Jointly Broadcasting Data and Power with Quality of Service Guarantees

Deekshith P K*, Trupthi Chougule†, Shreya Turmari†, Ramya Raju†, Rakshitha Ram† and Vinod Sharma*
*ECE Dept., Indian Institute of Science, Bangalore, India. †PES Institute of Technology, Bangalore, India.
*Email: {deeks, vinod}@ece.iisc.ernet.in,
†Email:{chouguletrupthi, shreya.turmari, ramyaraju93, rakshitharam.93}@gmail.com.

Abstract—**In this work, we consider a scenario wherein an energy harvesting wireless radio equipment sends information to multiple receivers alongside powering them. In addition to harvesting the incoming radio frequency (RF) energy, the receivers also harvest energy from its environment (e.g., solar energy). This communication framework is captured by a fading Gaussian Broadcast Channel (GBC) with energy harvesting transmitter and receivers. In order to ensure some quality of service (QoS) in data reception among the receivers, we impose a *minimum-rate* requirement on data transmission. For the setting in place, we characterize the fundamental limits in jointly transmitting information and power subject to a QoS guarantee, for three cardinal receiver structures namely, *ideal*, *time-switching* and *power-splitting*. We show that a time-switching receiver can switch between information reception mode and energy harvesting mode, *without* the transmitter's knowledge of the same and *without* any extra *rate loss*. We also prove that, for the same amount of power transferred, on average, a power-splitting receiver supports higher data rates compared to a time-switching receiver.**

I. Introduction

Intentionally transferring energy along with information, using radio frequency (RF) signals, is an attractive alternative to perpetually and remotely power energy harvesting sensors that have limited physical accessibility. It is foreseeable that in next generation wireless systems, a picocell or femtocell base station will be enabled to wirelessly charge low power communication devices within its range. These base stations themselves could be energy harvesting *green* base stations. Apart from the numerous system design challenges the problem offers, it also opens up a rich set of theoretically motivated research avenues. On this premise, we address the problem of characterizing the fundamental limits in jointly broadcasting data and power over a wireless medium with energy harvesting transmitter and receivers.

In this work, we consider the problem of Simultaneous Wireless Information and Power Transfer (SWIPT) over a fading Gaussian broadcast channel (GBC) with an energy harvesting transmitter, ensuring a certain quality of service (QoS) guarantee to the receivers. The QoS parameter we refer to is that of *minimum-rate* constraint. For the canonical fading GBC (non energy harvesting), the problem of characterizing the fundamental limits with minimum rate constraints as a means to ensure *fairness* among receivers is a well studied topic ([1]). In the context of SWIPT, the above constraint has the added advantage that the transmission ensures a *minimum instantaneous RF power* at the receivers at all times (which can potentially be harvested).

Recently, various studies have investigated the problem of characterizing the trade-offs involved in SWIPT in a multi-user communication system. Capacity-energy regions of a discrete memoryless multiple access channel and a multi-hop channel with a single relay is characterized in [2]. Achievable rates over an *uplink* channel wherein the transmitters are powered via RF signals in the *downlink* are provided in [3]. In [4], authors report a result on feedback enhancing the *rate-energy region* over a *constant gain* multiple access channel with simultaneous transmission of information and power.

As for broadcast systems, [5] studies MIMO SWIPT by a transmitter to two receivers (either *spatially separated* or *co-located*) in which one receiver harvests energy and the other receiver decodes information. In particular, for the co-located case, the authors derive an upper bound on achievable rate-energy region and compare it with that achievable using a *time-switching* and *power-splitting reveiver*. A broadcast MISO SWIPT system with a power-splitting receiver is considered in [6] wherein the authors jointly *design* the optimal transmit beamforming vectors and receiver power splitting ratios. In [7], authors propose an energy beamforming strategy and obtain the corresponding achievable rate-energy region for an intereference channel with time-switching receivers. For a comprehensive survey of recent advances in the domain of RF energy harvesting networks, refer [8].

In the fading GBC setting that we consider, we assume that both the transmitter and the receivers can harvest energy from a perennial ambient source. In addition, the receivers treat the transmitter as an RF energy source to meet additional energy requirements, if any. In this setting, we characterize the fundamental limits of SWIPT under a minimum-rate constraint. Our contributions are two-fold. Existing works, to the best of our knowledge, do not consider the information theoretic characterization of fundamental limits of SWIPT systems *with energy harvesting transmitter(s)* under a *non-quasi-static* fading environment. The corresponding results where the transmitter is not energy harvesting and the receivers in particular are not harvesting energy from a non-RF source, can thus be obtained as special cases. Second novel aspect in our model is the inclusion of minimum-rate constraints in characterizing the fundamental limits of SWIPT systems.

This paper is organized as follows. In Section II, we present the system model and notation. Section III is devoted to explain the main results of this work. We derive the minimum-rate capacity region of the SWIPT system under consideration, with ideal, time-switching and power-splitting receivers. Numerical results are provided in Section IV. We conclude in Section V. Proofs are sketched in the Appendices.

II. System Model and Notation

A. Transmission with QoS Constraints

Consider an energy harvesting transmitter equipped with an energy buffer (synonymous with battery or buffer) of infinite capacity. The transmitter could well be a *green* base station harnessing renewable energy, like solar or wind energy. Alternatively, in the context of energy harvesting sensor networks,

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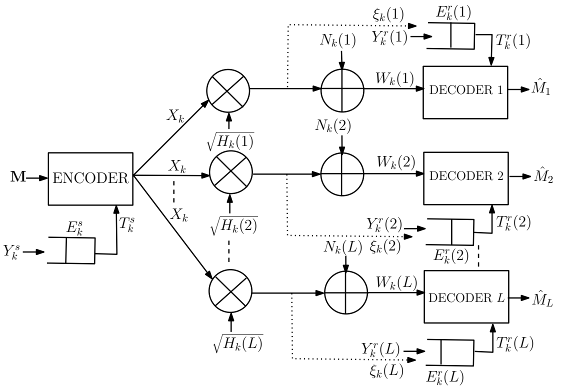

Fig. 1. A fading GBC with SWIPT.

transmitter could represent a *fusion centre* which multiple sensor nodes report to.

