Letter

Anomalous multifractality in quantum chains with strongly correlated disorder

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We demonstrate numerically that a robust and unusual multifractal regime can emerge in a one-dimensional quantum chain with maximally correlated disorder, above a threshold disorder strength. This regime is preceded by a mixed and an extended regime at weaker disorder strengths, with the former hosting both extended and multifractal eigenstates. The multifractal states we find are markedly different from conventional multifractal states in their structure, as they reside approximately uniformly over a continuous segment of the chain, and the lengths of these segments scale nontrivially with system size. This anomalous nature also leaves imprints on dynamics. An initially localized wave packet shows ballistic transport, in contrast to the slow, generally subdiffusive, transport commonly associated with multifractality. However, the timescale over which this ballistic transport persists again scales nontrivially with the system size.

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Multifractal wave functions in quantum systems, which are neither extended nor localized, are characterized by anomalous statistics of their amplitudes [1]. While the effective volume occupied by such states grows unboundedly with system size, it is a vanishing fraction of the system volume; as such, they are often dubbed nonergodic extended states. This is reflected in the scaling of the moments of the wave-function amplitudes with system size. For a wave function $\psi(x)$ defined on a discrete graph with L sites,

$$\sum_{x=1}^{L} |\psi(x)|^{2q} \sim \begin{cases} L^{-(q-1)} & \text{extended} \\ L^{-D_q(q-1)} & \text{multifractal} \\ L^{0} & \text{localized,} \end{cases}$$

where $0 < D_q < 1$ is the so-called multifractal dimension [1].

In the context of short-ranged disordered systems in finite dimensions, multifractality is often a feature of critical points, such as Anderson transitions [1–12] and quantum Hall plateau transitions [13–15], which are clearly fine-tuned points in parameter space. Multifractality is also realized, often robustly, in several long-ranged disordered hopping models, and fully connected random-matrix ensembles [16–29]. The presence of long-ranged physics, either emergently via diverging correlation lengths in the former, or explicitly via the structure of the models in the latter, unifies the two contexts. An interesting question thus arises: How can a robust multifractal phase be realized in a quantum system with inherently short-ranged interactions or hoppings? One possible avenue in a manifestly out-of-equilibrium setting is the time-periodic modulation of a quasiperiodic system with a mobility edge or a localization transition [30,31].

In this Research Letter, we demonstrate an alternative pathway to robust multifractality in an inherently short-ranged system, importantly in a time-independent Hamiltonian setting. The central ingredient is strong (in fact *maximal*, as clarified below) correlations in the disordered on-site potential of a one-dimensional chain. Interestingly, the origin and resultant structure of the multifractal states in such a system are markedly different from those of conventional multifractal states, which are associated with both rare large peaks and long polynomial tails of wave-function amplitudes. In contrast, the multifractal states we find in this Research Letter reside over continuous segments of length ℓ in the chain, and crucially, within the segments, the wave-function intensities are approximately *uniform*; $|\psi(x)|^2 \approx 1/\ell$, and 0 elsewhere. Due to this structure, we refer to the states as *tabletop* states. The multifractality of the wave functions is then encoded in the scaling of these tabletop lengths with system size. The anomalous nature of the multifractal states also has implications for dynamics. Multifractality of eigenstates is often accompanied by slow dynamics [30,32–35]. However, in this case, we find ballistic spreading of an initially localized wave packet, but over timescales that scale nontrivially with the system size.

As a concrete model, we consider a disordered tight-binding Hamiltonian on a chain of length L,

$$H = W\sum_{x=1}^{L} \epsilon_x |x\rangle\langle x| + \sum_{x=1}^{L-1} [|x\rangle\langle x+1| + |x+1\rangle\langle x|], \quad (1)$$

where W denotes the disorder strength and the on-site potentials are drawn from a multivariate Gaussian distribution, $\vec{\epsilon}_x \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$, with zero mean. The correlations in the

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FIG. 1. Schematic of the three regimes of the model, Eq. (1), as a function of disorder strength. For small disorder, all states are extended, and for large enough disorder all states are multifractal. In between these two phases is a mixed regime, where the disorder-averaged spectrum hosts a mixture of extended and multifractal states.

