Reduced Complexity Index Codes and Improved Upperbound on Broadcast Rate for Neighboring Interference Problems

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Abstract*—A single unicast index coding problem (SUICP) with symmetric neighboring interference (SNI) has K messages and K receivers, the kth receiver R_k wanting the kth message x_k and having the interference with D messages after and U ($D \geq U$) messages before its desired message. Maleki *et. al.* derived the lowerbound on the broadcast rate of this setting to be $D+1$. In our earlier work, for SUICP(SNI) with arbitrary K, D and U, we defined set S of 2-tuples and for every $(a,b) \in$ S, we constructed b-dimensional vector linear index code with rate $D+1+\frac{a}{b}$ by using an encoding matrix of dimension $Kb \times (b(D+1)+a)$. In this paper, we use the symmetric structure of the SUICP(SNI) to reduce the size of encoding matrix by partitioning the message symbols. The rate achieved in this paper is same as that of the existing constructions of vector linear index codes. More specifically, we construct b-dimensional vector linear index codes for SUICP(SNI) by partitioning the Kb messages into $b(U+1)+c$ sets for some non-negative integer c. We use an encoding matrix of size $\frac{Kb}{b(U+1)+c} \times \frac{b(D+1)+a}{b(U+1)+c}$ to encode each partition separately. The advantage of this method is that the receivers need to store at most $\frac{b(D+1)+a}{b(U+1)+c}$ number of broadcast symbols (index code symbols) to decode a given wanted message symbol. We also give a construction of scalar linear index codes for SUICP(SNI) with arbitrary K, D and U. We give an improved upperbound on the broadcast rate of SUICP(SNI).

I. Introduction and Background

The problem of index coding with side-information was introduced by Birk and Kol [1]. Ong and Ho [2] classified the index coding problem depending on the demands and the side-information possessed by the receivers. An index coding problem is single unicast if the demand-sets of the receivers are disjoint and the cardinality of demand-set of every receiver is one. A single unicast index coding problem (SUICP) has K messages $\{x_0, x_1, \ldots, x_{K-1}\}$ and K receivers $\{R_0, R_1, \ldots, R_{K-1}\}$ for some positive integer K. Receiver R_k wants the message x_k and knows a subset of messages in $\{x_0, x_1, \ldots, x_{K-1}\}$ as side-information.

In a single unicast index coding problem, the side-information is represented by a directed graph $G = (V,E)$ with $V = \{x_0, x_1, \ldots, x_{K-1}\}$ vertices and E is the set of edges such that the directed edge $(x_i, x_j) \in E$ if receiver R_i knows x_j. This graph G for a given index coding problem is called the side-information graph. In this paper, we use $\mathcal{W}_k$ to denote want set and $\mathcal{K}_k$ to denote side-information of the receiver R_k. The messages which are neither wanted by nor known to R_k is called interference $\mathcal{I}_k$ to R_k.

In the index coding, we assume that the messages belongs to a finite alphabet $\mathcal{B}$. The solution (includes both linear and nonlinear) of the index coding problem must specify a finite alphabet $\mathcal{B}_P$ to be used by the transmitter, and an encoding scheme $\varepsilon : \mathcal{B}^K \rightarrow \mathcal{B}_P$ such that every receiver is able to decode the wanted message from the $\varepsilon(x_0, x_1, \ldots, x_{K-1})$ and the known information. The minimum encoding length $l = \lceil log_2|\mathcal{B}_P|\rceil$ for messages that are t bit long ($|\mathcal{B}| = 2^t$) is denoted by $\beta_t(G)$. The broadcast rate of the index coding problem with side-information graph G is defined [3] as,

$$\beta(G) \triangleq \inf_t \frac{\beta_t(G)}{t}.$$

If $t = 1$, it is called scalar broadcast rate. For a given index coding problem, the broadcast rate $\beta(G)$ is the minimum number of index code symbols required to transmit to satisfy the demands of all the receivers. The broadcast rate $\beta(G)$ and capacity $C(G)$ are related as [4]