We consider a time slotted system. In slot k, let Y_k^s (s indicates sender) denote the energy harvested by the transmitter from a renewable source. We assume the energy harvesting process $\{Y_k^s,\ k \geq 1\}$ is stationary, ergodic. Let $\mathbb{E}[Y^s]$ denote the mean value of the energy harvesting process. Let E_k^s denote the energy available in the transmitter's buffer at the beginning of slot k. In the model we consider, the harvested energy Y_k^s can be used in the same slot and the remaining, if any, is stored in the buffer for future use. Let T_k^s denote the energy used up by the transmitter in slot k. Thus, $T_k^s \leq \hat{E}_k^s \triangleq E_k^s + Y_k^s$. The energy in the buffer evolves according to $E_{k+1}^s = \hat{E}_k^s - T_k^s$.

The transmitter has L messages to send, denoted by the message vector $\mathbf{M} \triangleq (M_1, \ldots, M_L)$, to L distant receivers, where M_l is the message corresponding to receiver $l \in [1 : L] \triangleq \{1, 2, \ldots, L\}$. Simultaneously, the transmitter is powering each of the receivers. The receivers, in practice, could either be user mobile devices or low power sensor motes. Corresponding to the message vector $\mathbf{M}$, a codeword of length n $\big(X_1'(\mathbf{M}), \ldots, X_n'(\mathbf{M})\big)$, is chosen. Since the transmitter is energy harvesting, transmitted symbol in slot k could be different from the codeword symbol because of insufficient energy at the transmitter. The channel input symbol in slot k is denoted as X_k. We note that the total energy used for transmission in slot k, $T_k^s = X_k^2 = \sum_{l=1}^{L} T_k^s(l)$, where $T_k^s(l)$ is the energy allocated for receiver l in slot k.

On account of the time varying nature of the underlying wireless channel, some users may be cut off from the transmitter for a certain duration of time depending upon the channel conditions. This is because the power allocation strategy which ensures the optimal *long term* data rates will allocate zero transmission power in certain time slots to those users with low channel gains [9]. At the same time, it is not desirable to transmit at a target rate irrespective of the channel gains (essentially by a multi user variant of *channel inversion*) as it reduces the permissible data rates [10]. An alternative to the above approaches is transmitting at a certain *minimum instantaneous rate* irrespective of the channel conditions (there by ensuring certain fairness among receivers) and use the additional power to maximize the long term achievable data rates [1]. Accordingly, let $\rho(l)$ be the minimum rate of transmission to be ensured to receiver l, irrespective of the channel conditions. The model parameter $\boldsymbol{\rho} \triangleq \big(\rho(1), \ldots, \rho(L)\big)$ dictates the quality of service guarantee on the joint data and power broadcast.

B. The Channel Model

The channel from the transmitter to the l^{th} receiver is a fading channel corrupted by an independent and identically distributed (i.i.d.) additive Gaussian noise process $\{N_k(l),\ k\}$ at the receiver. We denote the probability density function of $N_k(l)$ (having mean 0 and variance σ_l^2) by $\mathcal{N}(0, \sigma_l^2)$. The multiplicative channel gain from the transmitter to the l^{th} receiver in slot k is denoted as $H_k(l)$. We assume that the fading process $\{\mathbf{H}_k,\ k \geq 1\}$ is jointly stationary, ergodic, where $\mathbf{H}_k \triangleq \big(H_k(1), \ldots, H_k(L)\big) \in \mathcal{H}^L$ with stationary distribution $F_{\mathbf{H}}$. Here, $\mathcal{H} \subset \mathbb{R}^+$, the positive real axis and $\mathcal{H}^L$ is the Cartesian product $\mathcal{H} \times \ldots \times \mathcal{H}$ (L times). In addition, we assume that the channel gains $H_k(l)$ and $H_k(j)$ are statistically independent for $l \neq j$ and are known to all the receivers and the transmitter at time k. We consider a block fading channel model wherein the channel gain from the transmitter to receiver l remains fixed for the duration of a *channel coherence time* $T_c(l)$. The codeword length n is assumed to be an integer multiple of the least common multiple of $\{T_c(l),\ l \in [1 : L]\}$. If $W_k(l)$ is the channel output at receiver l in slot k, $W_k(l) = \sqrt{H_k(l)} X_k + N_k(l)$.

Note that, ensuring a certain minimum non-zero transmission power to all users in all time slots (dictated by $\boldsymbol{\rho}$), potentially requires infinite power if the fading process, with positive probability, can take values arbitrarily close to zero. As an example, for the same reason, the zero outage capacity region for a Rayleigh fading GBC is *null* ([10]). To encompass those transmission schemes that require a finite average power and ensure non-zero minimum rate, we make the assumption that $\mathbb{E}[\frac{1}{H_k(l)}] < \infty$ for all l and for all $k \geq 1$.

