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Modelling multiply connected heterogeneous objects using mixed-dimensional material reference features

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ABSTRACT

This paper proposes a general method for ab-initio modelling and representation of heterogeneous objects that are associated with complex material variation over complex geometry.

Heterogeneous objects like composites and naturally occurring objects (bones, rocks and meteorites) possess multiple and often conflicting properties (like high hardness and toughness simultaneously), which are associated with random and irregular material distribution. Modelling such objects is desired for numerical analysis and additive manufacturing to develop bio-implants, high-performance tools etc. However, it is difficult to define and map the arbitrary material distribution within the object as the material distribution can be independent of the shape parameters or form features used to construct its solid model.

This paper represents the source of random and irregular material distribution by mixed-dimensional entities with a focus on modelling compositional heterogeneity. The domain of effect of each material reference entity is defined automatically by using Medial Axis Transform (MAT), where the material distribution can be intuitively prescribed, starting from the material reference entity and terminating at the medial axis segment bounding the corresponding domain. Within such a domain, the spatial variation of the material is captured by a distance field from the material reference entity, which can be controlled locally and independently. These domains are stored using the neighbourhood relation for efficient operations like altering material distribution across the material reference entity and material evaluation for a given geometric location. Results from an implementation for 2.5D objects are shown and the extension to 3D objects is discussed.

1. Introduction

Heterogeneous objects are being used widely in various fields like biomedical, automotive, aerospace, nuclear and naval engineering (Miyamoto, Kaysser, & Rabin, 1999; Ghosh, Miyamoto, & Reimanis 1997; Suresh & Mortensen, 1997). Such applications require the synthesis of the objects with multi-fold functionality such as, high hardness and high toughness simultaneously. Such multiple functionalities can be achieved by using a combination of materials exhibiting different properties. These combinations could be of two kinds: discrete changes in material (multi-material) and smooth variation in material composition (functionally graded distribution) as shown in Fig. 1.

The discrete change in material distribution results in abrupt changes in material property across the common boundary of two sub-domains, where each sub-domain is associated with a unique material composition; this results into undesirable effects like thermal stress, and initiation of cracks. However, smooth variation in the material properties overcomes these limitations.

Heterogeneous objects need a computer model referred to as Heterogeneous Object Model (HOM) for computational analysis and structural optimization. Further, these models serve as input to the process planning task in Additive Manufacturing (AM) to realize the object. In a canonical AM process, the extrusion nozzle of the machine traverses over a geometric location; it queries the material details from the model that needs to be deposited at the site.

For various applications, the material variation within a geometric domain can be intricate. For example, design and manufacturing of implant devices and scaffolds (Hollister, 2005) need a model to tailor various conflicting properties like high hardness and high toughness, biocompatibility, and other bio-factors; drug delivery devices and wound covers (Liu, Maekawa, Patrikalakis, Sachs, & Cho, 2004) need the release of multiple drug molecules that can be controlled by defining various composition profiles of

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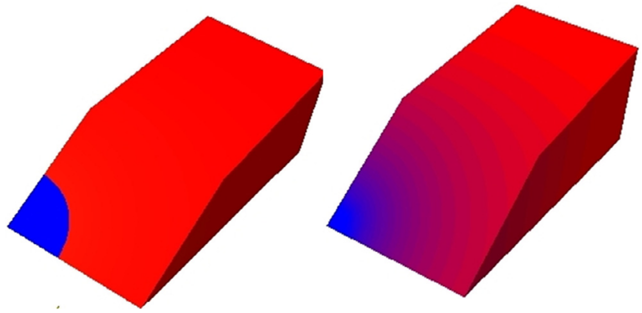

Fig. 1. Cutting tool having (a) Discrete material distribution, (b) Smooth material Distribution.

the drug molecules locally within different sub-domains; numerical analysis and simulation of fault-lines and rocks in the area of geo-science (Liu & Xing, 2013; Xing, Yu, & Zhang, 2009), simulation of growth of the organs for plants and animals (Durikovic, Czanner, Parulek, & Srámek, 2008) require the model to capture various properties across the layers and around different non-manifold entities. The main challenge in modelling these is that they are associated with random material distribution as shown in Fig. 2 where the material variation in material composition due to points and boundary edges are shown using different colours.

In the current state of art, heterogeneous objects have been modelled using evaluated or unevaluated models (Kou & Tan, 2007). An evaluated model or representation is one where the information available in the model is directly usable by applications. Evaluated models are required for numerical analysis and simulation. These models decompose the geometry of the object into simple cells and define the material composition within each cell; unevaluated model maps the material function to the geometry of the object. Evaluated models have the potential to represent the intricate heterogeneous objects by simplifying the geometric domain through the subdivision and associating the desired material composition to the subdivided domain. However, decomposition of the geometric domain based on the material distribution is an issue of ongoing research (Kumar, Burns, Dutta, & Hoffmann, 1999). In addition, these models are difficult to use, and suffers from the discretization error, lack of representational compactness and inefficient material interrogation. Unevaluated models, in contrast, carry information from which the data required by the application can be generated. Unevaluated models

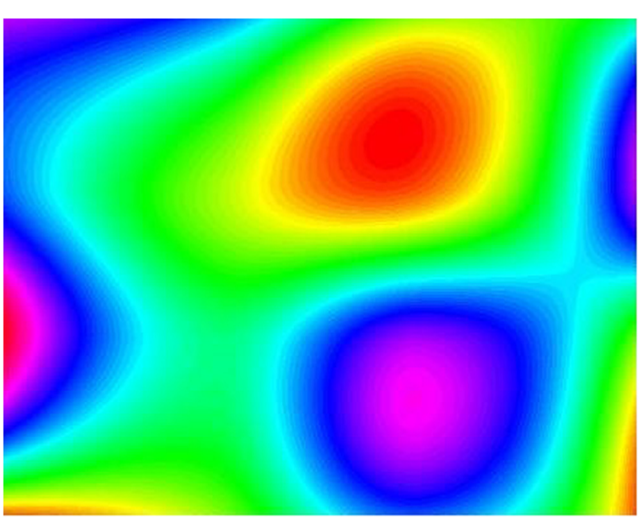

Fig. 2. Local material variations in different regions.

are easy to use, analytic and compact in representation but these models can model only a limited range of heterogeneous objects as these models define material functions mainly in terms of shape parameters, or form features. However, complex heterogeneous objects can have material distribution independent of the shape parameters or topology of the object. Unevaluated models using a weighted distance to represent the material distribution (Liu et al., 2004; Jackson, 2000; Liu et al., 2004; Siu & Tan, 2002; Shin & Dutta, 2001) blend the material composition at a point that is contributed by each material feature resulting into global effect in material property distribution. However, locally controlled material composition using such method is difficult to be achieved because of the difficulty in choosing the appropriate weight and its domain of effect corresponding to each material feature. For example in the domain in Fig. 2, arbitrary material features such as points or curves not related to any geometric feature cannot be supported till they are added to the geometric model. As mentioned earlier, non-manifold entities (point, curve) are not represented as part of the 3D model in most CAD systems. Moreover, these approaches cannot represent a situation where the material composition at a point is only a function of some of the material features and some material features do not affect the composition in some regions.