$$C(G) = \frac{1}{\beta(G)}.$$

In a vector linear index code $x_k = (x_{k,1}, x_{k,2}, \ldots, x_{k,p_k}) \in \mathbb{F}_q^{p_k}$, $x_{k,j} \in \mathbb{F}_q$ for $k \in [0 : K-1]$ and $j \in [1 : p_k]$ where $\mathbb{F}_q$ is a finite field with q elements. In vector linear index coding setting, we refer $x_k \in \mathbb{F}_q^{p_k}$ as a message-vector or message and $x_{k,1}, x_{k,2}, \ldots, x_{k,p_k}$ as the message symbols. An index coding is a mapping defined as

$$\mathfrak{C} : \mathbb{F}_q^{p_0+p_1+\ldots+p_{K-1}} \rightarrow \mathbb{F}_q^N,$$

where N is the length of index code. The index code $\mathfrak{C} = \{(c_0, c_1, \ldots, c_{N-1})\}$ is the collection of all images of the mapping $\mathfrak{C}$. We refer $c_0, c_1, \ldots, c_{N-1}$ as index code symbols, which are the symbols broadcasted by the transmitter. If $p_k = p$ for every $k \in [0 : K-1]$, then the index code is called symmetric rate p-dimensional vector linear index code. If $p = 1$, then the index code is called scalar index code.

A p-dimensional vector linear index code of length N is represented by a matrix $\mathbf{L}$ ($\in \mathbb{F}_q^{pK \times N}$), where the jth column contains the coefficients of the jth coded transmission and the $(ip+j)$th row L_{ip+j} ($\in \mathbb{F}_q^{1 \times N}$) contains the coefficients used for mixing message $x_{i,j}$ in the N transmissions for every

$i \in [0 : K-1]$ and $j \in [1 : p]$. The broadcast vector is

$$[c_0, c_1, \ldots, c_{N-1}] = [\underbrace{x_{0,1}, \ldots, x_{0,p}}_{x_0} \ldots \underbrace{x_{K-1,1}, \ldots, x_{K-1,p}}_{x_{K-1}}]\mathbf{L}$$
$$= \sum_{i=0}^{K-1} \sum_{j=1}^{p} x_{i,j} L_{ip+j}.$$

Example 1 given below illustrates the advantage of vector linear index codes.

Example 1. Consider the index coding problem with wanted message and side-information as given in Table below.

R_k	$\mathcal{W}_k$	$\mathcal{K}_k$
R_0	x_0	x_1, x_3
R_1	x_1	x_2, x_3
R_2	x_2	x_0
R_3	x_3	x_1, x_2

The side-information graph of this SUICP is given in Fig. 1. Let $x_{k,1}, x_{k,2}$ be the two generations of the message symbol

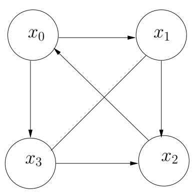

Fig. 1. side-information graph of SUICP given in Example 1

x_k for $k \in [0 : 3]$. A vector linear index code with symmetric rate $\frac{2}{5}$ for this index coding problem is

$$(c_0, c_1, c_2, c_3, c_4) = \{x_{0,1} + x_{1,1}, \quad x_{1,1} + x_{2,1}, \\ x_{0,2} + x_{3,1}, \quad x_{3,1} + x_{2,2}, \\ x_{1,2} + x_{3,2}\}.$$

It is easy to see that from the five broadcast symbols $(y_0, y_1, y_2, y_3, y_4)$, every receiver can decode its two wanted message symbols by using the available side-information with them.

A. Symmetric neighboring interference single unicast index coding problem

A symmetric neighboring interference single unicast index coding problem (SUICP(SNI)) with K messages and K receivers, each receiver has a total of $U + D < K$ interfering messages, corresponding to the D messages after and U $(D \geq U)$ messages before its desired message. In this setting, the kth receiver R_k demands the message x_k having the interference

$$I_k = \{x_{k-U}, \ldots, x_{k-2}, x_{k-1}\} \cup \{x_{k+1}, x_{k+2}, \ldots, x_{k+D}\}. \tag{1}$$

The side-information of this setting is given by

$$\mathcal{K}_k = (\mathcal{I}_k \cup x_k)^c. \tag{2}$$

All the subscripts in SUICP(SNI) are to be considered *modulo* K.

