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Matrix cracking in polymer matrix composites under bi-axial loading

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Abstract

A model to predict the matrix crack evolution in a continuous fiber polymer composite laminate under in-plane bi-axial static loading has been presented in the current work. Oblique co-ordinate based shear lag analysis was used to estimate the stress distribution inside the cracked $[0/90]_s$ cross-ply T300/934 carbon fiber reinforced plastic (CFRP) laminate. Weibull probability distribution has been used to account for the variation in ply transverse strength. Size-dependent strength due to variation in ply thickness has been accounted for by appropriate volume scaling based Weibull scale factor. The Weibull parameters have been estimated using a ‘master laminate’ crack evolution data. By applying incremental stress to the laminate, using the probabilistic variation of transverse strength and the stress at a material point, the new crack location has been identified using the Hashin matrix cracking criterion. The reciprocal of the normal distance between two cracks has been termed as crack density. The crack density evolution for cross-ply laminates with an increase in applied loading has been estimated for various bi-axial ratios and compared with the data available from the literature. A good correlation is found to exist between the literature evolution data and current simulation predictions.

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Keywords: Matrix cracking; Bi-axial loading; Weibull strength

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1. Introduction

The use of polymer matrix composites (PMCs) in structural airframes as compared to conventional aluminum alloys has been increasing steadily; today 50% by weight of the new Boeing 787 airframe is made from PMCs. The first indigenous Indian light combat aircraft (LCA) uses PMCs for more than 40% of its weight. Aircraft structures undergo time-varying structural loads during their operation and are subject to environmental degradation and external damage threats like impact, runway debris, hailstorm, etc. These events can cause initiation as early as the first cycle and evolution of multi-scale structural degradation steadily with loading due to their inherent inhomogeneous and distinctly anisotropic nature. The commonly observed damages are matrix cracking, interfacial fiber-matrix de-bonding, fiber breaks, delamination, fiber micro-buckling, etc., under static and fatigue loading, Berthelot (2003), Harris (2003). Matrix cracking happens to be the most dominant mode of damage to first appear in a laminate. Matrix cracking generally leads to loss of stiffness, local stress redistribution and most importantly, a path for moisture or other fluid ingress leading to further reduction in composite strength or loss of its integrity, Talreja and Singh (2012). Experimental investigation of matrix cracking and its effects on composite materials have been extensively reported and reviews on such findings are available in the literature, Berthelot (2003), Talreja and Singh (2012). Following matrix crack saturation, also referred to as ‘characteristic damage state’, delamination is observed to initiate at the matrix crack tips, growing slowly and steadily. Fiber breaking is also observed at all the above stages. At the final stages of life, linking up delamination, the complex interaction of matrix cracks and fiber breaking is observed leading to the final failure of the laminate. The damages are randomly distributed across various length scales and locations; oriented in different directions, Highsmith and Reifsnider (1982), Reifsnider and Jamison (1982).

Energy based models have been successfully used to predict the matrix crack initiation and evolution, Singh (2008). Energy-based models utilize the critical energy released during new matrix crack formation as a parameter to predict the matrix cracking, Nairn (2000). Various stress analysis methods have been developed to estimate the stresses or energy in the cracked laminate, Talreja and Singh (2012). In strength-based models, as the local stresses reach the strength at a point, a new matrix crack is assumed to initiate. Strength-based models fail to predict the crack initiation when deterministic single strength obtained from the uni-directional (UD) laminate experiments are used. However, successful prediction of matrix cracking behavior has been reported if statistical strength distribution is considered Jagannathan et al. (2016); studies have shown that the matrix cracks initiate at locations of largest voids created during the manufacturing process and the matrix crack evolution rate can be correlated to the statistical variation of flaws, Nairn (2000), Berthelot (2003), Talreja and Singh (2012). Cross-ply laminates matrix cracking has been extensively studied and reported in the literature, Berthelot (2003). Various models have been used to predict the stiffness of the cracked laminate as a function of crack density estimated using the above approaches. In the simplest model termed ply-discount method, the stiffness of cracked ply is assumed zero and discounted while estimating the stiffness of the laminate. An extensive review of different models can be found in the literature by Talreja and Singh (2012).

