analysis

This section graphically illustrate the power efficiency of proposed scheme compared to OMA, transmission scheme in [14] and [15] and conventional IC to meet minimum the QoS requirements as discussed in Section VII by considering the example scenario given by Table X.

Fig. 4 clearly shows that the proposed IC-NOMA provides improved power efficiency while meeting the QoS requirements of various users.

### C. Effect of Power Allocation Factor $\alpha$ on QoS Analysis of IC-NOMA System

From Section VII, it is clear that the total power needed for achieving the QoS requirements depends on the power allocation factor $\alpha$ and rate requirement (R).

The power requirement of conventional IC is independent of the power allocation factor, whereas the power requirement for NOMA transmissions in IC-NOMA varies with respect to $\alpha$. It is clear from the Fig. 5 that just like the conventional NOMA, the optimal selection of power allocation factor is important for our proposed system to increase the power efficiency.

Even though the value of optimal $\alpha$ depends on the channel gains of the users, the value of $\alpha$ generally varies between 0.2 and 0.3 for a conventional NOMA with 2 power levels as discussed in [13]. Hence in this analysis we have quantified the power efficiency of proposed IC-NOMA for $\alpha$ values of 0.2 and 0.3.

### D. IC-NOMA on Symmetric Neighboring and Consecutive Side Information Single Unicast Index Coding Problem (SNC-SUICP) [20]

In this section we considered a popular IC problem called symmetric neighboring and consecutive side information single

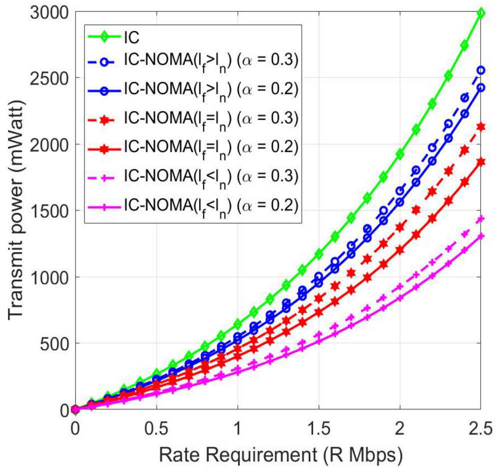

Fig. 5. The comparison of total power consumption for IC and three cases of IC-NOMA system as given in Table X for different power allocation factors ($\alpha$).

TABLE XII
TABLE REPRESENTING SNC-SUICP WITH $N = 6$ AND $D = 2$

| User $V_i$ | Want set $\mathcal{W}_i$ | Known set $\mathcal{K}_i$ |
|---|---|---|
| $V_1$ | $x_1$ | $\{x_2, x_3\}$ |
| $V_2$ | $x_2$ | $\{x_3, x_4\}$ |
| $V_3$ | $x_3$ | $\{x_4, x_5\}$ |
| $V_4$ | $x_4$ | $\{x_5, x_6\}$ |
| $V_5$ | $x_5$ | $\{x_6, x_1\}$ |
| $V_6$ | $x_6$ | $\{x_1, x_2\}$ |

unicast index coding problem (SNC-SUICP) [20] with number of users as $N$ and side information cardinality as $D$. In this case the optimal index code length is given by $l^{IC} = N - D$. Table XII represents SNC-SUICP with $N = 6, D = 2$ and $l^{IC} = 4$. When conventional OMA scheme is applied over the problem we need 6 trivial transmissions to serve the user demands. We have shown that applying the proposed IC-NOMA scheme over the problem reduces the number of transmissions required by the system to serve the user demands and reduces the total power consumption.

To apply IC-NOMA principles over the problem, consider the users $V_1, V_2, V_3$ as the near users; and the users $V_4, V_5, V_6$ as the far users. The index coded transmissions for far users are given by $y_{f_1} = x_6 + x_2, y_{f_2} = x_5 + x_1, y_{f_3} = x_4 + x_6$. Hence the far user index code length is $l_f = 3$. The index code length of near users is $l_n = 1$ and corresponding index code for near users is given by $y^c_{n_1} = x_3 + x_5$.

The mode selection scheme proposed in [14], [15] requires three cached NOMA transmissions for the problem under consideration. This is because none of the user pairs in SNC-SUICP has cached each other's data.

Figs. 6 and 7 indicate the performance gain of the proposed IC-NOMA system in terms of the reduced number of transmissions required and power savings of the system to meet the QoS requirements of users.

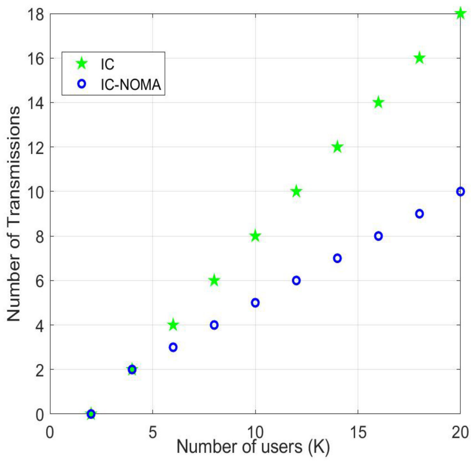

Fig. 6. The plot showing the reduction in number of transmission when IC-NOMA principles are applied over SNC-SUICP with $D = 2$.