B. Existing Results

Maleki *et al.* [4] found the capacity of SUICP(SNI) with $K \to \infty$. The capacity of SUICP(SNI) with $K \to \infty$ is

$$C = \frac{1}{D+1} \text{ per message.} \tag{3}$$

Maleki *et al.* [4] proved the outerbound for the capacity of SUICP(SNI) for finite K. The outerbound for the finite K is given by

$$C \leq \frac{1}{D+1}. \tag{4}$$

Blasiak *et al.* [3] found the capacity of SUICP(SNI) with $U = D = 1$ by using linear programming bounds. The capacity of this SUICP(SNI) with $U = D = 1$ is given by

$$\frac{\left\lfloor \frac{K}{2} \right\rfloor}{K}. \tag{5}$$

In [8], we give a construction of binary matrices with a given size $m \times n$ $(m > n)$, such that any n adjacent rows in the matrix are linearly independent over every field $\mathbb{F}_q$. We call these matrices as Adjacent Independent Row (AIR) matrices. In [10], for SUICP(SNI) with arbitrary K, D and U, we define a set $\mathbf{S}$ of 2-tuples as given below

$$\mathbf{S} = \{(a, b) : \gcd(bK, b(D+1) + a) \geq b(U+1)\} \tag{6}$$

and we show that for every $(a, b) \in \mathbf{S}$, the rate $D + 1 + \frac{a}{b}$ is achievable by using b-dimensional vector linear index codes and AIR matrices of size $Kb \times (b(D+1) + a)$.

Jafar [5] established the relation between index coding problem and topological interference management problem. The SUICP(SNI) is motivated by topological interference management problems. The capacity and optimal coding results in index coding can be used in corresponding topological interference management problems.

C. Contributions

The contributions of this paper are summarized below:

- We construct b-dimensional vector linear index codes for SUICP(SNI) by partitioning the Kb messages into $b(U+1)+c$ sets for some non-negative integer c satisfying the condition $\gcd(Kb, b(D+1)+a) = b(U+1)+c$. We use $\frac{Kb}{b(U+1)+c} \times \frac{(b(D+1)+a)}{b(U+1)+c}$ size matrix to encode each partition separately. The advantage of this method is that the receivers need to store at most $\frac{(b(D+1)+a)}{b(U+1)+c}$ received broadcast symbols to decode a given wanted message symbol.
- This proposed index code construction identifies the receivers which will be able to decode their wanted messages instantly (without using buffers).
- We give a construction of scalar linear index codes for SUICP(SNI) with arbitrary K, D and U.
- We give an improved upperbound on the broadcast rate of SUICP(SNI).

The paper is organized as follows. In Section II, we give reduced complexity encoding for SUICP(SNI) by partitioning the message symbols. In Section III, we give a scalar linear construction of index codes for SUICP(SNI). In Section IV, we give an improved upperbound on broadcast rate of SUICP(SNI). We summarize the paper in Section V.

The proofs of all Theorems and Lemmas in this paper have been omitted due to space constraints. All the proofs can be found in [12].

II. Partition Based Vector Linear Index Codes for SUICP(SNI) with arbitrary K,U and D

In this section, we use partition of message symbols to obtain a simple linear index code for SUICP(SNI) with arbitrary K, D and U. Let

$$\mathbf{S} = \{(a, b) : \gcd(bK, b(D + 1) + a) \geq b(U + 1)\} \qquad (7)$$

In a b-dimensional vector linear index code, receiver R_k wants to decode the b message symbols $x_{k,1}\ x_{k,2} \dots x_{k,b}$ for every $k \in [0 : K-1]$. In the b-dimensional vector linear index code construction for SUICP(SNI) given in [10], the Kb messages corresponding to K receivers are linearly combined to give $b(D+1)+a$ index code symbols by using an AIR matrix of size $Kb \times (b(D + 1) + a)$.

In Theorem 1, we partition the Kb message symbols into $b(U + 1) + c$ sets for some non-negative integer c such that each set comprises of $\frac{Kb}{b(U+1)+c}$ messages. Then, by using the symmetry in the SUICP(SNI), we show that we can use an AIR matrix of size $\frac{Kb}{b(U+1)+c} \times \frac{b(D+1)+a}{b(U+1)+c}$ to encode each partition separately.