In this paper, we extend the hybrid representation described in (Sharma & Gurumoorthy, 2017) to model complex heterogeneous objects. The main contribution of this paper lies in capturing and defining intricate material distribution by using mixed dimensional entities as material reference entities without being constrained by the shape or topology of the solid model, and tailor the material composition locally by using another reference entity called Medial Axis Transform (MAT), which uses proximity information to subdivide the domain of influence of material feature and uses rails derived from MAT to serve as distance function for material blending. In addition, this method possesses all the advantages of hybrid representation that include adaptive discretion based on the material distribution, efficient material interrogation for numerical analysis and manufacturing planning.

2. Literature review

HOM defines a material function over a geometric domain and represents them in either discrete (evaluated model) or continuous form (unevaluated model) as described by Kou and Tan (2007). Evaluated model subdivides a region exhaustively into regular sets and defines the material function over each regular set. Material functions are either a distance function from material reference features (principal axes, planes; cylindrical axis, sphere center etc) or analytic functions resulting from optimization and simulation.

Evaluated Models: Jackson (2000) divided a complex solid using tetrahedron decomposition methods and in each decomposed subregion, the material composition is obtained using analytic blending functions. Cho and Ha (2002) used quadratic elements to discretize the geometric domain and used an optimization scheme to determine the material composition of each element for relaxing the thermal stress.

Unevaluated Models: Unevaluated model defines material composition directly over the geometric domain without prior decomposition. Geometric domain has functional representation (B-rep, f-rep). Material representation is a distance-based function defined with respect to reference features or any other analytical function.

Samanta and Koc (2005) used surface parameters to define material functions for free-form surfaces. Qian and Dutta (2002) used form features in the solid model to define material features.

Biswas, Shapiro and Tsukanov (2004) proposed an intuitive means of modeling desired material distribution using distance function from material reference features. Based on generalized Taylor series expansion, a distance-based function with a canonical form is used to formulate the material distribution.

Kou and Tan (2005) proposed a hierarchal representation of HOM. They introduced a Hierarchal Feature Tree [HFT] structure to represent the different types of material gradations. HFT structure stored the geometric and material transition from one-dimensional entity to two-dimensional entity and from two-dimensional entity to three-dimensional entity.

Ozbolat and Koc (2011) presented a feature-based method to represent and design heterogeneous objects with material composition varying along multiple directions. They constructed the Voronoi diagram and the variation of the material composition from the bounding curve to matching internal curve is obtained using an optimization approach. The optimization approach uses visibility constraints to match point on one bounding curve to another internal curve. These matching lines (ruling lines) do not represent the minimum distance from the bounding curve. So, the distance-based material function cannot be mapped along the ruling lines. In addition, Voronoi diagram is not suitable to map across multiple features because many Voronoi segments will get generated depending on the sampling of points on bounding curve and some Voronoi segments may lie far from the surface of the object in three-dimensions.

Rvachev, Sheiko, Shapiro and Tsukanov (2001) utilized the theory of R-functions to construct smooth approximations to distance function for semi-analytic features. Pasko, Adzhiev, Schmitt and Schlick (2001) represented heterogeneous object using constructive tree built using the functional representation (F-rep) of primitives forming the leaves and operations forming the internal nodes of the tree. Frep is used as the basic model for point set geometry. Material composition is represented independently as attributes, using real-valued scalar functions.

Liu et al. (2004) proposed a feature-based method to model local composition control for HOM. Several material features like volume, surface, pattern, and transition were identified which used a function of distance from user-defined geometric features for editing and controlling material variation. Physics-based blending was used to smoothly vary the material across different boundaries. Siu and Tan (2002) also employ a weighted function of the distance from material feature(s) to describe the material composition at a point.

Liu, Duke and Ma (2015) developed a concurrent modeling approach to define heterogeneous objects using CAD and CAE modules. It uses CAD module to prescribe geometry and CAE module to define local material composition over a discrete geometry. The correspondence between these two modules is achieved through an associative feature model, supported by level set optimization to achieve the desired heterogeneous object.

Limitations of the current art can be summarized as follows:

1. Evaluated models use a mesh or voxel representation to approximate the geometry. It is difficult to prescribe the material distribution using geometric form features (like holes), or user-defined material reference entities as the geometric information about the features is lost after discretization. In addition, representation of material composition using these models depends on the resolution of the mesh or voxels that may not conform to the material distribution, leading to discretization error. In addition, any change in material function would lead to re-discretization of the whole geometry to re-approximate the new material distribution.
2. Unevaluated models are limited to modelling simple geometries and material variation. Most unevaluated models (Samanta & Koc, 2005; Qian & Dutta, 2002; Kou & Tan, 2005) use extant shape parameters or geometric form features to model material distribution. This ends up in restricting the material distribution to be defined according to the geometric form. This limits the user's freedom to design material distribution that does not depend on the geometric entities present. Also, the weighted distance-based blending approach (Liu et al., 2004; Jackson, 2000; Liu et al., 2004; Siu & Tan, 2002; Sharma, 2015) using multiple reference features is not suitable for a domain having multiply connected components and mixed dimensional reference features. This is because this approach considers the contribution of each feature for blending the material composition at a point; resulting in the influence of a material feature being global rather than local.

Various constructive representations (Pasko et al., 2001; Shin & Dutta, 2001; Kou, Tan & Sze, 2006) have been developed to build more intricate geometry and material distribution using these unevaluated models. But it is difficult to define the geometric and material primitives, and boolean set operations to be used to build a given HOM. In addition, the range of shapes and material distributions achievable get highly restricted with the use of Boolean set operations.

3. Background:

3.1. Material feature

According to Bidarra and Bronsvoort (2000), a feature is defined as a representation of the shape aspect of a product that is mappable to generic shape and functionally significant for some product lifecycle phase. Feature based design is more popular because features can be modified according to designer's intent to generate different instances of a design that makes designing convenient and fast. For example, a through hole is a feature on a cube where hole is generated through the subtraction of cylinder from cube (cylinders and cube are generic shapes). Varying the radius of cylinder or the size of the cube can generate variant models.

Similarly, material feature can be defined as representation of material distribution mappable to the given object. In this paper, material features use a material reference entity and distribute material as a function of distance from the reference entity. Thus, a material feature consists of two attributes: material reference entity and material distribution function.

Material reference entities may use entities in the boundary representation or mesh (Cheng, Dey, & Shewchuk, 2012). These may also be features/entities that are not otherwise present in the shape model These new reference entities may be point (0-D), line/curve (1-D), or plane/surface (2-D). The material distribution function can be polynomial, exponential, harmonic functions of the distance from material reference entities.

Material features are useful to generate variant material distribution.

3.2. Material composition vector and material composition function

Material composition vector (Siu & Tan, 2002) represents volume fraction of each constituent material at any point in the part, where the volume fraction of the constituent material forms the basis of the material space.

At any point (x, y, z), material composition vector is $[m_1, m_2, m_3, \ldots m_n]$ subject to the constraint of $m_1 + m_2 + m_3 + \ldots m_n = 1$ where n is the number of constituent materials and m_i is the volume of fraction of the i^{th} constituent material.