In most practical applications, a multi-axial loading scenario exists. Most of the models developed in the literature are limited to uni-axial loading condition and have not addressed matrix cracking under bi-axial loading. It has been reported that when a cross-ply laminate is subjected to in-plane bi-axial loading, splitting in 0° ply occurs in addition to matrix cracking in the 90° ply by Montesano and Singh (2015). In particular, such bi-axial loading conditions are expected to occur under thermal loading of composite cylindrical shells. There have been limited studies to account for cross-ply laminates under bi-axial loading conditions by Spain (1971), Adams et al. (1986), Maddocks (1995), Montesano and Singh (2015). All the models developed have used some form of energy parameter to predict the matrix cracking in the cross-ply laminate. In this work, an attempt has been made to predict the matrix cracking in cross-ply laminates using statistical strength-based approach under bi-axial loading condition.

2. Matrix crack evolution under bi-axial loading

A $2p$ ply symmetric $[0^\circ/90^\circ]_s$ laminate with an arbitrary number of 0° and 90° orientation plies have been chosen in the present study. The laminate has a thickness t and length L . The general cracking pattern in such cross-ply laminate has been shown in Fig.1. Stress analysis based on the oblique coordinate shear lag model has been adopted. Detailed stress analysis methodologies can be found in Jagannathan et al. (2016). The stress analysis of cracked laminate has been carried out using commercial software MATHEMATICA ver 9.

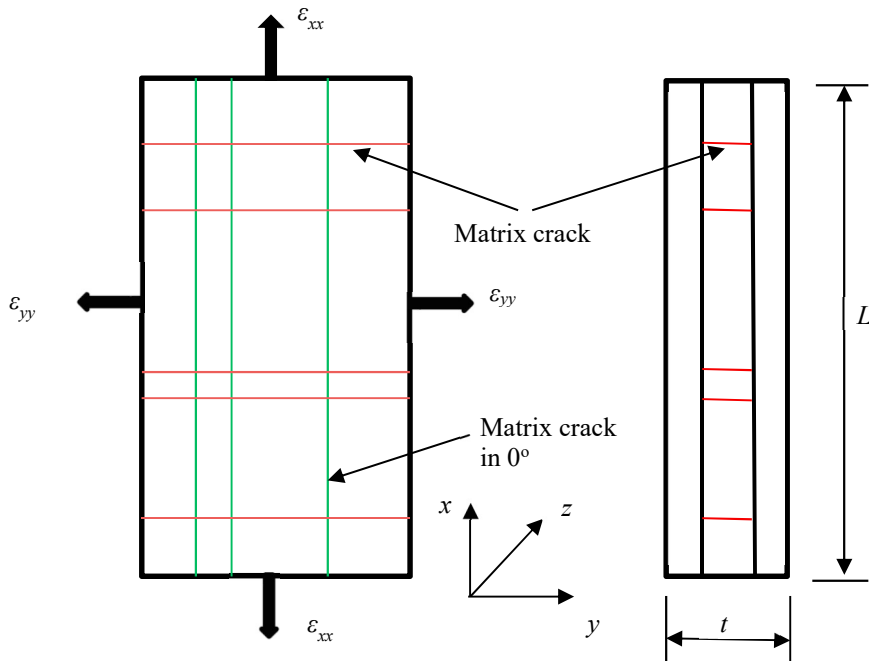


Fig. 1. Cracking of cross-ply laminate under bi-axial loading

2.1. Material properties and Weibull parameters

$[0^\circ/90^\circ]_s$ laminate made of T300/934 CFRP material has been used in this study. The mechanical properties for the above material system have been experimentally obtained by Zhang et al. (1992) as shown in Table.1.

Table 1. Material property for the laminate used in the current analysis

Property	Values
E_{11} , GPa	144.8
E_{22} , GPa	11.4
G_{12} , GPa	6.5
ν_{12}	0.3
Laminate length, mm	150
Ply thickness, mm	0.132

The transverse strength (Y_t) of the lamina typically follows a Weibull distribution, Sun et al. (2003). The Weibull probability function for failure strength of the uni-axially loaded material can be expressed as follows:

$$P(Y_t) = 1 - \exp \left[- \left(\frac{Y_t}{\beta} \right)^m \right] \quad (1)$$

Where β is the scale parameter and m is the shape parameter. The above parameters are obtained from a master laminate crack density evolution data. However, if the thickness of the ply undergoing matrix cracking is different from the ‘master laminate,’ there have been differences between predicted and experimentally observed crack density evolution curves. Suitable thickness correction has been proposed to account for such thickness effects and details can be found in Jagannathan et al., (2017). The 90° ply has been divided into D number of material elements; wherein each element has been assumed a constant strength and capable of cracking. The overall strength has been varied for each element randomly to represent the Weibull distribution of strength across the lamina.

