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# Hawking radiation from acoustic black holes in hydrodynamic flow of electrons

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**Abstract** – Acoustic black holes are formed when a fluid flowing with subsonic velocities accelerates and becomes supersonic. When the flow is directed from the subsonic to supersonic region, the surface on which the normal component of fluid velocity equals the local speed of sound acts as an acoustic horizon. This is because no acoustic perturbation from the supersonic region can cross it to reach the subsonic part of the fluid. One can show that if the fluid velocity is locally irrotational, the field equations for acoustic perturbations of the velocity potential are identical to that of a massless scalar field propagating in a black hole background. One, therefore, expects Hawking radiation in the form of a thermal spectrum of phonons. There have been numerous investigations of this possibility, theoretically, as well as experimentally, in systems ranging from cold atom systems to quark-gluon plasma formed in relativistic heavy-ion collisions. Here we investigate this possibility in the hydrodynamic flow of electrons. The resulting Hawking radiation in this case should be observable in terms of current fluctuations. Further, current fluctuations on both sides of the acoustic horizon should show correlations expected for pairs of Hawking particles.

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**Introduction.** – Laboratory analogues of cosmic/astrophysical phenomena have proved to be of great importance. One of the most important examples of this is the laboratory analogue of black holes, the so-called acoustic black hole [1,2]. Black holes are probably the most exotic objects known to occur in the Universe. There is ample observational evidence of accretion disks around astrophysical black holes, but the region close to the horizon has not been readily accessible. (Though, with gravitational wave observations of black hole collisions, even this regime of black hole physics should be within reach of future experimental investigations). Laboratory investigations with acoustic black holes can be very useful to get insight into this regime. Probably the most intriguing phenomenon associated with black holes is Hawking radiation arising from the behavior of quantum fields in the background of a black hole spacetime [3,4]. It does not seem possible, in any foreseeable future experiment,

to probe this phenomenon for any astrophysical black hole which typically has a Hawking temperature lower than about  $10^{-7}$  K (for a stellar mass black hole), much smaller than the temperature of cosmic microwave background radiation. It is well appreciated that Hawking radiation raises deep conceptual issues related to unitary evolution and information loss in the formation and subsequent evaporation of black holes. Any experimental probe of the physics of Hawking radiation will be an important step towards understanding this important phenomenon. It is not surprising that many investigations with acoustic black holes have focused on the possibility of observing Hawking radiation in these laboratory analogues.

An acoustic black hole is a specific case of a more general result related to the propagation of acoustic perturbations in the velocity potential of an inviscid, barotropic fluid. It was shown by Unruh that such acoustic perturbations obey an equation which is identical to the Klein Gordon equation for a massless scalar field in a curved Lorentzian spacetime [1], with the spacetime metric determined by the flow velocity, density, and pressure of the fluid. An  $n$ -dimensional analogue system gives rise to an

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$(n + 1)$  dimensional analogue spacetime. As the acoustic perturbations propagate with the speed of sound in the fluid, it is clear that if on a surface the normal component of fluid velocity equals the local speed of sound, and becomes supersonic beyond it, then no acoustic perturbation can cross this surface from the supersonic region to the subsonic region. One can then expect such a flow geometry to correspond to a black hole with this special surface being identified with the horizon of the black hole. Indeed, as shown by Unruh [1], for a spherically symmetric, stationary, convergent background fluid flow, one finds the effective metric seen by acoustic perturbations of the velocity potential to be the Schwarzschild metric, with the horizon coinciding with the surface where the fluid velocity becomes supersonic. It was then predicted in [1] that in a fluid where acoustic perturbations can be quantized, one should expect Hawking radiation in terms of thermal bath of acoustic phonons emitted from this sonic horizon.