Hence, at any point P (x, y, z), the material composition vector is given by $[m_1, m_2, m_3, \ldots m_n]$ where each component in the material composition vector is defined as a function of distance from some reference entity. Each of these functions are referred to as material composition function (Siu & Tan, 2002).

The terms material composition vector and material coordinate, and the terms material composition function and material distribution function are used interchangeably in this paper.

3.3. Representation

In this paper, Heterogeneous Object Model (O) is defined as a tuple of material reference entities (G), material composition function (A) and a discretization factor (N) for approximating material composition function as piece-wise linear function.

$$O = (G, A, N) \tag{1}$$

Operating over this representation O, physical quantities like density, temperature, stress, strain etc. can be evaluated for multi-material or functionally graded objects. The representation constructed for HOM in this paper, is intended to be used for efficient querying and planning for additive manufacturing only.

3.4. Decomposition of domain using Medial Axis Transform

Medial Axis Transform (MAT) of a domain is the locus of the centre of a maximal disc/ball, which touches the boundary of the domain at its foot-points (point of tangency). Fig. 3 shows foot-points a′, b′ and c′. Blum and Nagel (1978) subdivided the MAT based on different type of points that are normal point, branch point and end point.

i. **A normal point:** A point whose maximal disc touches the object border in exactly two separate contiguous sets of points.
ii. **A branch point:** A point whose maximal disc touches the object border in three or more separate contiguous sets; Fig. 3 has 2 branch points (a, c).
iii. **An end point:** A point whose maximal disc touches the object border in exactly one contiguous set; Fig. 3 has 4 end points (p, q, r, s).

Subdivided MAT bound by instances of these points is referred as a MAT segment. In Fig. 3(a) the red lines indicate the medial axis, the black lines the boundary of the object and the green circles are the maximal discs, with their centres at a, b, c, and their foot points at a′, b′ and c′ respectively. The line joining a foot-point to its corresponding MAT point is called a rail. End point of a medial axis segment is referred as Medial axis vertex (MA-VERT) as shown in Fig. 3(b). Projection of MA-vertices on the boundary is referred as MAP-VERT, and projection of Medial Axis Segment (MA-SEG) on the boundary is called as MAP-SEG. These terminologies are reproduced from LayTrack algorithm (Quadros, Ramaswami, Prinz & Gurumoorthy, 2004).

Subdividing the domain by placing points MAP-VERT on the boundary and MA-VERT forms a cell, where the rails will be inserted. A domain consists of a set of cells, where each cell is referred as C_m and m is index of the cell. The region bound by a pair of adjacent rails is called tracks (Refer Fig. 4(a)). Quadrilateral elements can be obtained by placing sleepers/ties at the appropriate spacing along the rails from the boundary towards the interior till

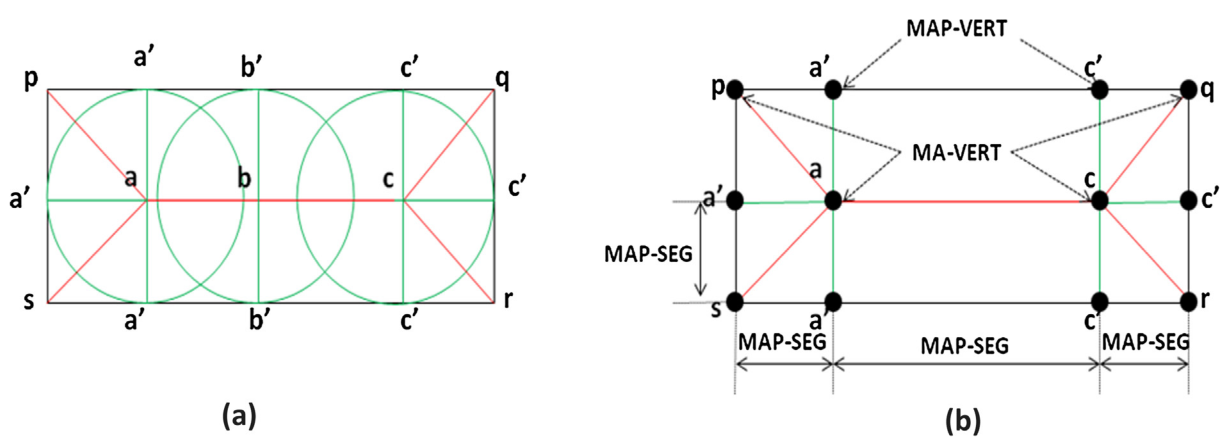

Fig. 3. (a) shows 2D medial axis transform 3, (b) shows mapping between medial axis and boundary.

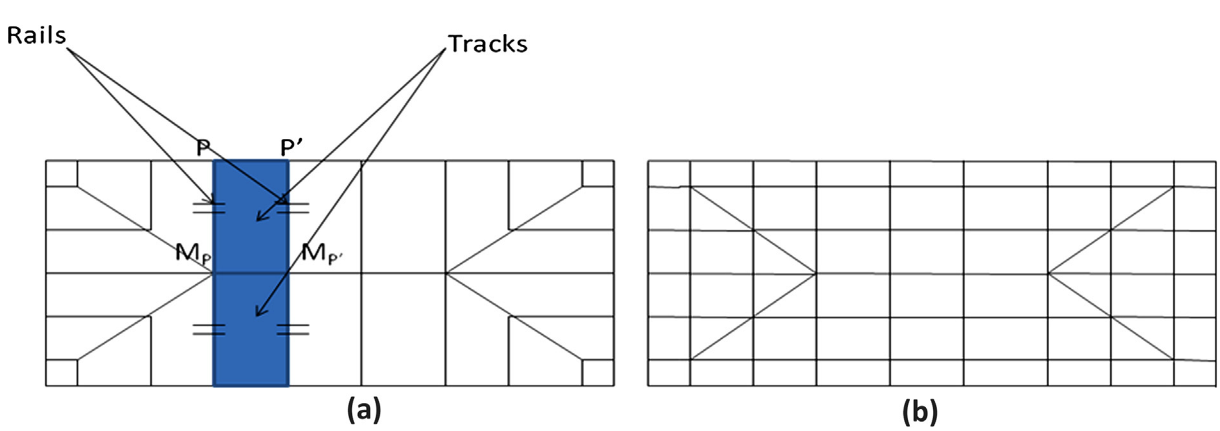

Fig. 4. Segmentation of a 2D domain using rails.

the medial axis as shown in Fig. 4(b). LayTrack Algorithm (Quadros et al., 2004) is used for generating these elements.

4. Overview

Given a heterogeneous object with locally varying material composition, user can model it by identifying material reference entities as the source of the material and then define the material composition within a domain by interpolating the material composition of these material reference entities. In order to achieve locally controlled composition, this paper first defines the domain of effect of each material reference entity by partitioning the domain using MAT. Within each sub-divided domain, material composition is blended and controlled locally starting from a material reference entity and terminating at the corresponding MAT segment.