C. Receiver

The co-located receiver architectures that we consider for SWIPT serve a dual purpose. There is a communication receiver to receive and decode the incoming data, and a rectenna module to harvest the RF energy. In slot k, receiver l harvests $Y_k^r(l)$ (r denotes receiver) from a surrounding perennial source. We assume that $\{Y_k^r(l),\ k \geq 1\}$ is a stationary, ergodic process for each l and is independent across receivers. Let $\mathbb{E}[Y^r(l)]$ denote its mean value. The receivers have an energy buffer of infinite capacity. Let $E_k^r(l)$ denote the energy in l^{th} receiver's buffer at the beginning of slot k. Let $\hat{E}_k^r(l) \triangleq E_k^r(l) + Y_k^r(l)$. There are various sources of energy consumption at the receivers. The front end of the communication receiver requires energy for filtering and other processing operations. This energy requirement, at receiver l, is modelled by a stationary, ergodic process $\{T_k^r(l),\ k \geq 1\}$. We refer to $\Delta_l \triangleq (\mathbb{E}[T^r(l)] - \mathbb{E}[Y^r(l)])^+$ as the average energy deficit at receiver l, where $(.)^+ = \max\{0, .\}$. Receiver l harvests, on an average, Δ_l units of RF energy so as to compensate for the deficit.

We now provide a brief description of the various receiver architectures considered in this work. An ideal receiver can harvest the incoming RF energy without *distorting* the noise corrupted data symbol. Thus, the total energy harvested $D_k^r(l)$ at receiver l in slot k is $Y_k^r(l) + \xi_k(l)$, where $\xi_k(l) = \eta H_k(l) X_k^2$. Here, η denotes the efficiency factor of the energy harvesting system ([8]). In contrast, a time-switching receiver harvests RF energy in a slot, at the expense of erasing the corresponding noise corrupted data symbol. Let $\mathcal{I}_{l,k}$ denote the indicator function of the event that RF energy is harvested by receiver l in slot k. Then, $D_k^r(l) = Y_k^r(l) + \mathcal{I}_{l,k}\xi_k(l)$. A power-splitting receiver *divides* the incoming RF power between the communication module and the rectenna, non-adaptively. We refer to it as the constant fraction power-splitting receiver. If $0 \leq \pi_{\mathcal{E}}(l) \leq 1$ is the fraction of power harvested in every slot, $D_k^r(l) = Y_k^r(l) + \pi_{\mathcal{E}}(l)\xi_k(l)$.

In general, among the L receivers, some receivers could be ideal and some others could be time-switching or power-splitting. But for the sake of exposition, we derive results assuming that all the receivers belong to one of the above kind. Our proof techniques readily yield the corresponding results for the general case as well.

In this work, we derive the fundamental limits in the framework propounded in [11]. Specifically, the channel input and output processes need not be stationary, since at time $k = 0$, the transmitter and the receivers start operating with *arbitrary* initial energy in their buffers. We consider *power control policies* (to combat fading) at the transmitter such that the stochastic process $\{T_k^s,\ k \geq 1\}$ is an asymptotic mean stationary (AMS), ergodic process [12]. We prove that, for the SWIPT system in place, the minimum-rate *AMS capacity region* is equivalent to that of a non energy harvesting system with the same average power constraints.

III. Minimum-Rate Capacity Region with Various Receiver Architectures

In this section, we derive the minimum-rate capacity regions of the SWIPT system for the three receiver models. We begin with the following definitions. An energy management policy T_k^s is called a *Markovian policy*, if it is exclusively a function of the variables $\hat{E}_k^s$ and $\mathbf{H}_k$. In this work, we only consider policies that are Markovian. We refer to such policies as Markovian because, if the processes $\{Y_k^s\}$, $\{\mathbf{H}_k\}$, $\{Y_k^r(l), l \in [1:L]\}$, $\{T_k^r(l), l \in [1:L]\}$ are i.i.d., adopting Markov policies make the joint process $\{\big(Y_k^s, E_k^s, X_k^s, W_k(1), \ldots, W_k(L)\big)\}$ a Markov process. We prove that such policies are *optimal* among the class of AMS, ergodic policies. A rate tuple $\mathbf{R} = \big(R(1), \ldots R(L)\big)$ is *achievable* if there exists a sequence of $\big((2^{nR_1}, \ldots, 2^{nR_L}), n\big)$ codes, an encoding function, a power controller so that for each joint fading state $\mathbf{h} = \big(h(1), \ldots h(L)\big)$, the instantaneous rate vector $\mathbf{R}(\mathbf{h}) \triangleq \big(R_1(\mathbf{h}), \ldots R_L(\mathbf{h})\big)$ satisfies $R_i(\mathbf{h}) \geq \rho(i)$ and $\mathbb{E}_{\mathbf{H}}[R_i(\mathbf{H})] \geq R(i)$, L decoders and energy harvesters, such that the average probability of decoding error (averaged over all possible realizations of codebooks) $P_e^{(n)} \to 0$ as $n \to \infty$. We follow the definition of *minimum-rate capacity region* provided in [1] (Definition 1, equation (7)). Since this definition explicitly involves *instantaneous rates*, the maximum achievable rate is not the capacity region in the usual Shannon theoretic sense. However, this is also the capacity region considered in [1], and we will continue to call it the capacity region in the following.

We note that the minimum-rate vector $\boldsymbol{\rho}$ should be within the zero outage capacity region [10] of a fading GBC with the peak power constraint corresponding to the minimum peak power imposed by the energy harvesting process. Since non zero minimum rates can be ensured only if the energy harvesting process $\{Y_k^s\}$ at the transmitter is such that $Y_k^s > \delta$ a.s. for some small $\delta > 0$ for all k, we assume the same.