Numerous studies have been carried out to probe this possibility ([5–12] and references therein). Many investigations with cold atom systems have focused on the signature of Hawking radiation in terms of correlated pairs of Hawking particles emitted from the sonic horizon, with the two partners of the pair propagating on the two sides of the sonic horizon [13–17]. Interestingly, though one can calculate properties of correlations among such pairs for the Hawking radiation of a real black hole, its experimental investigation is simply out of the question as the region inside the event horizon is causally disconnected from the physically accessible region outside the horizon. For acoustic black holes, in contrast, it is simply a matter of observing acoustic perturbations on the two sides of the sonic horizon, with both sides equally accessible to experiments. It has been claimed that the observations are in agreement with the theoretical predictions for Hawking radiation. Such observations are very important, providing the first ever experimental evidence of the basic physics underlying Hawking radiation. It will be highly desirable to find some experimental situation where the Hawking radiation can be observed directly in terms of thermal spectrum of acoustic phonons. There have been some investigations in this direction [5]. It has also been proposed by some of us that an acoustic black hole metric may be constructed in the flow of quark-gluon plasma (QGP) in relativistic heavy-ion collisions [18]. In that case, the resulting thermal radiation of acoustic phonons may be observable in terms of modification of the rapidity dependence of the transverse momentum distribution of various particles.

In this work, we propose another possible analogue model for acoustic black holes where the resulting Hawking radiation may be observable directly as thermal radiation of emitted phonons. We consider a hydrodynamic flow of electrons. The possibility of electron hydrodynamics was first proposed by Gurzhi [19,20] for a system where electron-electron scattering dominates over momentum non-conserving scattering of electrons, *e.g.*,

with impurities and with phonons. Electron-electron scattering conserves the net momentum of electron system thus leading to conservation equations, namely the hydrodynamical equations for electron flow. Theoretically, it is a clean argument, but the situation with experiments has not been so clean. It took several decades to achieve ultra-clean systems where this regime of dominant electron-electron scattering could be achieved. Hydrodynamical flow of electrons is believed to have been achieved in a quasi 2-dimensional electron gas in high mobility heterostructures (*e.g.*, (Al,Ga)As heterostructures [21,22]), in graphene [23–25], as well as in Dirac and Weyl semimetals in 3 dimensions [26]. In such systems, observations related to viscous effects of the Navier-Stokes equation in electronic transport, such as Poiseuille-like flow profile, flow pattern of vortices, etc. have been reported. There are also proposals for probing non-linear hydrodynamical effects, *e.g.*, Bernoulli effect, Eckart streaming, and Rayleigh streaming of vortices [27].

We will focus on an entirely different aspect of hydrodynamical flow of electrons. We will consider a specific geometry of the sample which allows the flow to become supersonic beyond a surface. To be specific, we will consider an example of a quasi 2-dimensional electron gas, *e.g.*, in ultra-clean heterostructures, assuming the system to have sufficient thickness that it may be treated as 3-dimensional. This allows us to establish correspondence with a  $(3 + 1)$ -dimensional black hole. We will then write down the analogue black hole metric and estimate the resulting Hawking temperature for specific system parameters. We will argue that the Hawking temperature in this system will manifest in terms of electric current oscillations with thermal spectrum which may be observable. We mention that for the Hawking radiation from an acoustic black hole, it is important that the fluid should have quantum nature as Hawking radiation results from the quantized modes of the relevant field. This is what is achieved in Bose-Einstein condensate (BEC) systems [7,13,14] and in the proposed quark gluon plasma (QGP) system produced in relativistic heavy-ion collisions [18]. This is also true for the present electron-hydrodynamics system expected to be manifest in ultra-clean systems with strong quantum correlations. Please note that our results can be smoothly extended to lower dimensions. We could equally well have taken the sample to be exactly 2-dimensional, *e.g.*, graphene, in which case the resulting analogue black hole spacetime would have been  $(2 + 1)$ -dimensional. The occurrence of Hawking radiation and the estimate of Hawking temperature rely on the behaviour of quantum fields in a curved spacetime with appropriate boundary conditions and so remain valid in lower dimensions too.

We begin by briefly reviewing how an acoustic metric can be constructed in a non-relativistic fluid and argue that this smoothly applies to the case of electron transport in the hydrodynamic regime. Next, we write down the expression for the Hawking temperature of an

acoustic black hole formed in a suitably chosen model of one-dimensional fluid flow. Following this, we discuss sample geometries, in particular that of a de Laval nozzle, that can give rise to flow configurations necessary for the formation of acoustic black holes and rewrite Hawking temperature in terms of shape of the de Laval geometry. Taking typical parameter values available in the literature, we give an estimate of the Hawking temperature and the amplitude and frequency spectrum of resulting current oscillations. We also discuss the possibility of observing Hawking pair correlations in such a system in terms of current-current correlations between two sides of the acoustic horizon. Finally, we conclude with a discussion of the limitations of our analysis and various future possibilities.