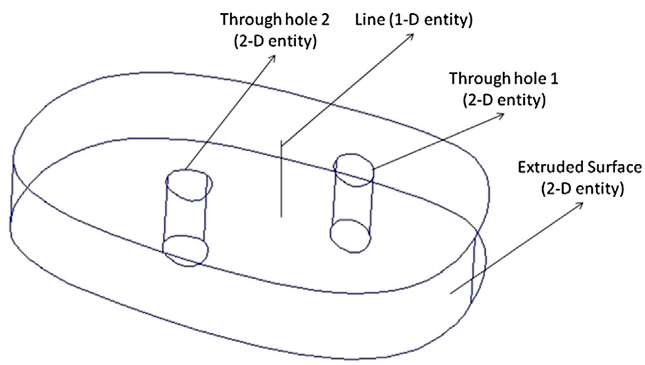

Fig. 5. CAD model with three entities which are two-dimensional (surface) and one entity is one-dimensional (line).

Fig. 5 shows a heterogeneous object model having multiply connected components associated with a set of mixed dimensional entities. It contains four material reference entities that are two holes (2-D entity), one shell face (2-D entity) and one line (1-D entity) respectively. For these entities, MAT is generated using the method presented in (Sharma, 2015).

Fig. 6 considers a 2-D view (Top view) of Fig. 5 to simplify the illustration. Fig. 6(a) shows the material reference entities and corresponding MAT; different colours denoting different material composition are shown in Fig. 6(b). Within each sub-domain, the material composition function is defined over rails as shown in Fig. 6(c); the material composition function is linear along the rails in this example. Material composition function for a shell face decreases from boundary to MAT for one constituent material (blue) and increases for the other constituent (green). Fig. 6(d) shows the last step where material distribution within each track

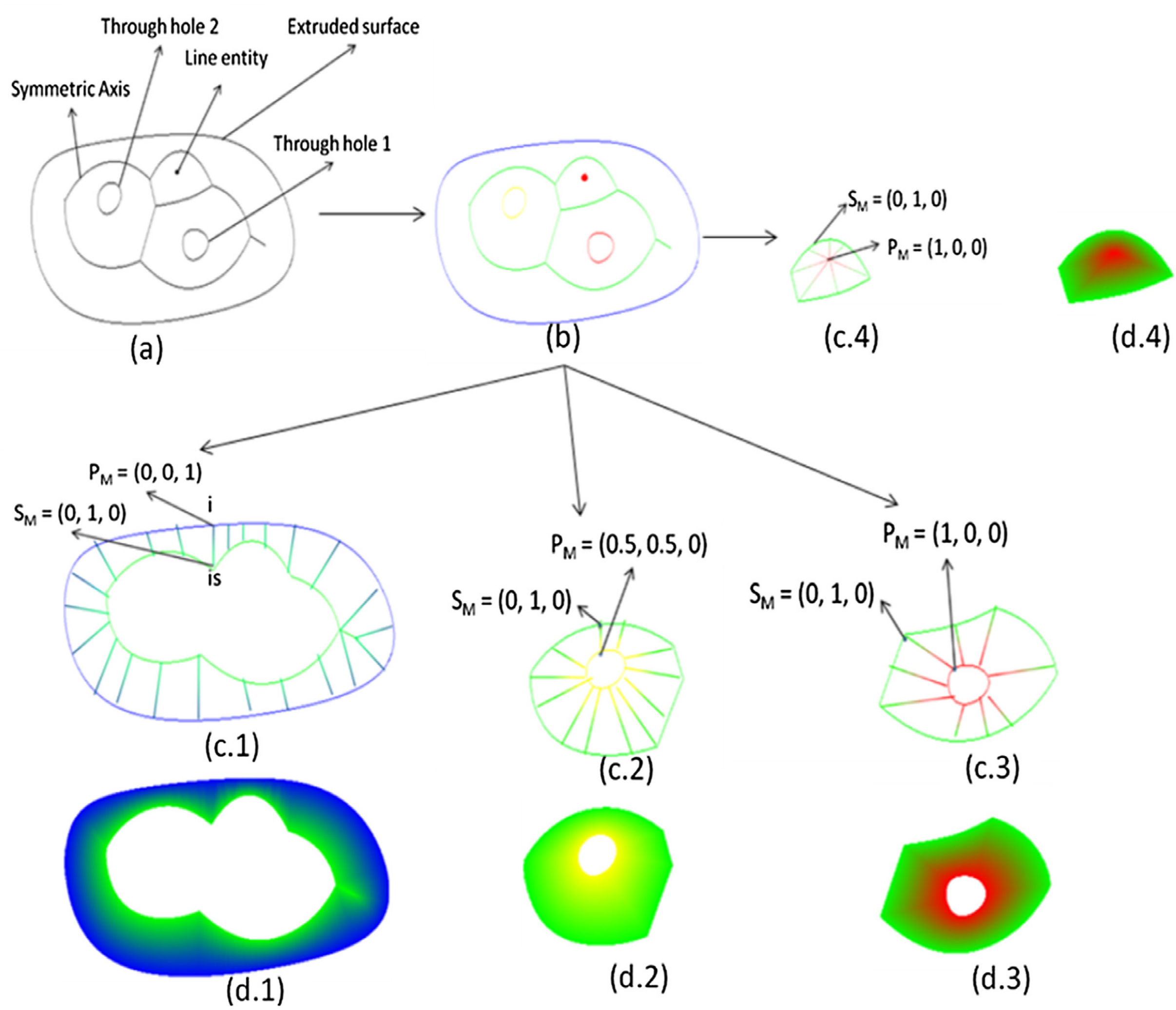

Fig. 6. Schematic Process of Heterogeneous Object Modelling. 2-D view is shown for better clarity.

bound by rails is represented by an appropriate interpolation scheme (bilinear interpolation in this illustration).

5. Methodology

Section 5.1 describes the specification of material reference entities, Section 5.2 defines partitioning or segmentation of a domain using MAT, Section 5.3 describes prescription of material within each sub-domain and construction of the hybrid representation.

5.1. Specify the material reference entity

This step defines the geometry and material composition of material reference entities. The geometry of a material reference entity can be a point (0-D), curve/line (1-D), surface/plane (2-D). It can be represented using Boundary representation (B-rep) or Piecewise Linear Complex (PLC) (Cheng et al., 2012). PLC is a discrete representation while B-rep is smooth representation (surface/curve). Both share graph-based structures (V, E, F) where, V is the set of vertices, E the set of edges, and F is the set of the set of facets.

The material composition vector of a reference entity is represented as an n-dimensional tuple $[m_1, m_2, m_3, \ldots m_n]$. It is assumed that the material reference entity has a uniform material composition.

Thus, a point-set defined on the material reference entity can be coupled with material composition vector as

$g = (P, P_M)$, where P is the geometric coordinate (x, y, z) of points and P_M is the corresponding material composition vector.