2.2. Crack spacing distribution and stiffness degradation

The laminate has been loaded from zero in steps to estimate the crack density at every load step. The cracks are assumed to form instantaneously spanning the width and the thickness of the lamina termed as ‘tunneling cracks.’ The cracks are always forming along the fiber direction. The following form of Hashin’s strength criterion has been used in the current analysis.

$$\left(\frac{\sigma_{22}}{Y_t} \right)^2 \geq 1 \quad (2)$$

The normal distance between two adjacent cracks is termed as crack spacing. The reciprocal of crack spacing is called crack density. Due to the random nature of cracking, there have been statistical variations in the crack density at every load level applied on the laminate. The average crack density has been estimated from the crack density statistics and used to estimate the stiffness degradation. The stiffness of a laminate for a given crack density has been analytically derived and available in the literature, Yokozeki and Aoki (2005). Initial crack spacing equivalent to the length of lamina has been assumed for the analysis.

3. Results and discussion

3.1. Weibull parameters estimation from master curve

The estimation of Weibull parameters via calibration from a ‘master curve’ has been carried out using the following methodology.

- In general, experimental matrix crack evolution under static loading has been carried out by loading the specimen continuously and monitoring the crack spacing/crack density at discrete stress/strain intervals. Following the above, the simulation is also carried out by applying a discrete stress/strain increment to the laminate and estimating the crack density at those points. The stress/strain increment used for measurement of crack density in the experiment may be different from that of simulations.
- To find the correlation between the experimental and simulated curves, the following procedure has been employed. Matrix crack evolution curve can be better represented by 4th order polynomial function. All the experimental and simulated points are fitted with a 4th order polynomial, and the coefficients were estimated.
- The matrix crack density has been re-estimated using the above polynomial fit at 50 discrete points between zero to maximum strain value observed in the experiment for every simulated curve, and for the experimental curve at the same points.
- The correlation coefficient (r) between these 50 experimental (E) and simulated values (S) has been estimated using the following relation. The bar represents the mean values of the quantities.

$$r = \frac{\sum_{i=1}^{50} (E_i - \bar{E})(S_i - \bar{S})}{\sqrt{(\sum_{i=1}^{50} (E_i - \bar{E})^2)(\sum_{i=1}^{50} (S_i - \bar{S})^2)}}$$

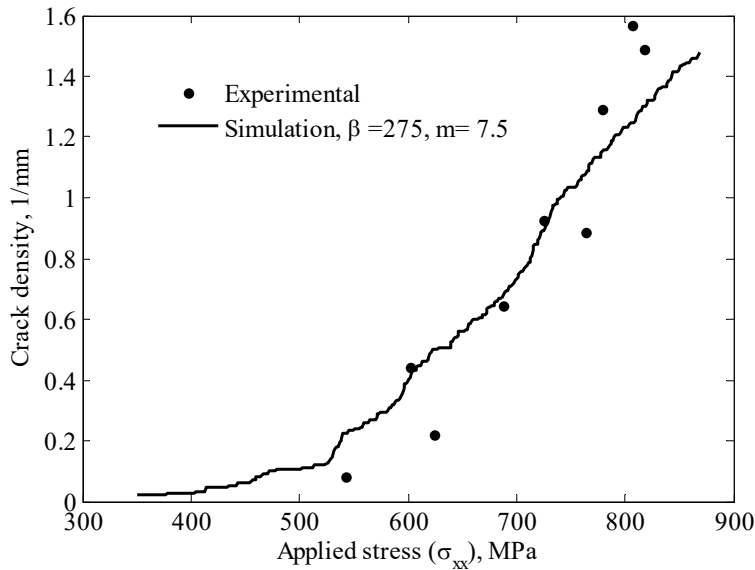


Fig. 2. Weibull parameter estimation for $[0^\circ/90^\circ]_s$ laminate under uni-axial loading. Experimental values are from (Zhang, Fan, & Soutis, 1992).