Theorem 1. Consider a SUICP(SNI) with arbitrary K, D and U. Let $(a, b) \in \mathbf{S}$ and $\gcd(bK, b(D + 1) + a) = b(U + 1) + c$ for some $c \in Z_{\geq 0}$. Let $\frac{Kb}{b(U+1)+c} = t$ and $\frac{b(D+1)+a}{b(U+1)+c} = \gamma$. An index code for this SUICP(SNI) with rate $D + 1 + \frac{a}{b}$ is obtained by using a $t \times \gamma$ AIR matrix.

Note 1. In [6], in the noisy communication channels where the received broadcast symbols are error prone, it is shown that the message probability of error in decoding a message at a particular receiver decreases with a decrease in the number of transmissions used to decode the message among the total of broadcast transmissions. The encoding and decoding method given in Theorem 1 indicates that every receiver uses at most $\frac{b(D+1)+a}{b(U+1)+c} = \gamma$ broadcast symbols to decode its wanted message symbol.

Note 2. Another application of the construction method given in Theorem 1 is related to Instantly Decodable Network Coding (IDNC). IDNC deals with code designs when the receivers have no buffer and need to decode the wanted messages instantly without having stored previous transmissions. A recent survey article on IDNC with application to Device-to-Device (D2D) communications is [7]. These results are valid for index coding since it is a special case of network coding. In Theorem 1, if $\gamma = 1$, then every receiver uses at most one broadcast symbol to decode a message symbol and hence the code is instantly decodable.

Example 2. Consider a SUICP(SNI) with $K = 13, D = 4, U = 1$. For this SUICP(SNI), we have $(a = 1, b = 5) \in \mathbf{S}$ and in [10], we showed that the rate $D + 1 + \frac{a}{b} = 5.2$ can be achieved by using AIR matrix of size 65×26 and 5-dimensional vector linear index coding. However, by using the partition method given in this paper, in this example, we show that the AIR matrix of size 5×2 is adequate for achieving a rate of $D + 1 + \frac{a}{b} = 5.2$.

For this SUICP(SNI), we have

$$t = \frac{Kb}{b(U + 1) + c} = 5,$$
$$\gamma = \frac{b(D + 1) + a}{b(U + 1) + c} = 2,$$
$$\tau = b(U + 1) + c = 13.$$

and the thirteen partitioned sets $\mathcal{A}_i$ for $i \in [1 : 13]$ are

$\{x_{0,1}, x_{2,4}, x_{5,2}, x_{8,0}, x_{10,3}\}, \{x_{0,2}, x_{2,5}, x_{5,3}, x_{8,1}, x_{10,4}\},$
$\{x_{0,3}, x_{3,1}, x_{5,4}, x_{8,2}, x_{10,5}\}, \{x_{0,4}, x_{3,2}, x_{5,5}, x_{8,3}, x_{11,1}\},$
$\{x_{0,5}, x_{3,3}, x_{6,1}, x_{8,4}, x_{11,2}\}, \{x_{1,1}, x_{3,4}, x_{6,2}, x_{8,5}, x_{11,3}\},$
$\{x_{1,2}, x_{3,5}, x_{6,3}, x_{9,1}, x_{11,4}\}, \{x_{1,3}, x_{4,1}, x_{6,4}, x_{9,2}, x_{11,5}\},$
$\{x_{1,4}, x_{4,2}, x_{6,5}, x_{9,3}, x_{12,1}\}, \{x_{1,5}, x_{4,3}, x_{7,1}, x_{9,4}, x_{12,2}\},$
$\{x_{2,1}, x_{4,4}, x_{7,2}, x_{9,5}, x_{12,3}\}, \{x_{2,2}, x_{4,5}, x_{7,3}, x_{10,1}, x_{12,4}\},$
$\{x_{2,3}, x_{5,1}, x_{7,4}, x_{10,2}, x_{12,5}\}.$

From the partition, in $\mathcal{A}_i$ for every $i \in [1 : 13]$, any receiver wanting a message in $\mathcal{A}_i$ knows three other consecutive messages in $\mathcal{A}_i$. In an AIR matrix of size 5×2, every two adjacent rows are linearly independent. Hence, AIR matrix of size 5×2 can be used as an encoding matrix for $\mathcal{A}_i$ for $i \in [1 : 13]$. AIR matrix of size 5×2 is given below.