potential are encoded in the covariance matrix **C**. We take the correlations in the potential to decay with distance as

$$C(r) \equiv [\mathbf{C}]_{x,x+r} = \langle \epsilon_x \epsilon_{x+r} \rangle = f(r/L), \quad f(0) = 1. \tag{2}$$

The key point here is that the correlation is a function of r/L, which implies that in the thermodynamic limit $C(r) \to 1$ for all subextensive r, as $\lim_{L\to\infty}(r/L) \to 0$. In other words, the potentials of two sites a subextensive distance r apart are completely slaved to each other in the thermodynamic limit. We refer to this as *maximal correlations* in the disorder [36,37]. For specificity we choose $f(r/L) = \exp[-r/(\lambda L)]$, but emphasize that the specific functional form of f is immaterial [38]. In the following we set $\lambda = 1$. Note that the limit of $\lambda \to 0$ is singular, as there the model becomes the conventional one-dimensional (1D) Anderson model with all eigenstates exponentially localized [39,40].

The form of the correlations, Eq. (2), endows the disordered potential with an extensive length scale (λL in this case), such that the potential fluctuations on subextensive scales are heavily suppressed and only those at extensive scales survive for large L. This is already suggestive that the eigenstates can be extended over length scales which scale nontrivially with L, resulting in multifractality. In fact, as demonstrated below, we find three distinct regimes as a function of W. For sufficiently strong disorder a robust multifractal phase is found, where the average or typical tabletop lengths scale as L^α with $\alpha < 1$; in contrast, for weak disorder we find $\alpha = 1$ for all eigenstates, indicating an extended phase. For a range of W between these two regimes a mixed phase is found, where for a given energy some realizations host extended states and some host multifractal states. Establishing these robust multifractal and extended regimes with the intervening mixed regime in a model with maximally correlated disorder is the central result of this work and is summarized in Fig. 1.

Before delving into a detailed analysis of the statistics of eigenstate tabletop lengths and the consequent multifractality, in Fig. 2 we show explicitly the tabletop nature of eigenstates. Operationally, we extract the tabletop edges for an eigenstate by scanning the chain for sites where the $|\psi_x|^2$ jumps from zero (within numerical precision) and the tabletop length ℓ is then simply the distance between two such sites. It is evident from Fig. 2 that the eigenstates are approximately uniform over the tabletop segments. Hence it is natural to study the distribution of the tabletop lengths ℓ, or equivalently of $\tilde{\ell} = \ell/L$, denoting these distributions by P_ℓ and $P_{\tilde{\ell}}$, respectively. Since we are interested, in particular, in how ℓ scales with system size L, we define the exponents α_m and α_t from the mean and typical tabletop lengths as

$$\ell_m \equiv \langle \ell \rangle \sim L^{\alpha_m}, \quad \ell_t \equiv \exp[\langle \ln \ell \rangle] \sim L^{\alpha_t}, \tag{3}$$

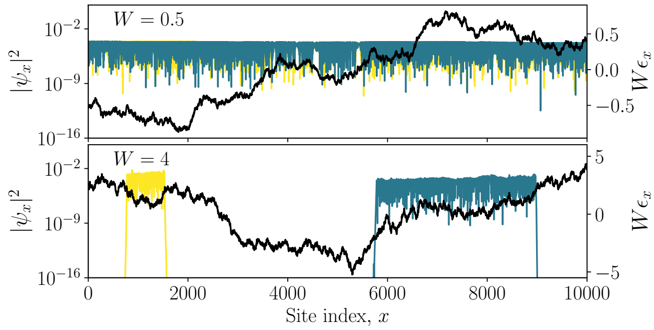

FIG. 2. For weak and strong disorder strengths W (top and bottom, respectively), representative eigenstates are shown in blue and yellow. In the former, tabletops span the entire chain, whereas in the latter they span a subextensive segment. Black traces denote a realization of the maximally correlated disorder potential (labels on right axis) wherein short-distance fluctuations are visibly suppressed.

where $\langle \ell \rangle = \int d\ell\, \ell P_\ell(\ell)$ and similarly for $\langle \ln \ell \rangle$. In addition, we also define the exponent $\alpha = \ln \ell / \ln L$ and study its distribution, which we denote by P_α.