A. SWIPT System: Ideal Receivers

We now provide a characterization of the minimum-rate capacity region when all receivers are assumed to be ideal. Let $\Sigma_l(\mathbf{H}) \triangleq H(l)T_l'^s(\mathbf{H})$, $\nu_l(\mathbf{H}) \triangleq \sigma_l^2 + \sum_{j=1}^{L} H(j)T_j'^s(\mathbf{H})\mathbb{1}_{\mathcal{E}_{l,j}}$, where $\mathbb{1}_{\mathcal{E}_{l,j}}$ is the indicator function corresponding to the event $\mathcal{E}_{l,j} \triangleq \{\sigma_l^2 H(j) > \sigma_j^2 H(l)\}$, $T_l'^s$ is an energy allocation policy corresponding to receiver l and $\text{SNR}_l(\mathbf{H}) \triangleq \Sigma_l(\mathbf{H})/\nu_l(\mathbf{H})$. Define

$$\mathcal{C}_i(\mathbf{T}'^s) = \Big\{\mathbf{R}:\ \rho(l) \leq R(l) \leq \mathbb{E}_{\mathbf{H}}\Big[\mathbf{C}_{i,l}(\mathbf{H})\Big],\ l \in [1:L]\Big\}.$$

Here, $\mathbf{C}_{i,l}(\mathbf{H}) \triangleq \frac{1}{2}\log\big(1 + \text{SNR}_l(\mathbf{H})\big)$ and $\mathbf{T}'^s = (T_1'^s, \ldots T_L'^s)$. For $\boldsymbol{\Delta} \triangleq (\Delta_1, \ldots, \Delta_L)$, let

$$\mathcal{T}^s(\boldsymbol{\Delta}) \triangleq \Big\{\mathbf{T}'^s : \mathbb{E}_{\mathbf{H}}\big[\sum_{l=1}^{L} T_l'^s(\mathbf{H})\big] \leq \mathbb{E}[Y^s],\ R_l(\mathbf{H}) \overset{\text{a.s.}}{\geq} \rho(l),$$
$$\mathbb{E}_{\mathbf{H}}\big[\eta H(l)T_l'^s(\mathbf{H})\big] \geq \Delta_l,\ l \in [1:L]\Big\}.$$

Theorem 1. (Capacity Region with Ideal Receivers)*: The minimum rate capacity region is*

$$\mathcal{C}_i(\boldsymbol{\Delta}) = \overline{Conv}\Bigg(\bigcup_{\mathbf{T}'^s \in \mathcal{T}^s(\boldsymbol{\Delta})} \mathcal{C}_i(\mathbf{T}'^s)\Bigg),$$

where $\overline{Conv}(S)$ is the closure of convex hull of the set S.

Proof. See Appendix A. □

Since the capacity region $\mathcal{C}_i(\boldsymbol{\Delta})$ is convex, we can obtain the boundary points of $\mathcal{C}_i(\boldsymbol{\Delta})$ by solving the following optimization problem:

$$\max_{\mathbf{T}'^s(.)} \sum_{l=1}^{L} \mu(l)\mathbb{E}_{\mathbf{H}}\big[R_l\big(T_l'^s(\mathbf{H})\big)\big],$$
$$\text{s.t. } \mathbb{E}_{\mathbf{H}}\big[\sum_{l=1}^{L} T_l'^s(\mathbf{H})\big] \leq \mathbb{E}[Y^s],\ \forall l,$$
$$R_l(\mathbf{H}) \overset{\text{a.s.}}{\geq} \rho(l),\ \forall\ l,$$
$$\mathbb{E}_{\mathbf{H}}\big[\eta H(l)T_l'^s(\mathbf{H})\big] \geq \Delta_l,\ \forall\ l.$$

Let $\Pi(.)$ be a permutation function on $[1:L]$ such that $H\big(\Pi(1)\big)/\sigma^2_{\Pi(1)} \geq H\big(\Pi(2)\big)/\sigma^2_{\Pi(2)} \geq \ldots \geq H\big(\Pi(L)\big)/\sigma^2_{\Pi(L)}$. Also, let $T'_{m,l}$ (m indicates minimum) denote the energy expended for maintaining the minimum rate $\rho(l)$ and $T'_{e,l}$ (e denotes excess) be the excess energy such that $T'_{m,l} + T'_{e,l} = T_l'^s$. Then, with additional algebraic manipulation, it is easy to show that the above optimization is equivalent to the optimization problem:

$$\max_{\mathbf{T}'_e} \sum_{l=1}^{L} \mu(l)\Big[\rho(l) + \mathbb{E}_{\mathbf{H}_{\text{ef}}}\big[C_l^{\text{ef}}(\mathbf{H}_{\text{ef}})\big]\Big],$$
$$\text{s.t. } \mathbb{E}_{\mathbf{H}_{\text{ef}}}\big[\sum_{l=1}^{L} T'_{e,l}(\mathbf{H}_{\text{ef}})\big] \leq \mathbb{E}[Y^s],\ \forall l,$$
$$\mathbb{E}_{\mathbf{H}_{\text{ef}}}\big[\eta H_{\text{ef}}(l)T'_{e,l}(\mathbf{H}_{\text{ef}})\big] \geq \Delta_{l,\text{ef}},\ \forall\ l,$$