5.2. Automated segmentation of the geometric domain to generate multiple material features

In general, the material composition at a point p is blended by a weighted sum of material composition of each material reference entities, where weight functions W_i represents the contribution of i^{th} material reference entity and M_i represents the Material composition of i^{th} material reference entity. Weights functions can be distance-based functions (Shin & Dutta, 2001) or any other user-defined function.

$$M(p) = \frac{\sum_{i=1}^{N} W_i \cdot M_i}{\sum_{i=1}^{N} W_i}, \quad \sum_{i=1}^{N} W_i = 1 \tag{2}$$

This method of blending allows us to control the contribution of each material reference entity using weight functions, but reference entities that need to take part in the material blending cannot be specified. As a result, each reference entity makes a global contribution in material blending at a point especially for a domain associated with mixed-dimensional material reference entities having multiply connected topology and concavities. Using the proximity information provided by MAT, not only the choice of appropriate reference entity can be made but also local composition control can be defined within each segmented domain by using rail parameters starting from reference entity and terminating at corresponding MAT segment.

MAT of a domain with multiple material reference entities, segments the domain into independent geometric domains symmetrically as shown in Fig. 6(a). Within each independent domain (corresponding to material features), material reference entity is connected to the medial axis through rails as shown in Fig. 6(b).

Each rail is represented by geometric coordinates and material composition vector at its endpoints. If (P, P_M) represents a foot-point and (S, S_M) represents the corresponding MAT point, then the rail can be represented as $g = (P, S, P_M, S_M)$.

Let the domain of effect of each material reference entity (material featured) be indexed using h.

If g_k represents a set of rails in the domain of effect of material reference entity h, then it can be stored as $G_h^{dh} = \{g_k\}_{k=1}^{I} G_h^{dh} = \{g_k\}_{k=1}^{I}$, where G_h^{dh} is the set of rails for material reference entity h, d_h the dimensionality of the corresponding material reference entity, and I is the number of rails corresponding to the material reference entity.

Thus, for the entire set of material reference entities, it can be generalized as

$$G = \left\{G_h^{dh}\right\}_{h=1}^{H} \tag{3}$$

where H is the total number (cardinality) of material reference entities.

Fig. 5 shows four material reference entities; extruded boundary surface or shell face which is 2-D (G_1^2), two through hole internal surfaces that are also 2-D (G_2^2, G_3^2), and a line feature which is 1-D (G_4^1). The set of material reference entities can be written as $G = \{G_1^2, G_2^2, G_3^2, G_4^1\}$.

For example, Fig. 6(c.4) shows 7 rails for the line feature in Fig. 5 that is written as $G_4^1 = \{g_k\}_{k=1}^{7}$. Each rail consists of a foot-point and corresponding medial axis point written as $g_i = (P, S, P_M, P_S)$, where $P = (x_i, y_i, z_i)$, $S = (x_{is}, y_{is}, z_{is})$, $P_M = (1, 0, 0)$, $S_M = (0, 1, 0)$.

In this example, material composition along the MAT is chosen to be constant, but one can define the composition at MAT points using a weighted composition of the material reference entities using Eq. (2) in order to consider the influence of these entities, where the contribution of only those reference entities have to be considered that contain the foot-points corresponding to the MAT point, while for others, weight function remains zero.

5.3. Specification of material distribution function in each segmented domains

The material composition within a segmented domain obtained in the previous step is defined along the rail as a function of the rail parameter as described in (Sharma & Gurumoorthy, 2017). Material blending within a track formed by adjacent rails is achieved using an appropriate interpolation scheme like barycentric interpolation, bilinear interpolation or radial distribution function (Sharma & Gurumoorthy, 2017).

For all rails corresponding to the material reference entity h, the material distribution function is stored as $A_h = \{f(r_{hij}/r_{hi})\}_{i=1}^{I}$.

As defined above, the rail parameter for any point on i^{th} rail in the domain corresponding to the material reference entity h is given by r_{hij}/r_{hi}, where r_{hij} is the distance of j^{th} point from the foot-point on i^{th} rail, and I is the number of the rails of material feature h.

It can be generalized for all material reference entities as

$$A = \{A_h\}_{h=1}^{H} = \left\{ \left\{ f\left(\frac{r_{hij}}{r_{hi}}\right) \right\}_{i=1}^{I} \right\}_{h=1}^{H} \tag{4}$$

Similarly, all the rails corresponding to the domain of influence of each material reference entity h is denoted by G_h^{dh}. Set of G_h^{dh} (over all rails) is denoted by G.

Thus, for a HOM of object O, the representation can be written as O = (G, A).

5.4. Evaluated form of the hybrid representation

An evaluated form of material distribution is desired for applications such as optimisation using finite-element analysis, manufacturing, and visualization. This step describes the generation of the evaluated form of hybrid representation for random and irregular material distribution (modelled using mixed-dimensional material features). Assuming that the given material distribution is captured using two fields (G, A), for each material feature, the evaluated form of the prescribed material distribution function is calculated at nodes inserted in the rails as described in (Sharma & Gurumoorthy, 2017). Field N is used to store the nodes inserted in the rail. Thus, subdivisions on the i^{th} rail belong to material feature h, can be stored using N_{hi} written as $N_{hi} = \left\{\frac{r_{hij}}{r_{hi}}\right\}_{j=0}^{J}$.

Iterating over all the rails of a material feature h is stored using N_h, and iterating N_h over all the material features is stored using N given by

$$N = \{N_h\}_{h=1}^{H} = \left\{\left\{\left\{\frac{r_{hij}}{r_{hi}}\right\}_{j=0}^{J}\right\}_{i=0}^{I}\right\}_{h=0}^{H} \quad (5)$$

where H is the total number of material features. The hybrid representation O for HOM is written in expanded form by combining Eqs. (3), (4) and (5):

$$O = (G, A, N) = \left\{\{g_{hi}\}_{i=0}^{I}, \left\{\left\{f\left(\frac{r_{hij}}{r_{hi}}\right)\right\}_{j=0}^{J}\right\}_{i=0}^{I}, \left\{\left\{\left(\frac{r_{hij}}{r_{hi}}\right)\right\}_{j=0}^{J}\right\}_{i=0}^{I}\right\}_{h=1}^{H} \quad (6)$$

It is to be noted that the material distribution function corresponding to different material features may be different. The material composition at a node is then calculated as and when required.

The tracks formed by adjacent rails are subdivided to form elements by matching and connecting nodes on adjacent rails. Over each element, specified interpolation can be used for representing the composition in the interior.

A sacral slice is used to illustrate the modelling of complex and irregular material distribution. Fig. 7(a) shows a CT scan slice of sacral. The bright green portion marks the high-density region, while the dark portion represents the low-density region. Two low-density regions can be identified at the left interior and right interior, which can be considered as emanating from a curve. The user needs to roughly identify these curves, which can be viewed as the source of material emanating from the scan. It can also be done automatically using an extrema graph (Itoh & Koyamada, 1995). The boundaries from the scan can be identified using edge detection algorithms. The boundaries (blue) and internal open curves (green) are considered as the material reference entities and shown in Fig. 7(b).

The next step is the construction of MAT to segment these entities into independent regions (Fig. 8a). Within each region, the material distribution function is fitted on each rail, which uses the grey level associated with pixels on the rail. The function is parameterized along the rail. Thus, the material distribution for i^{th} rail can be written as $(f_i(r_{ij}/r_i))$. These functions can be stored in second field A for each rail. At this step, the representation is complete with two fields (G, A). This representation can be discretized for FEA and visualization. Each rail can be adaptively discretized by inserting nodes to achieve the desired accuracy and stored using field N. Each track formed by adjacent rails is subdivided into elements and within each element; barycentric interpolation is used for material distribution. The procedure of adaptive subdivision of the rails is shown in Fig. 8 (Figures (a), (b) and (c)), and material distribution on each element in Fig. 8(d).