$$\mathbf{L}_{5\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

The 26 broadcast symbols for this SUICP(SNI) is obtained by multiplying each of the 13 partitions above with AIR matrix of size 5×2. Let $[c_j\ c_{j+13}]$ be the two code symbols obtained by encoding the five message symbols in the partition $\mathcal{A}_{j+1}$ with AIR matrix of size 5×2 for $j \in [0 : 12]$. Let $k = 3$. R_3 wants to decode $x_{3,1}, x_{3,2}, x_{3,3}, x_{3,4}$ and $x_{3,5}$. We have $x_{3,j} \in \mathcal{A}_{2+j}$ for every $j \in [1 : 5]$. R_3 decodes $x_{3,j}$ from $[c_{1+j}\ \ c_{14+j}]$. In $\mathcal{A}_{2+j}$, R_3 knows three messages for every $j \in [1 : 5]$. Hence, after substituting the known messages, R_3 sees $[c_{1+j}\ \ c_{14+j}]$ as two equations with two unknowns and solves the wanted message $x_{3,j}$ for every $j \in [1 : 5]$.

Note 3. In Example 2, the size of the AIR encoding matrix used is 5×2. Hence, the encoding and decoding method given in Theorem 1 guarantees that every receiver uses at most 2 broadcast symbols to decode its wanted message symbol.

Example 3. For SUICP(SNI) with $K = 71$, $U \leq D \leq 10$, the upperbound on β and lowerbound on β are shown in Table I. Note that the 9th column of Table I gives the maximum

K	D	U	$a_{\min}$	$b_{\min}$	$D+1$ (lowerbound on β)	$D+1+\frac{a_{\min}}{b_{\min}}$ (upperbound on β)	AIR matrix size required in [10]	AIR matrix size required in this paper by using partition
71	1	1	1	35	2	2.0285	2485×71	$35 \times 1^{**}$
71	2	1,2	2	23	3	3.0869	1633×71	$23 \times 1^{**}$
71	3	1	2	35	4	4.0571	2485×142	35×2
71	3	2,3	5	17	4	4.1764	1207×71	$17 \times 1^{**}$
71	4	1,2,3,4	1	14	5	5.0714	994×71	$14 \times 1^{**}$
71	5	1	3	35	6	6.0857	2485×71	$35 \times 1^{**}$
71	5	2	4	23	6	6.1739	1633×142	23×2
71	5	3,4,5	5	11	6	6.4545	781×71	$11 \times 1^{**}$
71	6	1,2,. . .,6	1	10	7	7.1000	710×71	$10 \times 1^{**}$
71	7	1	4	35	8	8.1142	2485×284	35×4
71	7	2,3	6	17	8	8.3529	1207×142	17×2
71	7	4,5,6,7	7	8	8	8.8750	568×71	$8 \times 1^{**}$
71	8	1	5	31	9	9.1612	2201×284	31×4
71	8	2	6	23	9	9.2608	1633×213	$23 \times 1^{**}$
71	8	3	7	15	9	9.4666	1065×142	15×2
71	8	4,5,6,7,8	1	7	9	9.1428	497×71	$7 \times 1^{**}$
71	9	1,2,. . .,9	1	7	10	10.1428	497×71	$7 \times 1^{**}$
71	10	1	3	32	11	11.0937	2272×355	32×5
71	10	2	4	19	11	11.2105	1349×213	19×3
71	10	3,4,. . .,10	5	6	11	11.8333	426×71	$6 \times 1^{**}$

TABLE I
PARTITION ENCODING FOR SUICP(SNI) WITH $K = 71$ AND $U \leq D \leq 10$ (**INSTANTLY DECODABLE INDEX CODES).

number of broadcast symbols used by any receiver to decode its wanted message. The 9th column of Table I also indicates instantly decodable codes.

III. SCALAR LINEAR INDEX CODES FOR SUICP(SNI)

In this section, we give scalar linear index codes for SUICP(SNI) with arbitrary K, D and U.