### Hydrodynamics of electrons and acoustic black hole metric.

– The possibility of a hydrodynamic regime for electron transport was first discussed by Gurzhi [19,20] for ultra-clean crystals where electron-electron scattering, which conserves the momentum of the electron fluid, dominates over the scattering of electrons with impurities and with phonons which do not preserve the momentum of electrons. As mentioned in the introduction, this remarkable possibility has been realized in some ultra-clean systems (to suppress electron-impurity scattering) at appropriately low temperatures (to suppress electron-phonon scatterings, but still allowing significant electron-electron scattering). There have been numerous investigations [19–28] discussing constraints on impurity concentration and the regime of temperature that make electron-electron scattering dominant over the other, momentum non-conserving, scattering modes. One also requires constraints on the system size as scattering of electrons with the boundaries of the sample, in general, leads to momentum loss from the electron fluid. With sufficient evidence available for validity of this regime in these ultra-clean materials, we assume validity of hydrodynamic description of electron transport in these systems and write down the Navier-Stokes equations for the electron fluid. An important point we mention here is that we restrict our discussion to the case of an inviscid fluid. While experiments show that electron hydrodynamical flow has non-zero viscosity, the viscous effects do not appear to be dominant compared to other relevant effects like scattering of electrons with impurities, phonons, and the sample boundary. For certain cases, *e.g.*, graphene, viscous effects may actually be negligible [28].

For non-relativistic hydrodynamics the basic equations of fluid flow are the following: the continuity equation

$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (1)$$

and the Euler equation

$$\rho [\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v}] + \vec{\nabla} p = 0. \quad (2)$$

This is in the absence of any external force. For electron fluid, there should be a term involving electrostatic

potential. We are not including that for simplicity. Such a term can be absorbed in the pressure term [27]. Further, usually these systems have very high conductivity, so even for reasonably high currents, the potential can be taken to be almost constant in the relevant region (which will be close to the sonic horizon).

We consider the case where the fluid is locally irrotational, so that one can write  $\vec{v} = \vec{\nabla} \phi$  where  $\phi$  is the velocity potential which is locally well defined in the regions where the fluid is irrotational. We also assume that the equation of state is barotropic so that  $\rho$  is a function of  $p$  only. We can then define the specific enthalpy

$$h(p) = \int_0^p \frac{dp'}{\rho(p')}. \quad (3)$$

With this, we get  $\vec{\nabla} h = \vec{\nabla} p / \rho(p)$ . In terms of  $h$  and  $\phi$ , the Euler equation can be reduced to

$$\partial_t \phi + h + \frac{1}{2} (\vec{\nabla} \phi)^2 = 0. \quad (4)$$

Consider now small perturbations  $(\rho_1, p_1, \phi_1)$  on a background flow  $(\rho_0, p_0, \phi_0)$ . Then, it can be shown that the linearised evolution equation for  $\phi_1$  can be written compactly as [1,2]

$$\partial_a (\sqrt{-g} g^{ab} \partial_b \phi_1) = 0. \quad (5)$$

Here,  $g^{ab}$  is a matrix whose elements are functions of the background velocity, density and local speed of sound,  $c_s$ , in the fluid.  $g_{ab}$  is the inverse matrix of  $g^{ab}$  and  $g = \det(g_{ab})$ . Notice that eq. (5) is structurally the same as the relativistic wave equation for a massless scalar field ( $\phi_1$ ) propagating in a curved spacetime with metric  $g_{ab}$ . Thus, we can identify  $g_{ab}$  as an effective acoustic metric seen by acoustic perturbations in the velocity potential of the fluid. It is given by the following line element:

$$\begin{aligned} ds^2 &= g_{ab} dx^a dx^b \\ &= \Omega [-c_s^2 dt^2 + (dx^\alpha - v^\alpha dt)(dx^\beta - v^\beta dt) \delta_{\alpha\beta}]. \end{aligned} \quad (6)$$

$\Omega = \rho/c_s$  is a conformal factor, the local speed of sound,  $c_s = \sqrt{\frac{\partial p}{\partial \rho}}$  and  $\vec{v}$  is the background flow velocity. (Here on, we use  $\rho, \vec{v}, p$ , etc. to denote background values, without the subscript 0. Latin letters  $a, b, \dots$  denote spacetime indices while Greek letters  $\alpha, \beta, \dots$  denote spatial indices.) As long as the above equations of fluid dynamics and the conditions imposed on them hold good, this derivation of an acoustic metric will remain valid in any dimension.  $\Omega, \vec{v}$  and  $c_s$  appearing in the metric all depend on the specific nature of the electron flow, *e.g.*,  $c_s$  is determined by electron-electron interactions in the fluid.