where, $C_l^{\text{ef}}(\mathbf{H}_{\text{ef}}) \triangleq \frac{1}{2}\log\big(1 + \text{SNR}_{l,\text{ef}}(\mathbf{H}_{\text{ef}})\big)$, $\text{SNR}_{l,\text{ef}}(\mathbf{H}_{\text{ef}}) = \Sigma_{l,\text{ef}}(\mathbf{H}_{\text{ef}})/\nu_{l,\text{ef}}(\mathbf{H}_{\text{ef}})$, $\Sigma_{l,\text{ef}}(\mathbf{H}_{\text{ef}}) = H_{\text{ef}}\big(\Pi(l)\big)T'_{e,\Pi(l)}(\mathbf{H}_{\text{ef}})$ and $\nu_{l,\text{ef}}(\mathbf{H}_{\text{ef}}) = \sigma^2_{l,\text{ef}} + \sum_{j<l} H_{\text{ef}}\big(\Pi(j)\big)T'_{e,\Pi(j)}(\mathbf{H}_{\text{ef}})$. We refer to $\mathbf{H}_{\text{ef}}$, $(\sigma^2_{l,\text{ef}}, l \in [1:L])$ as the effective fading coefficients and noise variances respectively, and can be obtained as in [1]. Also, $\mathbf{T}'_e \triangleq (T'_{e,1}, \ldots, T'_{e,L})$, $\Delta_{l,\text{ef}} = \Delta_l - \mathbb{E}_{\mathbf{H}}\big[\eta H(l)T'_{m,l}(\mathbf{H})\big]$. As an example, for the two receiver case, let q denote the probability of the event $\mathcal{E}_{1,2}$ and let, $q_c = (1-q)$. Denote, for $l \in \{1,2\}$, $p_l = (e^{2\rho(l)} - 1)$. Then, under the event $\mathcal{E}_{1,2}$, $\sigma^2_{1,\text{ef}} = \sigma_1^2$, $\sigma^2_{2,\text{ef}} = (\sigma_2^2 - \sigma_1^2)e^{-2\rho(1)} + \sigma_1^2$, $H_{\text{ef}}(l) = H(l)e^{-2\rho(1)-2\rho(2)}$, $\Delta_{1,\text{ef}} = \Delta_1 - \sigma_1^2 p_1 - \sigma_2^2 p_1 p_2 q_c$,

$\Delta_{2,\text{ef}} = \Delta_2 - \sigma_2^2 p_2 - \sigma_1^2 p_1 p_2 q$. For the complement of the event $\mathcal{E}_{1,2}$, the indices are swapped to the obtain corresponding expressions.

Remark 1. *As a consequence of Theorem 1, we can recover various important results as special cases. For instance, the capacity region of a fading GBC with an energy harvesting transmitter, and without power transfer and minimum rate constraints, is readily obtained. We also obtain the capacity of a fading AWGN channel with energy harvesting transmitter, sending simultaneously a delay sensitive data (at a pre specified rate ρ) and a delay tolerant data. The result can be obtained using the proof of Theorem 1, but using two separate codebooks (for each class of data) in conjunction with the rate splitting argument [13].*

B. SWIPT System: Time-Switching Receivers

In this section, we consider the SWIPT system with time-switching receivers. The corresponding capacity region is referred to as the minimum-rate erasure capacity region. The terminology signifies the fact that harvesting energy from a data bearing symbol (using time-switching receiver) erases its information content. In time-switching case, even though there is *no minimum rate* at those times, the constraint ensures that receivers can harvest a certain minimum RF power. An important aspect of our model is that, without loss of *optimality*, each receiver can decide when to harvest RF energy independent of other receivers' decision and the transmitter not knowing the same. The probability with which receiver l decides to harvest in any slot is dictated by Δ_l. Let $\pi_{\mathcal{E}}(l) \triangleq \Delta_l / \mathbb{E}_{\mathbf{H}}\big[\eta H(l) T_l'^s(\mathbf{H})\big]$ and denote $\pi_{\mathcal{E}}^c(l) = 1 - \pi_{\mathcal{E}}(l)$. Let

$$\mathcal{C}_t^e(\mathbf{T}'^s) \triangleq \Big\{\mathbf{R}: \ \rho(l) \le R(l) \le \mathbb{E}_{\mathbf{H}}\Big[\mathbf{C}_{t,l}(\mathbf{H})\Big], \ l \in [1:L]\Big\},$$

where, $\mathbf{C}_{t,l}(\mathbf{H}) \triangleq \frac{\pi_{\mathcal{E}}^c(l)}{2} \log\big(1 + \text{SNR}_l(\mathbf{H})\big)$.

Theorem 2. (Capacity Region with Time-Switching Receivers)*:*

$$\mathcal{C}_t^e(\mathbf{\Delta}) = \overline{Conv}\left(\bigcup_{\mathbf{T}'^s \in \mathcal{T}^s(\mathbf{\Delta})} \mathcal{C}_t^e(\mathbf{T}'^s)\right),$$

is the minimum-rate erasure capacity region.

Proof. See Appendix B. □

C. SWIPT System: Power-Splitting Receiver

At receiver l, let $\pi_{\mathcal{E}}(l)$ fraction of energy be harvested in every slot, where $\pi_{\mathcal{E}}(l)$ is defined as in the time-switching case. Let $\tilde{\nu}_l(\mathbf{H}) = \sigma_l^2 + \sum_{j=1}^{L} \pi_{\mathcal{E}}^c(j) H(j) T_j'^s(\mathbf{H}) \mathbb{1}_{\tilde{\mathcal{E}}_{l,j}}$, where $\mathbb{1}_{\tilde{\mathcal{E}}_{l,j}}$ is the indicator function corresponding to the event $\{\sigma_l^2 \pi_{\mathcal{E}}^c(j) H(j) > \sigma_j^2 \pi_{\mathcal{E}}^c(l) H(l)\}$. Also, let $\widetilde{\text{SNR}}_l(\mathbf{H}) = \Sigma_l(\mathbf{H}) / \tilde{\nu}_l(\mathbf{H})$. Define

$$\mathcal{C}_p(\mathbf{T}'^s) \triangleq \Big\{\mathbf{R}: \ \rho(l) \le R(l) \le \mathbb{E}_{\mathbf{H}}\Big[\mathbf{C}_{p,l}(\mathbf{H})\Big], \ l \in [1:L]\Big\},$$

where, $\mathbf{C}_{p,l}(\mathbf{H}) \triangleq \frac{1}{2} \log\big(1 + \pi_{\mathcal{E}}^c(l) \widetilde{\text{SNR}}_l(\mathbf{H})\big)$.