- It has been observed from the simulation that the shape parameter m controls the rate of crack evolution, and the scale parameter β decides the crack initiation strain. At the start of the analysis, an arbitrary β value was chosen and crack density curves were simulated for various values of m . All the crack density curves were overlaid with experimental data and the optimal m value corresponding to the highest correlation coefficient was chosen as the material parameter m .
- Upon fixing the m value, the optimal β was estimated using trial and error such that the simulated crack initiation point coincided with an experimental point. The above procedure is iterated to find out the optimal β and m value for the best fit that gives the highest r^2 . However, it is recognized that further studies may be needed to establish a more reliable methodology to estimate the Weibull parameters.
- These calibrated values for Weibull parameters have then been used for prediction of crack density evolution for other layups.

The estimation of Weibull parameters via calibration from a master curve has been carried out using 100,000 material elements and 1 MPa stress increment. The matrix crack evolution under uniaxial static loading on $[0^\circ/90^\circ]_s$ laminate has been carried out by Zhang et al. (1992). Using the approach described in the previous sections, the Weibull parameters have been estimated using this $[0^\circ/90^\circ]_s$ laminate data (Table. 1). The matrix crack data along with the best-fit Weibull parameter is shown in Fig. 2.

3.2. Matrix cracking under bi-axial loading

Simulations have been carried out on $[0^\circ/90^\circ]_s$ laminate under various bi-axial ratios. Experimental crack-density evolution and stiffness degradation data under biaxial loading has not been reported in the literature. Instead, matrix cracking predictions using energy based approach by Montesano and Singh (2015) has been used for comparison.

Montesano and Singh used the following approach to predict matrix cracking in cross-ply laminate under bi-axial loading:

- $[0^\circ/90^\circ]_s$ laminate uniaxial loading experimental data Zhang et al. (1992) has been used as a reference laminate.
- Micro-mechanics based finite element analysis (FEA) has been used to estimate the crack opening displacement (COD) values of the matrix crack, and the corresponding energy release rates (ERR) have been estimated.
- Using the experimental crack evolution reference curve and ERR values, critical energy release rate for initiation and statistical variation of the critical energy for cracking has been estimated using Weibull statistics.
- Using statistical variation of critical ERR, the matrix cracking has been estimated for cross-ply laminates under different bi-axial ratios.

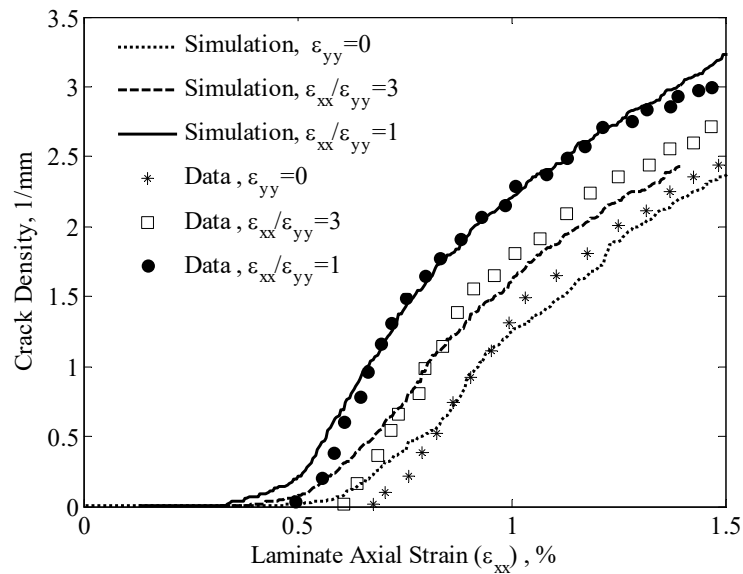


Fig. 3. Crack density evolution in a $[0^\circ/90^\circ]_s$ CFRP laminate under different bi-axial static loading, Data- (Montesano & Singh, 2015)

The matrix cracking under different bi-axial ratios have been carried out using the statistical strength-based approach described in the previous sections. The crack evolution simulation with different bi-axial loading conditions has been shown in Fig. 3. The simulations carried out by Montesano and Singh (2015) has also been superimposed for comparison. An excellent agreement can be seen in the simulations from both energy and strength based simulations. Using the crack densities predicted, the stiffness degradation has been estimated. The stiffness degradation curves under different bi-axial ratios have been shown in Fig.4-6. The following observations can be made from the stiffness degradation curves.