We turn now to numerical results, which unless stated otherwise refer to band center states (results for other energies remain qualitatively the same). For weak disorder, we find that the tabletops span not only an extensive segment of the chain, but the entirety of it, such that $P_\ell(\ell) = \delta(\ell - L)$ or, equivalently, $P_\alpha(\alpha) = \delta(\alpha - 1)$ [Figs. 3(a) and 3(d)]. This is the extended regime indicated in Fig. 1. On increasing W, the distribution $P_\ell(\ell)$ develops finite weight at subextensive ℓ while retaining the rest of the weight at extensive ℓ [Figs. 3(b) and 3(e)]. This is the mixed regime referred to in Fig. 1. Note that it is important to distinguish the multifractal states with subextensive ℓ from those that occupy an extensive $\ell = aL$ with $a \leqslant 1$, and hence confirm the presence of the former. That this is indeed the case is evidenced in Fig. 3(b), where the

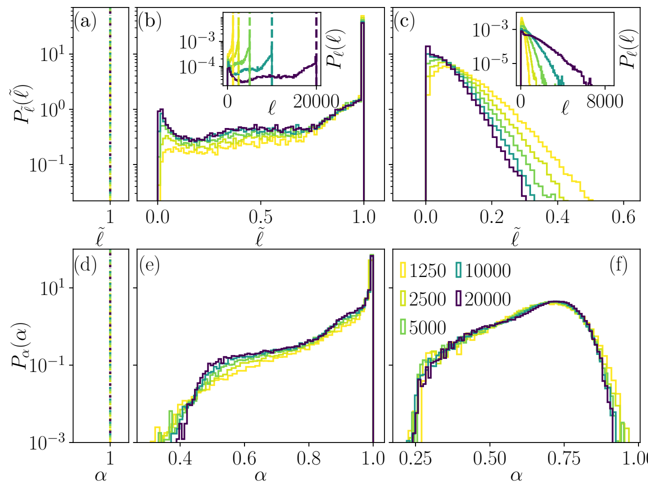

FIG. 3. (a)–(c) Distributions $P_{\tilde{\ell}}(\tilde{\ell})$ of $\tilde{\ell} = \ell/L$ for representative disorder strengths in the extended ($W = 0.5$), mixed ($W = 2$), and multifractal ($W = 8$) regimes, for different system sizes L [indicated in (f)]. Insets show the corresponding $P_\ell(\ell)$ distributions. (d)–(f) Distributions $P_\alpha(\alpha)$ for the corresponding W values. Statistics are obtained over 20 000 disorder realizations.

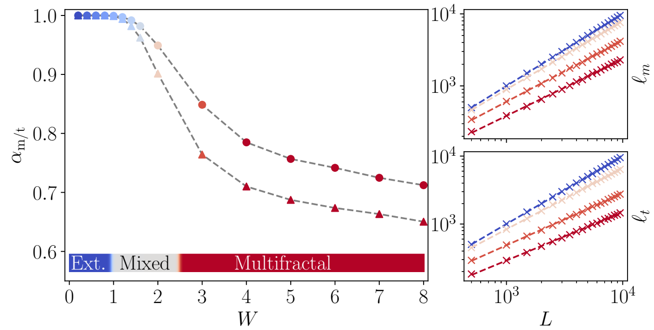

FIG. 4. Left: Exponents α_m (circles) and α_t (triangles), defined in Eq. (3), as a function of disorder strength W. Right: Fits of ℓ_m and ℓ_t vs L on logarithmic axes, used to extract the exponents. Data are shown for $W = 0.5$, 2, 3, and 4.

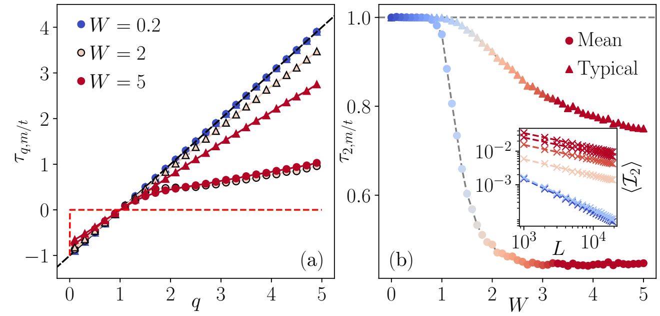

FIG. 5. (a) Exponents $\tau_{q,m}$ (circles) and $\tau_{q,t}$ (triangles), defined in Eq. (4), for representative W in the three regimes. Black and red dashed lines denote extended and localized behavior, respectively. (b) Exponents $\tau_{2,m}$ and $\tau_{2,t}$ as a function of W. The inset shows representative plots of $\langle \mathcal{I}_2 \rangle$ vs L, used to extract $\tau_{2,m}$, for $W = 0, 1, 2, 3, 4, 5$.