Theorem 3. (Capacity Region with Power-Splitting Receivers)*: The closure of*

$$\mathcal{C}_p(\mathbf{\Delta}) = \overline{Conv}\left(\bigcup_{\mathbf{T}'^s \in \mathcal{T}^s(\mathbf{\Delta})} \mathcal{C}_p(\mathbf{T}'^s)\right),$$

is the minimum-rate capacity region with constant fraction power-splitting receivers.

Proof. The proof follows from the proof of Theorem 1 with the channel gain from the transmitter to l^{th} receiver scaled by a factor $\pi_{\mathcal{E}}^c(l)$. □

The boundary points of $\mathcal{C}_t^e(\mathbf{\Delta})$ and $\mathcal{C}_p(\mathbf{\Delta})$ should be obtained by solving optimization problems similar to that for the ideal case. Since the time switching and power splitting ratios depend upon the energy management policy at the transmitter, the optimization problem is more involved and is non-convex. Hence, in the next section, we compute achievable rate regions for the time switching and power splitting cases which are close to the optimal boundary points.

Remark 2. *In the absence of energy harvesting constraints, it is well known that a GBC and a Gaussian multiple access channel (GMAC) are duals of each other [14]. A similar result can be proved for these channels when powered by energy harvesting sources. On account of space constraints, we choose to avoid the technical details. Rather, we provide a numerical example in Section IV.*

IV. Numerical Results

In this section, we provide numerical examples to compare the minimum-capacity region of the SWIPT system for the ideal, time-switching (TS) and power-splitting (PS) receivers. The time slot is considered in multiples of 1μsec. We consider a 2 user GBC with $\sigma_1^2 = 0.8$, $\sigma_2^2 = 1.6$. We assume i.i.d. fading, independent across users. The fading distribution at each user is chosen such that $\mathbb{E}[H(1)] = 0.8$, $\mathbb{E}[H(2)] = 0.5$. We consider a *discretized* Rayleigh fading channel, obtained as follows: Fix an appropriate subset of the positive real axis. We choose the interval $[0, 10]$. We discretize this set in steps of .1. For the channel between the transmitter and receiver l, the probability $p_l(h)$, $h \in \{.1, .2, \ldots, 9.9\}$, is chosen such that $p_l(h) = Pr\big(H(l) \in [h - .1, h]\big)$ and $p_l(10) = Pr\big(H(l) \ge 9.9\big)$, where $H(l)$ is exponentially distributed. We take $\mathbb{E}[Y^s] = 10W$. Fix $\rho(1) = 300$ Kbps, $\rho(2) = 150$ Kbps. Let $\mathbb{E}[T^r(1)] = 90\mu$W, $\mathbb{E}[T^r(2)] = 50\mu$W and the energy deficits $\Delta_1 = 60\mu\text{W} \approx -12$dBm, $\Delta_2 = 30\mu\text{W} \approx -15$dBm. The efficiency factor η is fixed to 10^{-4}.

In Figure 2, we compare the minimum-rate capacity regions of the SWIPT system with all receivers ideal, all TS and all PS, against the capacity region of *all ideal receivers* case without minimum rate constraints and achievable rates without RF power transfer. As mentioned in the previous section, rather than obtaining the boundary points of $\mathcal{C}_t^e(\mathbf{\Delta}), \mathcal{C}_p(\mathbf{\Delta})$ which is more involved due to the non-convexity of the problem, we compute two achievable regions denoted respectively as $\mathcal{A}_t^e(\mathbf{\Delta}), \mathcal{A}_p(\mathbf{\Delta})$. Specifically, these achievable regions are computed using the optimal energy management policy for the case of *all ideal receivers*. It can be observed that, by comparing with $\mathcal{C}_i(\mathbf{\Delta})$ (which is an upper bound for $\mathcal{C}_t^e(\mathbf{\Delta})$ and $\mathcal{C}_p(\mathbf{\Delta})$), the achievable regions closely approximate the actual capacity regions. Also, RF power transfer with ideal receivers enhances the data rates by 200% for receiver 1 and about 67% for receiver 2, with respect to that with no RF power transfer. Further, due to concavity of the achievable rates as a function of the power expended, $\mathcal{A}_t^e(\mathbf{\Delta}) \subseteq \mathcal{A}_p(\mathbf{\Delta})$.

We also compute the capacity region for a more realistic scenario in which receiver 1 is PS and receiver 2 is TS. We compare it with $\mathcal{A}_t^e(\mathbf{\Delta})$ and $\mathcal{A}_p(\mathbf{\Delta})$, in Figure 3, for two

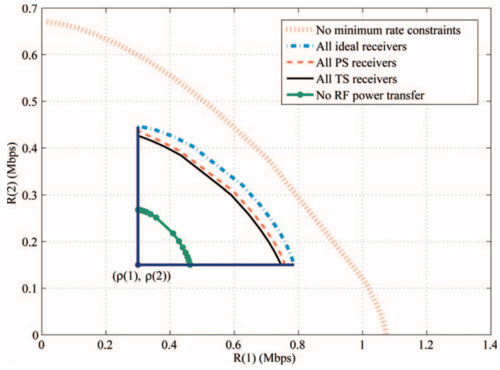

Fig. 2. $\mathcal{C}_i(\mathbf{\Delta})$, $\mathcal{A}_t^e(\mathbf{\Delta})$ and $\mathcal{A}_p(\mathbf{\Delta})$ versus capacity region without minimum rate constraints, capacity region without RF power transfer.