5.5. Neighbourhood relation across multiple-material features

The domain of influence of each material feature can be stored systematically within a graph structure using its neighbourhood relationship with other domains. Such structures are useful for efficient navigation across different domains and editing the material distribution within them. If the material distribution over the domain of one material feature is altered, the material distribution in the neighbourhood domains can be automatically changed to ensure material continuity.

Two domains are said to be in neighbourhood to each other if they share a common boundary. Two domains defined by G_i^{di} and $G_{i+1}^{d(i+1)}$ are said to be connected if $G_i^{di} \cap G_{i+1}^{d(i+1)} \neq \Phi$.

This connectedness information of the domains is stored using a graph structure as MF = (G, E), where G is the set of the domain of influence of each material reference entity and E is the edge connecting these domains.

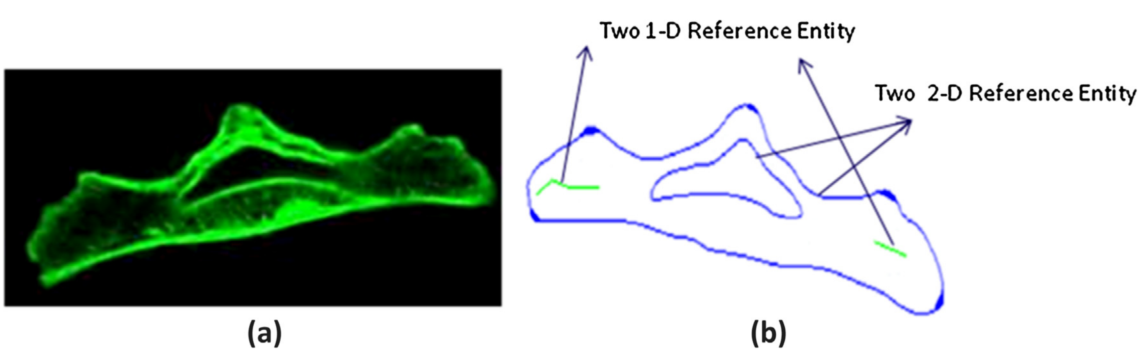

Fig. 7. (a) CT scan slice of sacrel, (b) Four material reference entities are defined.

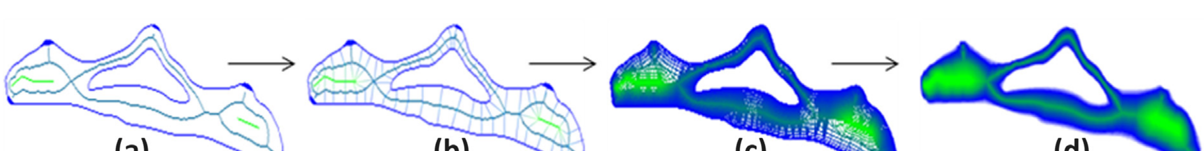

Fig. 8. (a–c) Adaptive subdivision of the rails and tracks, (d) shows the resultant distribution.

Editing the material distribution: Material distribution can be edited respecting the continuity constraints in two ways – either changing the material composition of the point-set of the domain or material composition functions defined over the rails.

If the material composition of a point P is changed from P_M to P'_M, the following procedure is followed:

(a) Find the domain G_i^{di} and corresponding cell C_m containing the point P.
(b) If the point P is inside the cell C_m, find the corresponding rail incident on the point P and subdivide it into two: one having P as the endpoint for the first rail and second having the starting point for the second rail.
(c) If the point P is on the boundary of the cell C_m, find the connected set of cells incident on P using breadth-first search on the graph MF. Find the rail incident on the point for each connected set of cells which are incident on point P, and edit the material composition vector of the corresponding point on the rail for each cell sharing point P.

Fig. 9(a) shows three domains D_1, D_2 and D_3 corresponding to the three material features that contain one outer boundary and two points respectively as material reference entities. Domain D_1 is connected to D_2 and D_3 as they share common boundaries while D_2 and D_3 are disjoint. Fig. 9(b) shows D_1 and MAT that are assigned with the same material composition [1 0 0] shown in red. Material reference entities corresponding to D_2 and D_3, which are point source are assigned with the material composition [0 1 0] and [0 0 1] respectively. Fig. 9(c) shows the change in the material composition of reference entity (i.e. boundary) corresponding to domain D_1 as [0 1 0]. The change in material composition behaves as a boundary condition, and the change automatically propagates across other domains as defined by the interpolation functions already stored in second tuple A of hybrid representation.

Material functions can be varied to alter material distribution. Fig. 10(c) shows further change in the same hole feature by altering the material distribution function on rail from linear function to cubic function.

5.6. Incremental addition of new material feature

Small changes in the material distribution can be made easily by altering the material composition vectors of point-set or material composition functions as described in the previous section. However, if a new material distribution has to be prescribed over a larger region, then it is better to define it as a new material feature to avoid the insertion of a large number of rails. New material features can be defined by inserting a new material reference entity into the existing domain, segmenting its domain of effect and defining corresponding material composition functions. It has to be noted that MAT needs to be regenerated for the newly inserted material reference entity, but this effort can be made inexpensive by re-computing MAT segments only for those MAT segments that are affected by the newly inserted reference entities

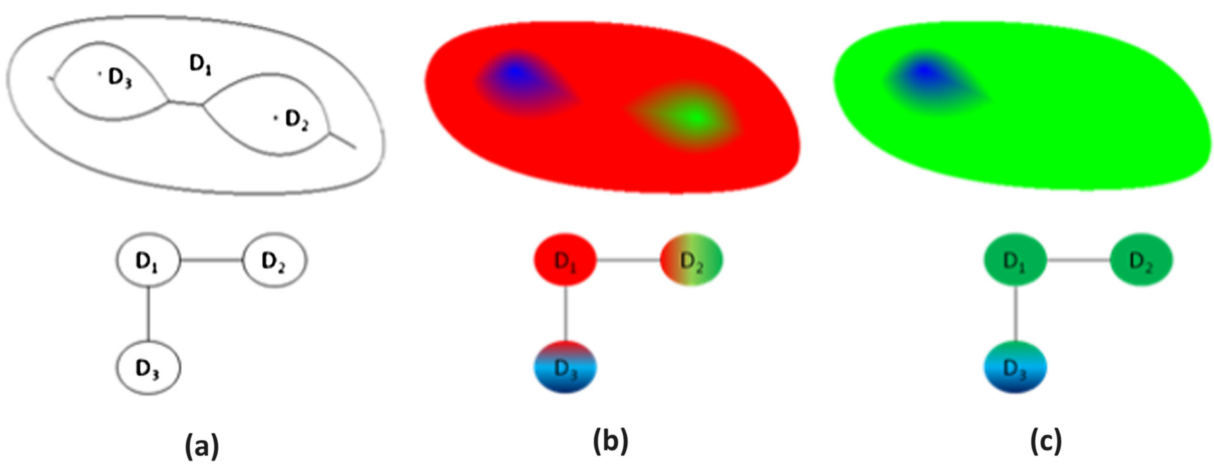

Fig. 9. Automated propagation of the change in material distribution from (b–c) using neighbourhood relation.