weight of the distribution $P_{\tilde{\ell}}(\tilde{\ell})$ at $\tilde{\ell} = 0$ grows with increasing L. Further clear evidence for the mixed regime is also seen in Fig. 3(e) by the fact that $P_\alpha(\alpha)$ retains weight at both $\alpha = 1$ and $\alpha < 1$ as $L \to \infty$. Finally, at strong disorder, the system enters a regime where all states in the spectrum are multifractal. As shown in Fig. 3(c), for finite $\tilde{\ell}$ the weight in $P_{\tilde{\ell}}(\tilde{\ell})$ ultimately decays with increasing L, suggesting that there exist no states with extensive tabletop lengths. That all states are indeed multifractal is further confirmed by the distribution $P_\alpha(\alpha)$ (which is well converged with L) having support strictly on $0 < \alpha < 1$ as shown in Fig. 3(f). The vanishing of the weight of $P_\alpha(\alpha)$ at $\alpha = 1$ and $\alpha = 0$ implies the absence of extended and localized states, respectively. We add that this three-phase picture is also consistent with results for the scaling of transmittances with L, obtained via a transfer-matrix calculation [38].

Having established the presence of a multifractal regime, along with an extended and a mixed regime, based on the distributions $P_{\tilde{\ell}}$ and P_α, we next present results for the scaling of mean and typical tabletop lengths with L. In particular, Fig. 4 shows results for α_m and α_t [defined in Eq. (3)] as a function of W. For weak disorder we find both $\alpha_m = 1$ and $\alpha_t = 1$, consistent with the presence solely of extended states. On increasing W and entering the mixed regime, α_t and α_m decrease from 1, indicating the emergence of multifractal states in the spectrum. Note that on entering the mixed regime, α_m deviates from 1 less markedly than α_t. This is natural, as in the presence of both extended and multifractal states the mean ℓ_m is dominated by the extended states with $\ell \sim L$, whence α_m is closer to 1 than α_t. Finally, on increasing W further into the regime where the spectrum has solely multifractal states, α_m and $\alpha_t < \alpha_m$ continue to decrease monotonically.

We close our analysis of the multifractal statistics of tabletop eigenstates with results of a standard probe of multifractality, the generalized inverse participation ratios (IPRs), defined as $\mathcal{I}_q(\psi) = \sum_{x=1}^{L} |\psi(x)|^{2q}$. We will be interested in the scaling with L of both the mean and typical IPR,

$$\mathcal{I}_{q,m} = \langle \mathcal{I}_q \rangle \sim L^{-\tau_{q,m}}, \quad \mathcal{I}_{q,t} = \exp[\langle \ln \mathcal{I}_q \rangle] \sim L^{-\tau_{q,t}}. \quad (4)$$

Extended states have $\tau_{q,m/t} = q - 1$, while for exponentially localized states $\tau_{q,m/t} = 0$ for $q > 0$. An intermediate behavior for τ_q indicates multifractal states [1]. Figure 5(a) shows results for $\tau_{q,m}$ and $\tau_{q,t}$, for representative W values in each of the three regimes. For the weakest disorder, which lies in the extended regime, we indeed find $\tau_{q,m/t} = q - 1$. The presence of multifractal states upon increasing W is borne out by $0 < \tau_{q,m/t} < q - 1$ for $q > 1$. In Fig. 5(b), we focus on $\tau_{2,m/t}$ as a function of W. Note that on increasing W and entering the mixed regime, $\tau_{2,m}$ deviates from the ergodic value of 1 more markedly than $\tau_{2,t}$, reflecting the fact that the mean IPR is dominated by the small fraction of multifractal states which have qualitatively larger IPRs than the extended ones.