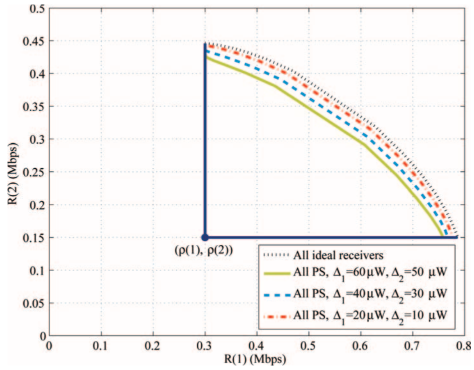

Fig. 5. Comparison of achievable regions for various values of $\mathbf{\Delta}$ with all PS receivers.

different values of $\mathbb{E}[Y^s]$. For the same amount of energy harvested at the transmitter, on average, a relatively wide range of energy deficit values at the receivers can be catered without *much* rate loss. Next, we fix $\mathbb{E}[Y^s]$ value and compare the

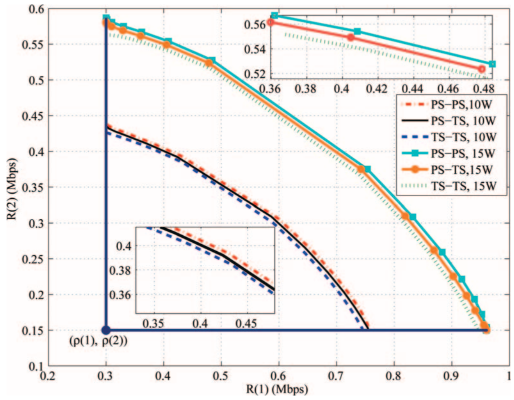

Fig. 3. Comparison of rate regions with receiver architectures all same and all different, for $\mathbb{E}[Y^s] = 10W$, $\mathbb{E}[Y^s] = 15W$.

data rates achievable for various values of energy deficits at the receiver. In Figure 4, we exemplify the change in $\mathcal{A}_t^e(\mathbf{\Delta})$ as a function of $\mathbf{\Delta}$. A similar plot for $\mathcal{A}_p(\mathbf{\Delta})$ is provided in Figure 5.

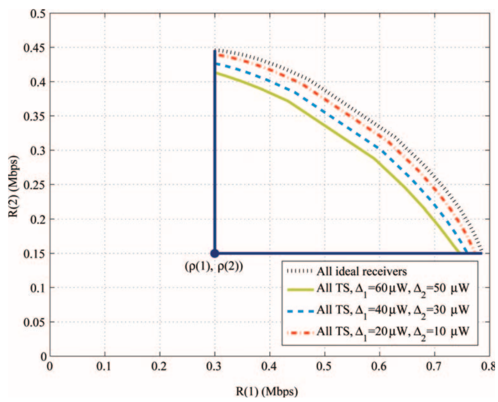

Fig. 4. Comparison of achievable regions for various values of $\mathbf{\Delta}$ with all TS receivers.

Finally, in Figure 6, we obtain the minimum-rate capacity region (with minimum rates as before) of a 2 user fading GMAC, with average energy harvested at the transmitters $\mathbb{E}[Y^s(1)] = 6$W, $\mathbb{E}[Y^s(2)] = 4$W, average energy consumed at the receiver (assumed to be ideal) $\mathbb{E}[T^r] = 90\mu$W, energy deficit at the receiver $\Delta = 60\mu$W, receiver noise variance $\sigma^2 = 1$, from that of a GBC with $\mathbb{E}[Y^s] = 10$W, $\mathbb{E}[T^r(l)] = 90\mu$W, $\Delta_l = 60\mu$W and $\sigma_l^2 = \sigma^2$, $l = 1, 2$. Even though the capacity regions are readily obtained via duality, the method does not bring out the structure of the corresponding optimal power control policies.

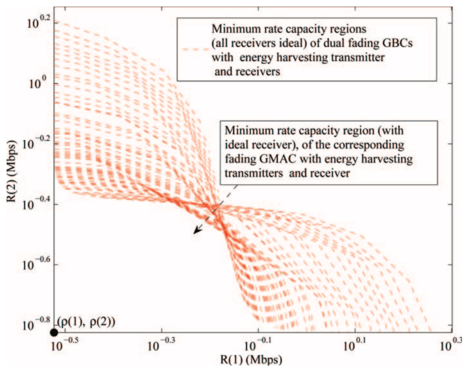

Fig. 6. The minimum-rate capacity region (in log scale) of a fading GMAC with energy harvesting constraints and SWIPT, via duality.

V. Conclusion

In this work, we have considered a fading GBC with an energy harvesting transmitter and receivers. We characterize the minimum-rate capacity region of the channel with SWIPT for the ideal, time-switching and power splitting receiver architectures. The resultant power control policies obtained are optimal within a general class of permissible policies for energy harvesting SWIPT systems. The minimum-rate capacity region that we have derived for the SWIPT system can serve as a benchmark for evaluating the performance of a communication system which should ensure fairness among its users (for instance, cellular downlink). Also, using our results in this work, via duality arguments, it is possible to numerically compute the corresponding capacity regions of a fading GMAC.

Appendix A

Proof sketch of Theorem 1: For the sake of clarity and brevity, we explain the proof technique for fading processes with finite support set $\mathcal{H}$. The extension to the continuous fading distributions can be handled in a standard way as in [1].