Now, we consider an effectively one-dimensional steady flow of an electron fluid. We can orient the axes of our coordinate system such that the flow is parallel to the  $z$ -axis and the velocity vector points in the direction of decreasing  $z$ . So,  $v^\alpha(t, z) = (0, 0, -v^z(z))$  (which is irrotational).

So, the acoustic line element simplifies to

$$ds^2 = \Omega \left[ - (c_s^2 - (v^z)^2) dt^2 + 2v^z dt dz + dx^2 + dy^2 + dz^2 \right]. \quad (7)$$

This is actually qualitatively similar to the Schwarzschild metric written in Painlevé-Gullstrand coordinates except that the metric coefficients here are functions of  $z$  instead of the radial coordinate, as in the spherically symmetric Schwarzschild black hole. If the velocity field of the fluid is such that, given some value  $z = z_H$ ,

$$\begin{aligned} v^z < c_s, & \quad \text{for } z > z_H, \\ v^z = c_s, & \quad \text{for } z = z_H, \\ v^z > c_s, & \quad \text{for } z < z_H, \end{aligned}$$

then an acoustic horizon forms at  $z = z_H$ . The fluid flowing with supersonic velocities in  $z < z_H$  sweeps all acoustic perturbations away from the horizon. The supersonic region is thus acoustically disconnected from the subsonic region. Now, if  $v^z \rightarrow 0$  as  $z \rightarrow z_0$  ( $z_0 > z_H$ ), then we get back Minkowski metric there. Thus, an observer at  $z = z_0$  would serve as an ‘‘asymptotic observer’’ in ‘‘asymptotically flat’’ spacetime for our purposes. (The setup described here is similar to that of fig. 2. For convenience, as explained later, we have adopted a different orientation of the coordinate axes in fig. 1.) If the fluidic system were effectively two-dimensional, we would similarly get a  $(2 + 1)$  dimensional acoustic metric with the same structure.

A remarkable property of this system is that due to the presence of a purely absorbing boundary condition at the horizon, there would be a spontaneous emission of phonons (quantised acoustic perturbations) near the horizon in the form of *acoustic Hawking radiation*. This radiation is expected to be thermal and its temperature is given by

$$T = \frac{\kappa}{2\pi} = - \frac{1}{2\pi} \left. \frac{\partial v^z}{\partial z} \right|_{z_H}. \quad (8)$$

Here,  $\kappa$  is known as the surface gravity at the acoustic horizon. After reinstating the fundamental constants  $\hbar, k_B$  which had otherwise been set equal to unity, the above equation becomes

$$k_B T = - \frac{\hbar}{2\pi} \left. \frac{\partial v^z}{\partial z} \right|_{z_H}. \quad (9)$$

The conformal factor  $\Omega$  does not affect the value of the temperature here [18]. The acoustic metric has been derived starting from fluid equations that allow the freedom to multiply the metric by an overall constant. We can utilize this to replace the conformal factor  $\Omega(z)$  in eq. (7) by  $\Omega(z)/\Omega(z_0)$  where  $z_0$  denotes the location of the asymptotic observer. Since we are considering a steady state flow where  $\Omega = \rho(z)/c_s$  is a function of  $z$  only, the new normalised conformal factor remains unity at  $z = z_0$  at all times. A conformal factor with this asymptotic behavior does not affect the asymptotic Hawking temperature [18].

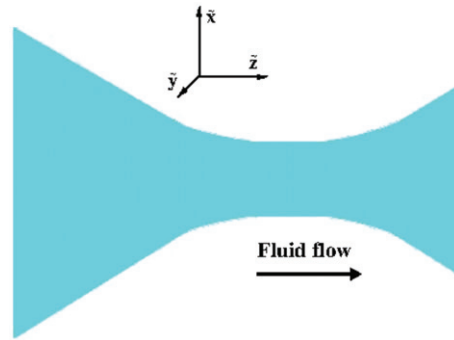


Fig. 1: de Laval geometry with converging-diverging flow.