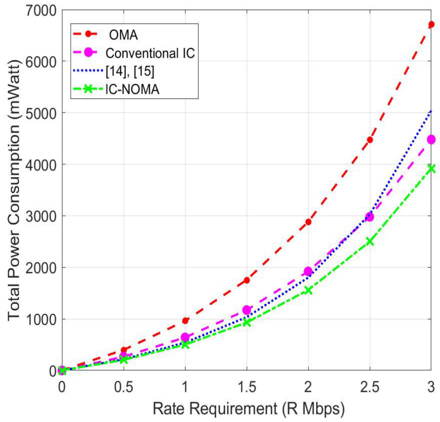

Fig. 7. Plot showing the improved power efficiency of proposed IC-NOMA for SNC-SUICP with $N = 6$ and $D = 2$.

## X. CONCLUSION

In this paper, we proposed a spectral efficient transmission strategy called IC-NOMA for VANETs. IC-NOMA improves spectral efficiency by linearly superposing index coded packets in the power domain at the transmitter and decoding the desired data by exploiting the side information and SIC at the receiver. We showed that IC-NOMA requires a unique index code design to fit in with the NOMA scenario and developed an algorithm to design the same. The scenarios where IC-NOMA offer improved bandwidth efficiency compared to IC are studied. Based on this, the power saving of the proposed system is discussed by analytical studies.

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**Sreelakshmi Pazhoor** (Student Member, IEEE) was born in Kerala, India. She received the B.Tech. degree in electronics and communication engineering in 2016 from the NSS College of Engineering, Palakkad, Kerala, and the M.Tech. degree in communication engineering and signal processing in 2018 from Government Engineering College, Thrissur, Kerala. She is currently working toward the Ph.D. degree with the Department of Electronics and Communication Engineering, National Institute of Technology, Calicut, Kozhikode, Kerala. Her primary research interests include non-orthogonal multiple access, index coding, and vehicular communications.

**Jesy Pachat** (Member, IEEE) was born in Kerala, India. She received the B.Tech. degree in electronics and communication engineering, in 2004, from Mahatma Gandhi University, Kottayam, Kerala, and the M.Tech. degree in telecommunication from the National Institute of Technology, Calicut, Kozhikode, Kerala, in 2014. Since 2010, she has been a Faculty in Govt. college of engineering Kannur. She is currently working toward the Ph.D. degree with the Department of Electronics and Communication Engineering, National Institute of Technology, Calicut. Her primary research interests include network coding, index coding, and coding theory.

**Anjana Ambika Mahesh** (Student Member, IEEE) was born in Kerala, India. She received the B.Tech. degree in electronics and communication from Kerala University, Thiruvananthapuram, Kerala, and the master's degree in telecommunication from the Department of ECE, IISc and has previously held positions with Qualcomm India Private Limited, Bangalore and Lekha Wireless Solutions, Bangalore. She is currently working toward the Ph.D. degree with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore. Her research interests include index coding, coded caching and their applications. She was the recipient of the Qualcomm Innovation Fellowship.

**Deepthi P. P.** (Member, IEEE) was born in Kerala, India. She received the M.Tech. degree in instrumentation from the Indian Institute of Science, Bangalore, in 1997, and the Ph.D. degree in secure communication from the National Institute of Technology, Calicut, Kozhikode, Kerala, in 2009. She is currently a Professor with the Department of Electronics and Communication Engineering, National Institute of Technology, Calicut. Her interests include cryptography, information theory, coding theory, and multimedia security, and secure signal processing.

**B. Sundar Rajan** (Fellow, IEEE) was born in Tamil Nadu, India. He received the B.Sc. degree in mathematics from the Madras University, Chennai, Tamil Nadu, India, in 1979, the B.Tech. degree in electronics from the Madras Institute of Technology, Chennai, Tamil Nadu, in 1982, and the M.Tech. and Ph.D. degrees in electrical engineering from the IIT Kanpur, Kanpur, in 1984 and 1989, respectively. He was a Faculty Member with the Department of Electrical Engineering, IIT Delhi, New Delhi, from 1990 to 1997. He has been a Professor with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, since 1998. His research interests include space-time coding for MIMO channels, distributed space-time coding and cooperative communication, coding for multiple-access and relay channels, and network coding.

Dr. Rajan was the Editor of the IEEE Wireless Communications Letters (during 2012–2015), an Associate Editor for the Coding Theory of the IEEE Transactions On Information Theory (during 2008–2011 and 2013–2015), and the Editor of the IEEE Transactions on Wireless Communications (during 2007–2011). He was a Technical Program Co-Chair of the IEEE Information Theory Workshop (ITW'02), held in Bangalore, in 2002. He is a Fellow of the Indian National Academy of Engineering, New Delhi, India, the Indian National Science Academy, New Delhi, India, the Indian Academy of Sciences, Bangalore, Karnataka, India, and the National Academy of Sciences, India. He was the recipient of the Prof. Rustum Choksi Award by I.I.Sc for Excellence in Research in Engineering for the year 2009, the IETE Pune Center's S.V.C Aiya Award for Telecom Education in 2004, and Best Academic Paper Award at the IEEE WCNC 2011. He was a J.C. Bose National Fellow (during 2016–2020) and a member of the American Mathematical Society.