Achievability: *Codebook Generation:* Fix the power control policy $\mathbf{T}'^s$ obtained by solving the optimization problem in

Section III-A (with $\mathbb{E}[Y^s]$ therein, replaced by $\mathbb{E}[Y^s]-\epsilon$, for some small $\epsilon>0$). Fix message vector $\mathbf{M}$, blocklength n and a rate vector $\mathbf{R}$. The message vector is divided into independent messages $\mathbf{M_h}$ with rate $\mathbf{R}(\mathbf{h})$ such that $R(l)=\sum_{\mathbf{h}} R_l(\mathbf{h})$, $\mathbf{h}\in\mathcal{H}^L$. Corresponding to each joint fading state $\mathbf{h}$, there exists a unique order in which the channel is *degraded*. That is, the receivers can be ordered according to the increasing values of $h(l)/\sigma_l^2$, $l\in[1:L]$ such that the receiver with the lowest value of $h(l)/\sigma_l^2$ is the *weakest receiver* and that with the highest value is the *strongest*. Accordingly, for each joint fading state (and the corresponding order of degradation), generate an L level superposition codebook as per the *satellization process* ([15], Section III B). Each of the $2^{nR_l(\mathbf{h})}$ codewords of the l^{th} satellite codebook are generated i.i.d. according to $\mathcal{N}\big(0,T_l'^s(\mathbf{h})\big)$ and independent of other codebooks. The superposition codebooks generated are shared with all the receivers.

Encoding and Signalling Scheme: At time k, if the joint fading state is $\mathbf{h}_k$, the next untransmitted symbol in the codewords (to each of the receivers) corresponding to message $\mathbf{M_h}$ is chosen for transmission. Since the transmitter is energy harvesting, in a given slot k, it may not have the required amount of energy $T'^s(\mathbf{h}_k)=\sum_{l=1}^{L} T_l'^s(\mathbf{h}_k)$ in the buffer. In that case, transmission is done according to the following *truncated policy*:

$$T_k^s(l)=\begin{cases} T_l'^s(\mathbf{h}_k) & : T_l'^s(\mathbf{h}_k)\le \hat{E}_k^s,\\ \frac{\hat{E}_k^s}{T'^s(\mathbf{h}_k)}T_l'^s(\mathbf{h}_k) & : T_l'^s(\mathbf{h}_k)>\hat{E}_k^s.\end{cases}$$

Since the average power expended at the transmitter $\mathbb{E}[Y^s]-\epsilon$ is strictly less than the average harvested energy $\mathbb{E}[Y^s]$, $E_k^s\to\infty$ a.s. as $k\to\infty$ (Chapter 7, [16]). Accordingly, $T_k^s(l)\to T_l'^s$ a.s. as $k\to\infty$ for each l.

Decoding: Since the channel gains are known perfectly, receiver l can *demultiplex* its received sequence $w^n(l)$ into subsequences $\{w^{n_{\mathbf{h}}}(l)\}$ such that $n=\sum_{\mathbf{h}} n_{\mathbf{h}}$. Note that, by the law of large numbers, $(n_{\mathbf{h}}/n)\ge(1-\delta)p(\mathbf{h})$, for a large n and small $\delta>0$, where $p(\mathbf{h})$ is the probability of the joint fading state $\mathbf{h}$. Hence, for the demultiplexed subsequence corresponding to state $\mathbf{h}$ at each receiver, the decoding operation can be performed using a sub codebook of block length $n(1-\delta)p(\mathbf{h})$. Each receiver adopts successive cancellation decoding. Note that, each $\mathbf{h}$ corresponds to a particular channel degradation order. Successive cancellation decoding corresponding to the degradation order of state $\mathbf{h}$ is performed such that, each receiver decodes all the codewords (corresponding to message $\mathbf{M_h}$) of all the receivers degraded with respect it, *subtracts them off* and decodes its own codeword.

Analysis of Error Events: First note that, by ensuring $\eta\mathbb{E}[H(l)T'^s(l)]\ge\Delta_l$, the total mean harvested energy by an ideal receiver l, $\mathbb{E}[D^r(l)]\ge\mathbb{E}[T^r(l)]$. Thus, the probability of energy outages can be proven to *vanish* asymptotically as in the Transmitter's case. Next, note that AMS, ergodic sequences satisfy Asymptotic Equipartition Property (AEP) ([17]) under appropriate regularity conditions. These conditions hold good for the setting under consideration. In particular, the channel input and output random variables have finite variances. Also, the non energy harvesting channel with average transmitter power constraint equal to $\mathbb{E}[Y^s]$ has finitely bounded capacity region and is an upper bound to the capacity of the system model under consideration. Hence, the associated mutual information rates in our case are all finite. In addition, the AMS stationary mean is dominated by an i.i.d. Gaussian measure on a suitable Euclidean space. Thus the AEP result in ([17]) can be invoked in our context. Decoding is done with respect to the joint finite dimensional distribution induced by the stationary AMS mean distribution on the channel input and output processes. Taking into consideration these facts, error event analysis can be performed as in the standard case to obtain the required result. □

Appendix B

Proof of Theorem 2: At receiver l fix an appropriate $\pi_{\mathcal{E}}(l)\in[0,1]$. In each time slot, receiver l harvests RF energy with probability $\pi_{\mathcal{E}}(l)$. If energy is harvested, channel output is recorded as an erasure. Thus, the system with time-switching receiver can be equivalently thought of as a fading GBC concatenated with an erasure channel. Encoding is done as per in the proof of Theorem 1. Decoder discards the erasures and perform successive cancellation on the remaining. The erasures are independent of the channel output and the fraction of erasure instances converge almost surely to $\pi_{\mathcal{E}}(l)$. Hence, the achievability follows as in the case of Theorem 1. □

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