In the above discussion, the starting point has been non-relativistic fluid equations. The relativistic fluid case would be directly relevant for the case of Dirac materials like graphene, especially due to the expectation of very low viscosity in such systems. An acoustic metric can be derived for the relativistic hydrodynamics case also [29–31] and the basic physics of our proposal should carry over to the relativistic regime, though we do not discuss it in this article (see also [32–36]). Further, even for Dirac fermions in graphene, non-relativistic fluid equations have been used as an approximation [37]. So, we will work in the same spirit and assume that the basic physical idea behind our approach remains valid even for such systems.

**Supersonic flow of electrons in a de Laval nozzle.** – Hydrodynamics of electrons have been extensively investigated recently and specific experimental investigations/proposals have discussed specific geometries of the sample, focusing on different aspects of electron hydrodynamical flow [27,28]. Here, we discuss the specific example of a de Laval nozzle which has a converging-diverging geometry, as shown in fig. 1 [28]. We assume the flow of electron fluid to be along the  $\tilde{z}$ -axis. Note that we use here  $(\tilde{x}, \tilde{y}, \tilde{z})$  coordinates distinct from the  $(x, y, z)$  coordinate used above in the derivation of an acoustic metric. This is because a discussion of acoustic black holes is most conveniently done with a choice of  $z$ -axis such that the fluid velocity vector points towards  $-\hat{z}$  and its magnitude decreases with increasing values of  $z$  (with fluid velocity approaching zero at some large  $z = z_0$ ). This is the standard convention in the literature of acoustic black holes with 1-dimensional fluid flow. In contrast, for the discussion of fluid flow in nozzles, *e.g.*, a de Laval one, it is traditional to choose a  $\tilde{z}$  coordinate such that fluid velocity increases with increasing  $\tilde{z}$ .

The width of the quasi 2-dimensional electron gas system is taken to be along the  $\tilde{x}$ -axis, and the  $\tilde{y}$ -axis represents the thickness of the film (which is assumed to be small). We take the  $\tilde{x}$  dimension of the sample to initially decrease along the  $\tilde{z}$ -axis, *i.e.*, along the fluid flow. This will cause fluid velocity to increase as a function of  $\tilde{z}$ . With suitable values of system parameters, the flow can achieve sonic velocity at a specific value of  $\tilde{z} = z_H$



which represents the location of the horizon of the acoustic black hole. (To avoid using too many different notations, we denote the location of the sonic horizon by  $z_H$  throughout the article.) It is important to take note of the fact that within a converging or diverging shape, the flow of electrons cannot be strictly one-dimensional as the flow has to converge towards the narrowest part and diverge beyond it. Thus, flow velocity will have non-zero  $v^{\tilde{x}}$  components, which will be significant especially near the boundary of the sample. However, near  $\tilde{x} = 0$ , all along the  $\tilde{z}$ -axis,  $v^{\tilde{x}}$  will be negligible. Thus, our assumption of a one-dimensional flow remains a reasonably good approximation in this region and we shall neglect  $v^{\tilde{x}}$  in our discussion. (This is the same as the situation encountered in the discussion of Hawking radiation in Bose-Einstein condensates using a de Laval geometry in ref. [38].)

In the following, we derive the variation of the flow velocity along  $\tilde{z}$  in a de Laval geometry [38]. Let  $A(\tilde{z})$  be the cross-sectional area perpendicular to the  $\tilde{z}$ -axis. The continuity equation then gives

$$\frac{d}{d\tilde{z}}(\rho Av) = 0 \Rightarrow \rho' = -\rho \left[ \frac{A'}{A} + \frac{v'}{v} \right]. \quad (10)$$

Here, a  $'$  denotes derivation with respect to  $\tilde{z}$  and  $v = |\vec{v}| = v^{\tilde{z}}$ . The fluid acceleration  $a$  is given by