1. The axial modulus degradation is in the range of 5-8%. The statistical variation of modulus estimated from the uniaxial test is in the limit of <10%. Such small variations may be neglected for any design purposes. The increase in bi-axial loading does not alter degradation much, and it is less than 1-2%.
2. However, the in-plane shear modulus has a reduction of 30%, and the Poisson's ratio reduction of 40% has been observed.
3. An additional 10% reduction in in-plane shear modulus and Poisson's ratio has been observed with an increase in the bi-axial ratio of the loading.
4. The maximum degradation has been observed for equal bi-axial loading condition.

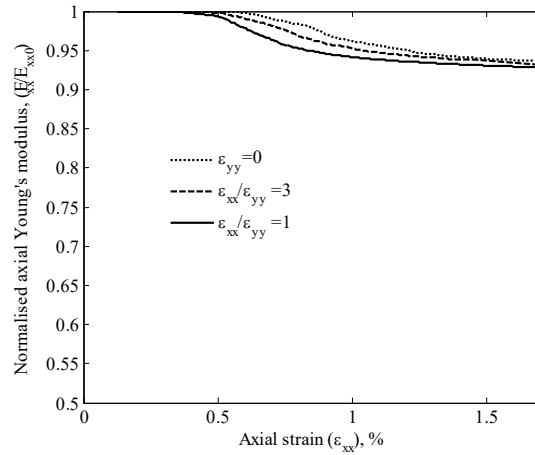


Fig. 4. Normalised axial modulus degradation in a $[0^\circ/90^\circ]_s$ CFRP laminate under different bi-axial static loading.

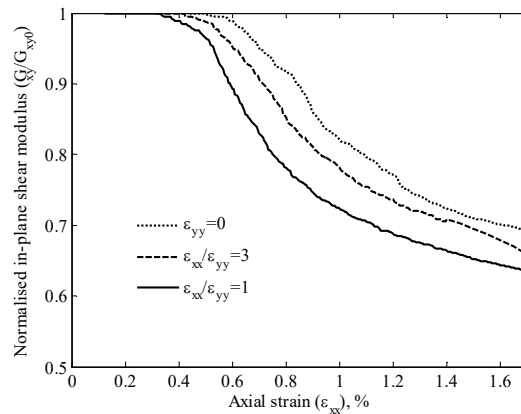


Fig. 5. Normalised shear modulus degradation in a $[0^\circ/90^\circ]_s$ CFRP laminate under different bi-axial static loading.

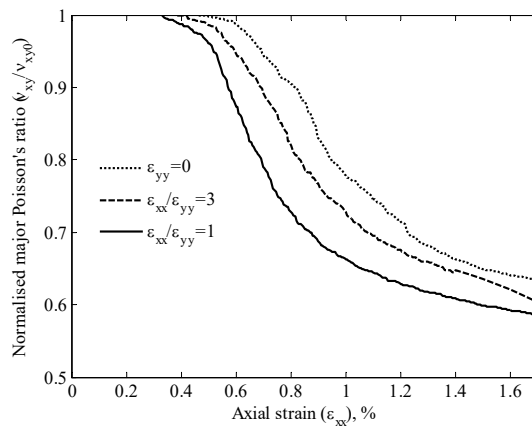


Fig. 6. Normalised Poisson's ratio degradation in a $[0^\circ/90^\circ]_s$ CFRP laminate under different bi-axial static loading.

4. Conclusions

Matrix cracking under different bi-axial ratio loading in a cross-ply laminate has been carried out using probabilistic strength-based models. The results have been compared with the energy-based simulations due to the lack of existing experimental data. The statistical strength-based methods can predict the crack evolution well compared to energy based approaches, and no significant difference has been observed. Stiffness degradation under different loading scenarios has been estimated. There has been a significant degradation in the in-plane shear modulus and Poisson's ratio values. Currently, the model has been verified with the other analytical/numerical formulations available in the literature. However, in future, the validity of the model under various biaxial loading conditions will be verified experimentally.

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