An explanatory comment is in order regarding the lack of energy resolution between extended and multifractal states in the mixed regime. The density of states (DoS) for the model (1) with the correlated disorder (2) can be shown to fluctuate strongly across disorder realizations [38]. For each realization we choose to refer energies relative to the center of the spectrum (tantamount to $H \to H - \mathrm{Tr}[H]$). However, the fluctuations in all higher moments of the DoS also remain finite in the thermodynamic limit. As a result, whether an eigenstate at a given energy (relative to the center of the spectrum) is extended or multifractal depends on the specific disorder realization (though for any given realization, multifractal and extended states do not of course coexist at the same energy). Averaging over disorder realizations therefore smears out any energy resolution in the mixed regime, thereby precluding the traditional notion of a mobility edge in the averaged DoS.

So far, we have focused on "static" properties of the model. Since multifractality often goes hand in hand with slow dynamics [30,32–34], it is worth asking what imprint the anomalous multifractal states leave on the dynamics in the present case. In order to answer this question, we consider the spreading of an initially localized wave packet, in particular, its second moment defined as

$$X^2(t) = \left\langle \sum_{x=1}^{L} x^2 |\psi(x,t)|^2 - \left[\sum_{x=1}^{L} x |\psi(x,t)|^2 \right]^2 \right\rangle, \quad (5)$$

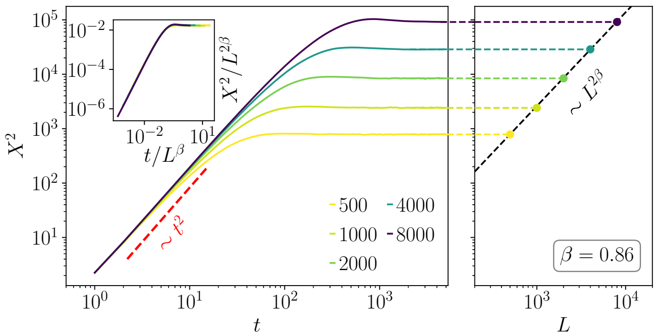

FIG. 6. Dynamics of an initially localized wave packet in the multifractal regime, for $W = 6$. Left: Ballistic transport of the wave packet as $X^2(t)$ [defined in Eq. (5)] initially grows as t^2. Right: The saturation value, $X^2(t \to \infty) \sim L^{2\beta}$, plotted as a function of L, with $\beta \simeq 0.85$. The inset shows the scale-collapsed form $X^2(t) = L^{2\beta} g(t/L^\beta)$, with $g(y) \sim y^2$ for $y \ll 1$.

with $\psi(x, t = 0) = \delta_{x,L/2}$. The simplest expected behavior for $X^2(t)$ is

$$X^2(t) \sim \begin{cases} t^{2z}, & t \ll t_* \\ L^{2\beta}, & t \gg t_*, \end{cases} \tag{6}$$

with crossover scale $t_* \sim L^{\beta/z}$. For example, $z, \beta = 1$ would correspond to ballistic transport until the wave packet spans the entire system; in contrast, for localized states, $z, \beta = 0$, indicating the absence of transport. Conventionally, multifractal states lead to $0 < z,\ \beta < 1$ [30,32–34].

Figure 6 shows results for the maximally correlated model in the pure multifractal phase. Remarkably, the transport is ballistic with $z = 1$, but the saturation of $X^2(t)$ scales as $L^{2\beta}$ with exponent $\beta < 1$. In fact, the inset in Fig. 6 shows the scale-collapsed behavior $X^2(t) = L^{2\beta} g(t/L^\beta)$ with $g(y) \sim y^2$ for $y \ll 1$, which confirms the ballistic spreading and subextensive saturation of the wave packet. We now show how a toy model of the anomalous multifractal states qualitatively rationalizes the above dynamics.