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = (\vec{v} \cdot \vec{\nabla})\vec{v}, \quad (11)$$

since  $\frac{\partial v}{\partial t} = 0$ . For the 1-dimensional flow, we get  $a = v \frac{dv}{d\tilde{z}} = vv'$ . With this, eq. (10) becomes

$$\rho' = -\rho \left[ \frac{A'}{A} + \frac{a}{v^2} \right]. \quad (12)$$

The Euler equation for time-independent 1-dimensional flow gives

$$\rho v \frac{dv}{d\tilde{z}} \equiv \rho a = -\frac{dp}{d\tilde{z}} = -\frac{dp}{d\rho} \rho'. \quad (13)$$

The last equality follows from a barotropic equation of state for the fluid. Using speed of sound  $c_s^2 = \frac{dp}{d\rho}$  and eliminating  $\rho'$  from the above two equations we get

$$a = \frac{-v^2 c_s^2}{(c_s^2 - v^2)} \frac{A'}{A}. \quad (14)$$

This is known as the *Nozzle equation*. This shows that for a focussing geometry with  $A' < 0$ , the fluid accelerates as long as  $v < c_s$ . Further acceleration of the fluid to supersonic velocities  $v > c_s$  can only be achieved if  $A' > 0$ . This happens in the diverging part of the Laval nozzle. Truncating a de Laval nozzle at the throat where  $A' = 0$  gives us what is known as a Venturi geometry. Though a fluid in this geometry would achieve sonic velocity at the neck giving rise to an acoustic horizon, we do not expect Hawking radiation in this configuration as it admits no supersonic region or acoustic black hole where negative energy Hawking partner modes of phonons can be absorbed.

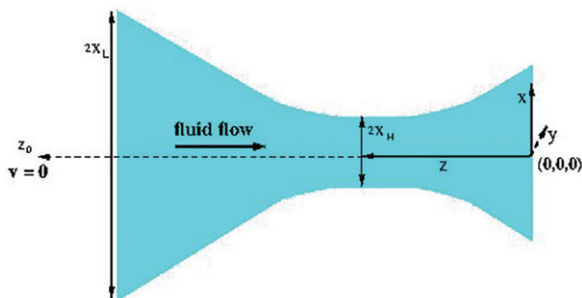


Fig. 2: Detailed picture of sample with de Laval geometry showing various dimensions of the sample. The neck is assumed to have a parabolic cross-section which should smoothly merge to the wedge-shaped geometry away from the neck. The thickness of the sample (in the  $y$ -direction) is  $d$ . The orientation of coordinate axes here are different from that of fig. 1.

Now, to evaluate fluid acceleration  $a(z_H)$  from eq. (14), with  $v(z_H) = c_s$  and  $A'(z_H) = 0$ , we use the l'Hospital rule. For simplicity, we consider the case of a constant speed of sound  $c_s$ . Then one obtains [38]

$$a(z_H) = \frac{c_s^3}{v'(z_H)} \frac{A''}{2A} \Big|_{z_H}, \quad (15)$$

$$\text{or, } \frac{dv}{dz} \Big|_{z_H} = c_s \sqrt{\frac{A''}{2A}} \Big|_{z_H}. \quad (16)$$

**Estimates of Hawking temperature.** – We now discuss specific values of system parameters and estimate the resulting Hawking temperature. Figure 2 shows a detailed picture of the proposed de Laval geometry where various dimensions of the sample are marked. Note that the  $z$ -axis of fig. 2 is oriented opposite to the  $\tilde{z}$ -axis of fig. 1. This choice of the  $z$ -axis is consistent with that made in the derivation of the acoustic black hole metric. The narrowest part of the neck of the nozzle, where the sonic horizon is located, is still denoted as  $z_H$ , now at  $z = z_H$ . Thus, eq. (16) written in  $z$  coordinate becomes

$$\frac{dv}{dz} = -\frac{dv}{d\tilde{z}} \Big|_{z_H} = -c_s \sqrt{\frac{A''}{2A}} \Big|_{z_H}. \quad (17)$$

With this, the Hawking temperature (restoring fundamental constants) is given by (8)

$$k_B T = -\frac{\hbar}{2\pi} \frac{dv}{dz} \Big|_{z_H} = -\frac{\hbar}{2\pi} c_s \sqrt{\frac{A''}{2A}} \Big|_{z_H}. \quad (18)$$