Theorem 2. Consider a SUICP(SNI) with arbitrary K, D and U. Let a and b be the non negative integers satisfying the relation

$$gcd(K + a, D + 1 + a + b) \geq U + 1 + a. \tag{8}$$

Then, for this SUICP(SNI), an AIR matrix of size $(K+a) \times (D+1+a+b)$ can be used as an encoding matrix to generate an index code with length $D + 1 + a + b$.

Example 4. Consider SUICP(SNI) with $K = 19, D = 13$ and $U = 3$. These K, D and U satisfy (8) with $a = 1$ and $b = 0$. Hence, AIR matrix of size $(K+a)\times(D+1+a+b) = 20\times15$ can be used as an encoding matrix for this SUICP(SNI). The length of the index code is 15.

Example 5. Consider SUICP(SNI) with $K = 71, D = 52$ and $U = 16$. These K, D and U satisfy (8) with $a = 1$ and $b = 0$. Hence, AIR matrix of size $(K+a)\times(D+1+a+b) = 72\times54$ can be used as an encoding matrix for this SUICP(SNI). he length of the index code is 54.

The following lemma guarantees that the length of index code for SUICP(SNI) with arbitrary K, D and U is less than $D + U + 1$.

Lemma 1. For an SUICP(SNI) with arbitrary K, D and U, AIR matrix of size $K\times(D+U+1)$ can be used as an encoding matrix.

IV. IMPROVED UPPER-BOUNDS ON THE BROADCAST RATE OF SUICP(SNI)

Let $\mathbf{S} = \{(a, b) : \gcd(bK, b(D + 1) + a) \geq b(U + 1)\}$. In Theorem 1, we gave reduced complexity index code for SUICP(SNI) with length $D + 1 + \frac{a}{b}$. In [10], we gave an algorithm to find out the values of a and b in $\mathbf{S}$ such that $\frac{a}{b}$ is minimum and gave an index code with length $D + 1 + (\frac{a}{b})_{\min}$. However, for a given K, for certain values of D and U, we get $D + 1 + (\frac{a}{b})_{\min} = K$ and this length does not give any advantage when compared with uncoded transmission. In Lemma 2, we give the values of D and U for a given K for which $D + 1 + (\frac{a}{b})_{\min} = K$.

Lemma 2. Consider an SUICP(SNI) with K, D and U. Let

$$\mathcal{D}_l = \left[\left\lfloor \frac{lK}{l+1} \right\rfloor : \left\lfloor \frac{(l+1)K}{l+2} \right\rfloor - 1\right], \tag{9}$$

$$\mathcal{U}_l = \left[\left\lfloor \frac{K}{l+2} \right\rfloor : \left\lfloor \frac{K}{l+1} \right\rfloor - 1\right], \tag{10}$$

for some non negative integer l. Let D and U be such that

$$D \in \mathcal{D}_l \quad \text{and} \quad U \in \mathcal{U}_l \tag{11}$$

for $l \in \mathbb{Z}^+$. For this SUICP(SNI), we get $D+1+(\frac{a}{b})_{\min} = K$.

The sets $\mathcal{D}_l$ and $\mathcal{U}_l$ for $K = 71$ and $l \in [1 : 5]$ are given in Table II. The value of $D + 1 + (\frac{a}{b})_{\min}$ is given in Table III.

Theorem 3. Let $l_1 = D + 1 + (\frac{a}{b})_{\min}$, where $(\frac{a}{b})_{\min}$ is the minimum value of a and b such that $(a, b) \in \mathbf{S}$ given in (7) and $\frac{a}{b}$ is minimum. Let $l_2 = D + 1 + (a + b)_{\min}$, where $(a + b)_{\min}$ is the minimum value of a and b satisfying (8) and $(a + b)_{\min}$ is minimum. The upperbound on the broadcast rate of