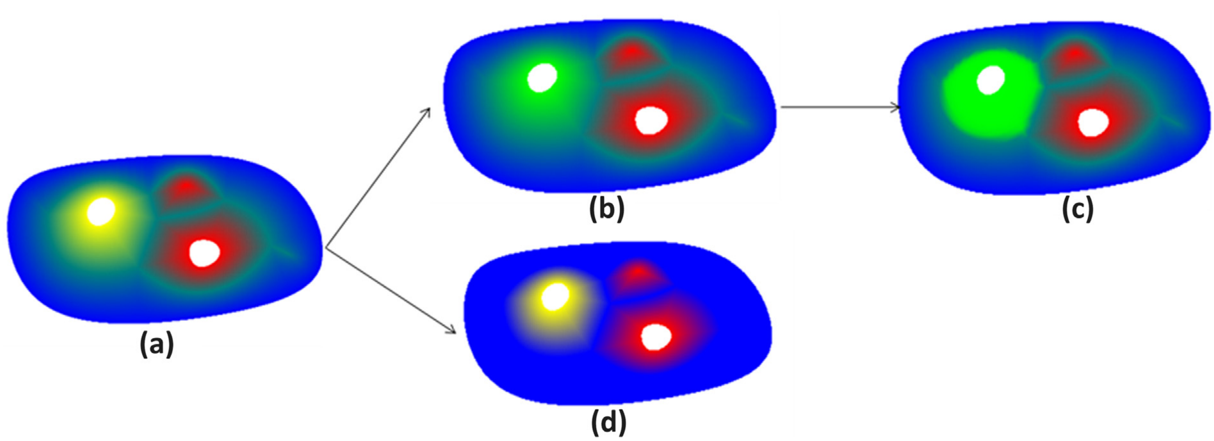

Fig. 10. (b) Change in the material composition vector of a material feature from [0.5 0.5 0] to [0 1 0], (c) Change of material composition function of a feature from linear to cubic degree on the parameterized rail, (d) Change in material composition vector of the MAT from [0 0.5 0.5] to [0 0 1].

by maintaining the history of MAT segments. This process is explained in the following three steps.

1. Let the points on MAT segments and corresponding foot-points in original representation before inserting new feature be $(P, Ps) = \left\{\{P_{hi}, Ps_{hi}\}_{i=0}^{I}\right\}_{h=0}^{H}$, and the new set of MAT points after insertion of the material feature be $(Q, Qs) = \left\{\{Q_{hk}, Qs_{hk}\}_{k=0}^{K}\right\}_{h=0}^{H+1}$.

   Find the set of common MAT points using the intersection of two sets as $(P, Ps) \bigcap (Q, Qs)$.

2. Using $(P, Ps) \bigcap (Q, Qs)$, extract the corresponding set of rails and its material composition from the hybrid representation O and express it as subset $(O_{Original})$. If the newly added feature is expressed as $O_{Newfeature}$, corresponding changes in its original features is expressed as subset $(O_{Original})$, then the new representation would be as follows:

$$O_{New} = \left(\text{subset}(O_{Original}),\ O_{Newfeature}\right)$$

3. Above two steps are repeated during the incremental addition of the material feature by setting $O_{Original} = O_{New}$, and increment H, at the end of step 2.

Fig. 11 shows the successive addition of material features, the corresponding change in MAT and subsequent change in the material distribution. The successive change in the material reference entity follows the boundary (i.e. green) as first reference entity shown in Fig. 11(a), point entity (i.e. red) as second reference entity shown in Fig. 11(b) and another point entity (i.e. yellow) as third reference entity shown in Fig. 11(d).

5.7. Data structure and material interrogation

A heterogeneous object model represented using hybrid representation is stored using graph-based structure, where the material features and its neighbourhood relations are stored as a set (G, E), where G is the domain of each material feature and E defines the connectivity of these domains. Each material feature G is internally stored using set of rails and material function defined over them using the data structure described in (Sharma & Gurumoorthy, 2017). This graph-based storage of the material feature allows efficient search (breadth-first) for material interrogation, where the first step is to find the domain containing the query point, and next step is to find the track containing the point. Once the track is identified, the remaining steps are to evaluate the rail parameter for the rail incident on the point and then evaluate the material composition from the material function stored using hybrid representation (for more details refer to the Section 5.2 in (Sharma & Gurumoorthy, 2017).

5.8. Material continuity

MAT maintains the material continuity across the domain of influence of each material reference entities or material features. This can be proved by limiting condition that the material composition vector across adjacent material features converge to the same value.

6. Results

The hybrid approach described above has been implemented in Windows 7 using Microsoft Visual Studio 2008. The input to the method can be a smooth boundary representation or a mesh model. The implementation uses ACIS Kernel (Portal:ACIS) for geometric computations. Results are rendered using OpenGL (Neider, Davis & Woo, 1993) and VTK (Schroeder, Avila, & Hoffman, 2000). Fig. 11 shows an example of a turbine blade modelled using point material features. These point features have a composition of low thermal conductivity materials or heat insulators like magnesium zirconate to alleviate from the hot-spot damage by building appropriate thermal barriers at the site, which would further enhance the life-cycle of the blade. Fig. 12 shows a mixed dimensional material distribution where the material composition vector on the extruded surface is [0 0 1], two through-holes and a line