(Note that  $A' \equiv \frac{\partial A}{\partial \tilde{z}} = -\frac{\partial A}{\partial z}$  and  $A'' \equiv \frac{\partial^2 A}{\partial \tilde{z}^2} = \frac{\partial^2 A}{\partial z^2}$ ). The width of the system in the  $x$ -direction is  $2x_L$  at the left edge, and it is  $2x_H$  at the nozzle neck, at  $z = z_H$ . The diagram only shows a small part of the nozzle for the supersonic region  $z < z_H$ . It seems reasonable to assume that the supersonic region should be larger than the typical wavelength of Hawking radiation. To calculate  $A''|_{z_H}$ , we need to know the detailed shape of the neck. For a

simple estimate, let us assume it is of a parabolic shape at the neck which smoothly changes to a wedge-shaped geometry little away from the neck. The Hawking radiation being only sensitive to near-horizon flow geometry, shape changes further away from the neck do not affect the estimates of Hawking temperature. Let the upper part of the parabolic region of this neck be characterized by

$$x = b(z - z_H)^2 + x_H. \quad (19)$$

Here,  $b$  is a positive constant. The thickness of the quasi 2-dimensional nozzle in the  $y$ -direction is taken to be  $d$ . Then the cross-sectional area  $A_{bh} = 2x_H d$  and  $(A''/A)|_{z_H} = 2b/x_H$ .

For sample parameters, we take values of the same order as used in the literature [27,28]. Thus, we take  $x_H = 1 \mu\text{m}$  and  $x_L = 5 \mu\text{m}$ . To have a reasonable value of  $a$ , we consider a parabolic shape such that when  $|z - z_H| = 2 \mu\text{m}$ , we get  $x = 2 \mu\text{m}$ . This gives  $b = 0.25 (\mu\text{m})^{-1}$ . For  $c_s$  we take typical Fermi velocity of electrons,  $c_s \sim 10^6 \text{ m/s}$ . Using eq. (16), we get  $dv/dz = 5 \times 10^{11} \text{ s}^{-1}$ . Finally, from eq. (18), we estimate the Hawking temperature to be about 0.6 K. The peak frequency for this black body Hawking radiation is about  $10^{10} \text{ Hz}$ , corresponding to the energy of Hawking phonons.

Few points need to be discussed here. The Hawking temperature in eq. (18) is the temperature that an asymptotic observer sitting in asymptotically flat space-time would measure. The acoustic metric becomes flat when the fluid velocity becomes zero. In our sample geometry in fig. 2, the leftmost part has width  $2x_L = 10 \mu\text{m}$ . As the flow velocity is  $v = c_s$  at the neck where the width is  $2x_H = 2 \mu\text{m}$ , the flow velocity at the left edge will be  $c_s/5$ , directed towards the horizon. Thus there is no asymptotic observer in our sample geometry. The observer at the left edge of the sample sees a blueshifted Hawking temperature. The Lorentz  $\gamma$  factor for  $v = c_s/5$  is about 1.02, thus introducing only a negligible correction to the value of the Hawking temperature.

The second point is about the effect of non-zero  $v_x$  components of flow. As we discussed above,  $v_x$  will be almost zero near the  $z$ -axis, while it will be significant near the sample boundaries in the  $x$ -direction. Due to the fact that  $A' = 0$  at the sonic horizon, we expect  $v_x$  not to play a significant role in that region. However, for the left edge, where the observer is located, one needs to restrict attention to the region near  $x = 0$  so that  $v_x$  components of the flow can be safely neglected.

**Observational aspects.** – The Hawking radiation here is composed of quanta of acoustic perturbations in the velocity potential of the electron fluid. It has a thermal spectrum with a peak frequency of about  $10^{10} \text{ Hz}$ . For observations at the left edge of the sample, we can estimate the flux of radiation as follows. We first calculate the area of the horizon. For this we need thickness  $d$  of the electron gas system. The peak frequency of  $10^{10} \text{ Hz}$ , with sound velocity  $c_s = 10^6 \text{ m/s}$  gives a phonon wavelength

of about  $100 \mu\text{m}$ . For a consistent picture of Hawking radiation, the thickness  $d$  should be of order of the peak phonon frequency, *i.e.*, about 100 microns. However, the typical thickness of heterostructures is of the order of few hundred nanometres, much smaller than the phonon wavelength. We shall ignore this issue for quasi 2-D materials. For 3-D materials this will not be an issue. In our estimates,  $d$  only enters in calculating total flux of Hawking radiation, the area of the horizon being  $A_{bh} = 2x_H d$ . The power of the Hawking radiation emitted is