For simplicity, let us assume that in a given realization the initially localized wave packet lies at the center of a tabletop segment of length ℓ. Approximating the eigenstates therein as fully extended over the segment, the wave packet spreads as a truncated Gaussian

$$|\psi(x, t)|^2 = \Theta(\ell - 2|x|)e^{-x^2/t^2}/(t\sqrt{\pi}\mathrm{Erf}(\ell/2t)), \tag{7}$$

where we have absorbed the velocity into the units of time. $X^2(t)$ for the wave packet (7) behaves in time as

$$X_\ell^2(t) = \tfrac{1}{2}t^2[1 - e^{-\ell^2/4t^2}(\ell/t)/(\sqrt{\pi}\mathrm{Erf}(\ell/2t))]. \tag{8}$$

Averaging over disorder is equivalent to integrating over the probability distribution of the initial site residing in a tabletop of length ℓ, denoted by $\Phi_\ell(\ell, L)$. Thus

$$X^2(t) = \tfrac{1}{2}t^2\left[1 - \frac{2}{\sqrt{\pi}} \int d\ell \Phi_\ell(\ell, L) \frac{\ell e^{-\ell^2/4t^2}}{2t\mathrm{Erf}(\ell/2t)}\right]. \tag{9}$$

For timescales much smaller than a typical tabletop length, the second term in Eq. (9) is negligibly small, and the ballistic behavior $X^2(t) \sim t^2$ is recovered. In the opposite limit where t is much larger than typical tabletop lengths [such that $\ell/2t \ll 1$ in (9)], the wave-packet spreading saturates, and $X^2 \simeq \int d\ell\, \Phi_\ell(\ell, L)\ell^2$, which scales nontrivially with L due to the scaling of $\Phi_\ell(\ell, L)$. In the Supplemental Material, we provide numerical evidence for the essential validity of this heuristic argument [38].

Finally, we provide a simple heuristic argument for the physics underlying the emergence of the multifractal regime. Note that with the form of the correlations, $C(r) = e^{-r/L}$, the sequence of site energies $\ldots, \epsilon_i, \epsilon_{i+1}, \ldots$ is readily shown to be a martingale. Thus, given ϵ_i, ϵ_{i+1} is a Gaussian random number with mean $e^{-1/L}\epsilon_i$ and variance $\simeq 2/L$. This leads to the site energies of nearby sites being very close to each other with high probability, to which we attribute the approximately uniform $|\psi(x)|^2$ across the tabletop segment. However, since the conditional distribution $P(\epsilon_{i+1}|\epsilon_i)$ is a Gaussian, it has unbounded support. As a result, with very low probability, there can be neighboring sites where the site energies are wildly different. The break in the tabletops may be attributed to such rare regions. Moreover, as the sequence of site energies is a martingale, the probability of a tabletop terminating at a site is independent of the energies of the sites prior to it. It is then intuitively natural to regard the tabletop breaks as a Poisson process. This in turn implies $P_\ell(\ell) \sim e^{-\gamma\ell}$, where γ is the density of tabletop breaks, which decays with increasing L [41]. An exponential $P_\ell(\ell)$ with a rate that decays with L is indeed consistent with the numerical results shown in Fig. 3(c) in the strong-disorder regime. Obviously, however, obtaining the precise multifractal exponents would require a theory which takes into account the fluctuations of $|\psi(x)|^2$ within the tabletop length.

In summary, we demonstrated numerically that a one-dimensional quantum chain (1), with maximally correlated disorder (2), hosts a robust multifractal regime. These multifractal eigenstates are strikingly unusual. They reside approximately uniformly over segments of the chain whose lengths scale nontrivially with L, whence the multifractal statistics arise. Such an anomalous structure also leaves imprints on the dynamics—in the multifractal phase an initially localized wave packet spreads ballistically, but over timescales that scale as L^β with $\beta < 1$, beyond which the expansion saturates.

We note that the behavior found here differs radically [42] from the same model considered on a tree with connectivity $K \geqslant 2$ [37], which hosts localized states but not a multifractal phase. The present work is also very different from earlier studies [43–45] of 1D Anderson localization with long-ranged, power-law (and hence scale-free) disorder correlations. These models too do not host a robust multifractal regime. This suggests that the system-size-dependent scale introduced by the correlations (2), along with the martingale nature of the site energies, is responsible for the robust multifractal regime in the 1D model with maximally correlated disorder.

Finally, it is interesting to note that the effective Fock-space disorder in a many-body localized system is likewise maximally correlated [36], and the many-body localized eigenstates indeed exhibit multifractality on the Fock space [46–49]. Whether a concrete connection between maximal disorder correlations and multifractality exists, and if so under what conditions, remains an open question.

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