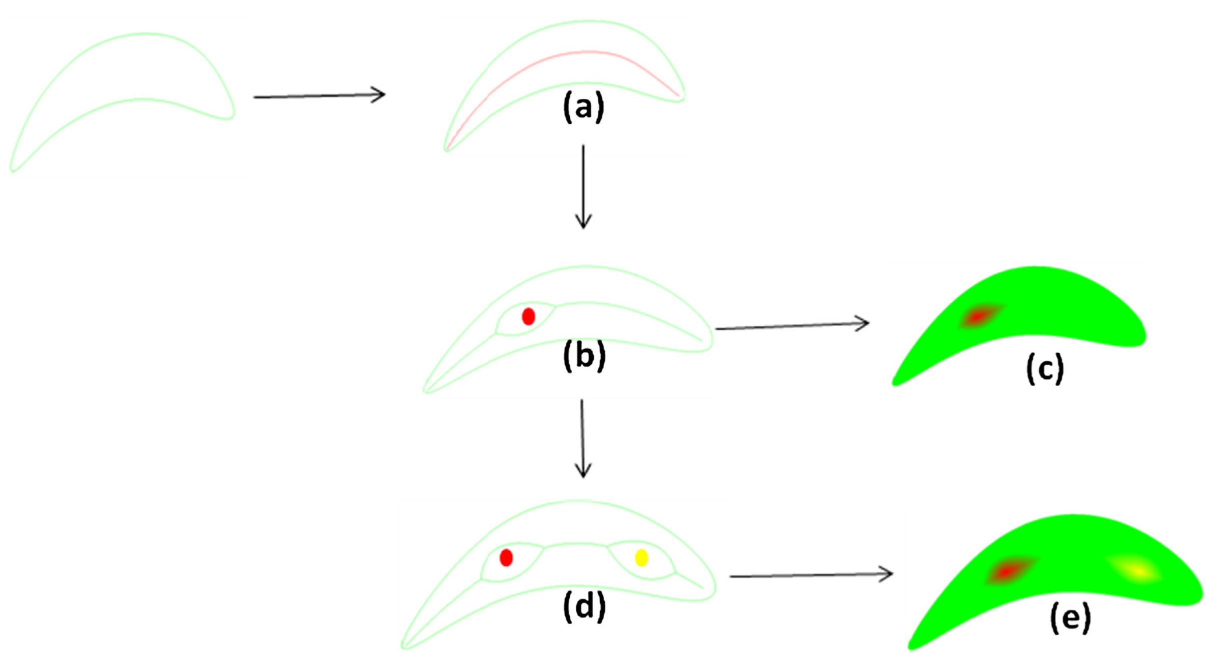

Fig. 11. Incremental Insertion of material features by inserting points as material reference entities.

feature having material composition [1 0 0] and MAT has the material composition vector [0 1 0] (refer to Fig. 5 for schematic information).

Fig. 13 has two material reference entities at the centre of two spheheres with composition $M_1 = [1\ 0\ 0]$ and $M_2 = [0\ 0\ 1]$ respectively. The material composition M at the boundary and MAT is given by weighted interpolation of M_1 and M_2 as $M = \frac{(\frac{M_1}{d_1^2} + \frac{M_2}{d_2^2})}{\frac{1}{d_1^2} + \frac{1}{d_2^2}}$, where d_1 and d_2 are the distance from the two centres. The material distribution within the track is achieved by subdivision and using barycentric interpolation.

Fig. 14 shows objects with geometric form features like holes. Fig. 14(a) has three material reference entities (two holes and one boundary). MAT is given the material composition of the boundary. Fig. 14(b) has two material reference entities (one hole and boundary).

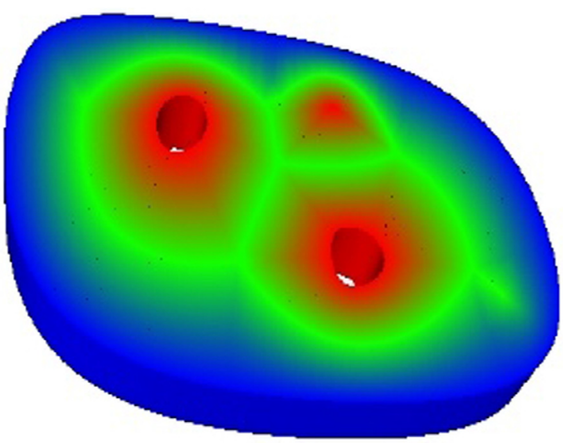

Fig. 12. Material distribution for an object with four material reference entities: Extruded surface, two through holes and a line feature.

7. Discussion

Medial Axis Transform has been central to this approach of modelling and representation. Medial Axis has the same homotopy type as any bounded open subset in R^n (Lieutier, 2004), which enforces that the connected open sets have a corresponding connected Medial Axis. Thus, the Medial Axis of a domain with a hole forms a cycle around that hole. In case the domain is associated with mixed-dimensional entities like point, line etc, this property still holds true by assuming lower dimensional entities like 0-D entity (point) as a ball in R^n of an infinitesimally small radius. The implemented results show the unique domain for each material reference entity bounded by medial axis segments without any overlap with other entities. Thus, any material distribution can be modelled by choosing the right dimension of material reference entities and its location. This way of modelling allows the decomposition of the geometric domain according to the desired material distribution, which has been a long sought issue (Kumar, 1999). The decomposition allows a locally controlled composition within each sub-domain by defining the variation along the rail.

Weighted distance methods (Liu et al., 2004; Jackson, 2000; Siu & Tan, 2002; Shin & Dutta, 2001) have been widely used for blending material for HOM. For a domain with multiply connected components associated with mixed-dimensional material reference features, it is important to use the proximity information to consider the appropriate material feature that should influence the material blending at a specified point. In this case, weighted distance blending can be used in two steps: one approximating the composition of points on Medial Axis using the weighted mean of the material composition of the adjoining material references and then further use weights to blend material within each segmented domain corresponding to each material feature.

The limitation of this approach is cost of generating the MAT of a domain. In order to support the smooth representation in the hybrid approach, resolution of MAT needs to be high, but this is a one-time effort. We recommend the use of discrete MAT (Sharma, 2015), which has a logarithmic complexity for generation. The sampling density of points on Medial Axis is similar to

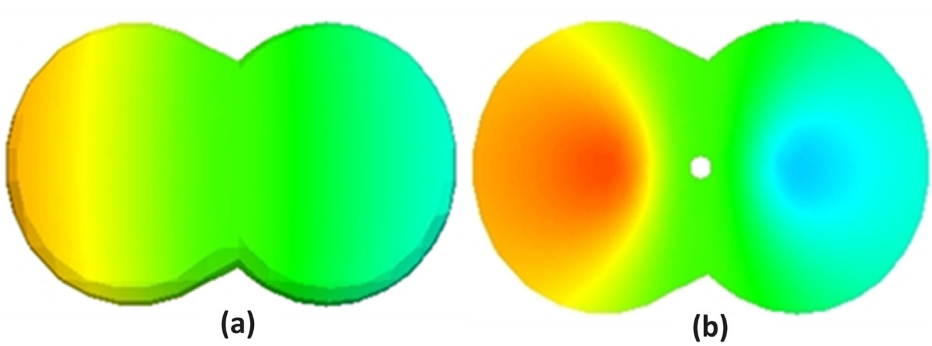

Fig. 13. Material distribution as a function of distance from the centre of two spheres.

Fig. 14. Material distribution on multiply-connected components.

the sampling density of input point set on material references, thus, avoiding the unnecessary cost of generating complete MAT.

While the representation scales to 3D shapes, some changes are required in the generation process. Rails in 2D will now be replaced by columns/pipes. The function defined to represent variation of the material composition will now be a function of two parameters. The present implementation to generate the representation addresses 2D and 2.5D shapes. Work is ongoing to handle 3D shapes.

8. Conclusion

A new modelling technique has been proposed that has been effective in capturing arbitrary material distribution using mixed-dimensional entities and represent it even when these entities are not part of the shape parameters or topological entities of the solid model of the part. Material composition within each domain of material feature can be controlled locally. This allows adaptive generation of a mesh in the direction of material blending for FEA, simulation, and path planning for additive manufacturing. These domains can be navigated using neighbourhood relations, which makes this method efficient for operations. This method is purely driven by material design rather than the shape parameter or geometric form (Samanta & Koc, 2005; Qian & Dutta, 2002). Thus, this method enhances the range of HOM that can be realized through Additive manufacturing. Future work is to generate the representation for 3D objects and to develop a process planning method for additive manufacturing using the proposed hybrid representation.

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