$$P_{bh} = \sigma T^4 A_{bh}. \quad (20)$$

We assume that this entire power is focussed towards the left edge of the sample, neglecting any phonon absorption at the sample boundary. This is in the spirit of neglecting momentum transfer from the electron fluid to the sample boundaries (necessary to get electron hydrodynamics regime in the first place). The flux of the Hawking radiation obtained at the left edge is then

$$F(z_L) = \sigma T^4 \frac{2x_H d}{2x_L d} = \sigma T^4 \frac{x_H}{x_L}, \quad (21)$$

where  $z_L$  denotes the  $z$ -coordinate at the left edge of the sample. This flux of the Hawking radiation is made up of quantized sound modes or phonons. Flux of energy in a sound wave with frequency  $f$  and amplitude  $A_{sound}$  is given by

$$F_{sound} = 2\pi^2 \rho c_s f^2 A_{sound}^2. \quad (22)$$

$\rho = n_e m_e$  where  $n_e$  and  $m_e$  are electron number density and effective electron mass in the sample. As a sample value [27], we take  $n = 10^{16} / \text{d m}^{-2}$ . For sample thickness, we take  $d = 100 \text{ nm}$ . With this we get  $n = 10^{23} \text{ m}^{-3}$ . For  $m_e$ , we take the free electron mass. For frequency  $f$ , we take the peak frequency of Hawking radiation,  $f \simeq 10^{10} \text{ Hz}$ . With these values, we equate the energy flux of the sound wave to the energy flux of Hawking radiation at the left edge of the sample to get

$$A_{sound} \simeq 3 \times 10^{-15} \text{ m}. \quad (23)$$

The ratio of the oscillatory part of the electric current to the average current at the left edge of the sample is given by

$$\frac{I_{oscl}}{I_0} = \frac{A_{sound} f}{v(z_L)} \simeq 10^{-10}. \quad (24)$$

For a background current  $I_0$  of order milliamperes,  $I_{oscl} \sim 10^{-13} \text{ amperes}$ . It is unclear to us whether such an oscillatory current can be observed through electromagnetic radiation. The total flux of microwave photons in this comes out to be too small. However, one may be able to observe this current oscillation directly. The important factor which distinguishes this current from a general background noise is its black body spectrum. Further, direct dependence on parameters like the shape of the neck and  $x_H/x_L$  ratio can help in identifying the signal.

**Conclusions.** – We have proposed the possibility of observing the Hawking radiation in an acoustic black hole system for electron hydrodynamics. This is expected to be realized in ultra-clean quasi 2-D materials as well as in Dirac and Weyl semi-metals in 3-D. For typical parameter values of such samples, our estimate gives a Hawking temperature of about 1 K. The resulting Hawking radiation will manifest in terms of sound modes of the electron fluid, hence in electric current oscillations. We estimate amplitude of current oscillations to be of order  $I_{oscl}/I_0 \simeq 10^{-10}$ . This current oscillation will have strictly black body spectrum of frequency as expected of Hawking radiation. Its specific dependence on system parameters, such as the curvature of the region near sonic horizon, can help in separating this signal from background noise.

We have made many strong simplifying assumptions. The peak wavelength of the Hawking radiation has been estimated to be about  $100 \mu\text{m}$ . For consistency, one should require all dimensions of the sample (subsonic region, supersonic region, and the thickness of the sample) to have at least this size. For standard experimental situations, this is not the case. It may not be easy to prepare ultra-clean samples of this size. For smaller system sizes, one may expect corrections to the estimates we have provided. However, the qualitative picture of acoustic black hole and resulting Hawking radiations should remain applicable.

It will be very interesting to calculate the current-current correlations for the subsonic and supersonic regions. This will carry signatures of the correlations between Hawking partners, just like density-density correlations in cold atom systems. For the system size we have considered, any electromagnetic radiation resulting from current oscillations is expected to be negligible. However, with suitably chosen parameters, *e.g.*, size/shape of the sample, it may be possible to observe imprints of this Hawking radiation in electromagnetic radiation resulting from current oscillations.

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