l	$\mathcal{D}_l$	$\mathcal{U}_l$
1	$\left[\left\lfloor\frac{K}{2}\right\rfloor : \left\lfloor\frac{2K}{3}\right\rfloor - 1\right] = [35:46]$	$\left[\left\lfloor\frac{K}{3}\right\rfloor : \left\lfloor\frac{K}{2}\right\rfloor - 1\right] = [23:34]$
2	$\left[\left\lfloor\frac{2K}{3}\right\rfloor : \left\lfloor\frac{3K}{4}\right\rfloor - 1\right] = [47:52]$	$\left[\left\lfloor\frac{K}{4}\right\rfloor : \left\lfloor\frac{K}{3}\right\rfloor - 1\right] = [17:22]$
3	$\left[\left\lfloor\frac{3K}{4}\right\rfloor : \left\lfloor\frac{4K}{5}\right\rfloor - 1\right] = [53:55]$	$\left[\left\lfloor\frac{K}{5}\right\rfloor : \left\lfloor\frac{K}{4}\right\rfloor - 1\right] = [14:16]$
4	$\left[\left\lfloor\frac{4K}{5}\right\rfloor : \left\lfloor\frac{5K}{6}\right\rfloor - 1\right] = [56:58]$	$\left[\left\lfloor\frac{K}{6}\right\rfloor : \left\lfloor\frac{K}{5}\right\rfloor - 1\right] = [11:13]$
5	$\left[\left\lfloor\frac{5K}{6}\right\rfloor : \left\lfloor\frac{6K}{7}\right\rfloor - 1\right] = [59:59]$	$\left[\left\lfloor\frac{K}{7}\right\rfloor : \left\lfloor\frac{K}{6}\right\rfloor - 1\right] = [10:10]$

TABLE II

K	D	U	a	b	$D+1$	$D+1+(\frac{a}{b})_{\min}$	Remark
71	44	1,2,3,4,5	2	11	45	45.1818	$D \in \mathcal{D}_l$ and $U \notin \mathcal{U}_l$
71	44	$6, 7, \dots, 22$	7	3	45	47.3333	$D \in \mathcal{D}_l$ and $U \notin \mathcal{U}_l$
71	44	23,24,25,26	26	1	45	71	$D \in \mathcal{D}_l$ and $U \in \mathcal{U}_l$
71	45	1,2	3	20	46	46.1500	$D \in \mathcal{D}_l$ and $U \notin \mathcal{U}_l$
71	45	$3, 4, \dots, 22$	4	3	46	47.3333	$D \in \mathcal{D}_l$ and $U \notin \mathcal{U}_l$
71	45	23,24,25,26	25	1	46	71	$D \in \mathcal{D}_l$ and $U \in \mathcal{U}_l$
71	27	27	15	28	28	28.535	$D \notin \mathcal{D}_l$ and $U \in \mathcal{U}_l$
71	33	25	3	34	46	34.0882	$D \notin \mathcal{D}_l$ and $U \in \mathcal{U}_l$
71	15	2	3	22	16	16.1363	$D \notin \mathcal{D}_l$ and $U \notin \mathcal{U}_l$
71	3	1	2	35	4	4.0571	$D \notin \mathcal{D}_l$ and $U \notin \mathcal{U}_l$

TABLE III
$D+1+(\frac{a}{b})_{\text{MIN}}$ FOR $K = 71$.

SUICP(SNI) is given by

$$\beta(G) \leq \min(l_1, l_2, D+U+1).$$

Example 6. Consider an SUICP(SNI) with $K = 71, D = 44$ and $U = 23$. From Lemma 2, we have $l_1 = 71$. From Theorem 2, we have $l_2 = 71$. From Lemma 1, we have $D+U+1 = 68$. Hence from Theorem 3, the upperbound is given by

$$\beta \leq \min(l_1, l_2, D+U+1) = 68.$$

Example 7. Consider an SUICP(SNI) with $K = 71, D = 52$ and $U = 16$. From Lemma 2, we have $l_1 = 71$. From Theorem 2, we have $l_2 = 54$. From Lemma 1, we have $D+U+1 = 69$. Hence, from Theorem 3, the upperbound is given by

$$\beta \leq \min(l_1, l_2, D+U+1) = 54.$$

V. Discussion

In this paper, we gave near-optimal vector linear index codes for SUICP(SNI) for receivers with small buffer size. We gave an improved upperbound on the broadcast rate of SUICP(SNI). The broadcast rate and optimal coding for SUICP(SNI) with arbitrary K, D and U is a challenging open problem.

Acknowledgement

This work was supported partly by the Science and Engineering Research Board (SERB) of Department of Science and Technology (DST), Government of India, through J.C. Bose National Fellowship to B. Sundar Rajan.

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