Plurality of wave–particle duality

Varun S. Bhatta*

National Institute of Advanced Studies, Indian Institute of Science Campus, Bengaluru 560 012, India and
Manipal Academy of Higher Education, Manipal 576 104, India

Wave–particle duality has been one of the guiding concepts in the development of modern physics. Even though duality has been extensively studied, the concept possessing multiple definitions has not received due attention. The article, to begin with, establishes the plurality associated with duality by highlighting the presence of multiple interpretations in the history of quantum mechanics and also demonstrates how prominent scientific pedagogic texts, drawing from these different threads of development, differ in the depiction of duality. This plurality has not been recognized for several reasons. Among these, the article criticizes the simplistic historical depiction of duality by showing how the narration of this concept undergoing linear development glosses over crucial historical details. Following that, the nature of the concerned plurality is analysed by comparing it with other kinds of plurality found in science. The article concludes by emphasising the relevance of this discussion for the current scientific research and for the pedagogy of quantum mechanics.

Keywords: History of quantum mechanics, plurality, wave–particle duality.

WAVE–PARTICLE duality has been one of the central concepts of 20th century physics. This concept was so pivotal for the initial development of quantum mechanics that the well-known historian of science Max Jammer[1] suggests that the 'birthday' of the theory should be determined based on the first proposal of wave–particle duality. Being a widely discussed topic, at present there are multiple views subsumed under the title 'wave–particle duality'. For some, duality is a question that arose during the first three decades of the previous century and was positively resolved by quantum electrodynamics (QED). As a brief review on the presence of duality in the *Nobel Archive* mentions, QED – whose inventors were awarded the Nobel prize for 'solving the duality problem' – goes 'beyond the everyday dialectic of wave and particle duality to the synthesis of a quantum field'[2]. According to this view, duality is nothing more than an important event in the historical development of quantum mechanics. Contrary to this, some consider duality as an intrinsic characteristic of the quantum domain. One of the prominent articulation of this stance has been the de Broglie–Bohm interpretation, according to which both matter and radiation physically have the dual characteristics. This view, which did not receive enough attention initially due to the Copenhagen 'hegemony', has gathered support in the last few decades[3]. In spite of this revival, the Copenhagen interpretation, based on the principle of complementarity, is still the dominant view about duality. This interpretation is still an active research topic where the principle gets verified in novel scenarios[4] and experiments on new entities are still conducted to observe the wave–particle complementarity[5].

As the above overview highlights, this concept – being almost a century old – has accrued several meanings over time. Because of this, some historians have even concluded that 'the wave–particle duality was and remains a rather vague concept that has neither been well defined nor used with sufficient consistency'[6]. A scientific concept having multiple connotations is not unfamiliar. This characteristic is observed even in the case of fundamental concepts like mass and space. And, in a recent article, the same aspect about the general concept of duality was demonstrated[7]. As these examples indicate, scientific concepts possessing multiple meanings is not unusual and also not problematic as the intended meaning becomes evident in the specific context of use. Even though this is largely true of wave–particle duality as well, the plurality associated with it has not received considerable attention. As pointed out above, wave–particle duality is depicted in some scientific texts as an obsolete historical artefact and in others, as a fundamental aspect of quantum entities. Not recognizing this situation about the concept is a point of concern since duality still plays a significant role in the pedagogy of quantum mechanics.

Given this, I will highlight the plurality of wave–particle duality and discuss few of the important characteristics of this plurality. I begin by introducing the numerous historical proposals that are generally identified as wave–particle duality claims. With these views laid out, I will illustrate the plurality by presenting a broad survey of contemporary science textbooks and exhibiting how variedly duality is dealt within these texts. There are various reasons because of which the plurality of duality has not yet been recognized. Among these, an important one has to do with the way the historical development of duality is narrated. I evaluate this often found portrayal of duality as a specific concept going through different stages of development and show how it overlooks the

*e-mail: varunsbhatta@protonmail.com

plurality present in the history of quantum mechanics. Subsequent to this, I engage with the nature of this plurality by comparing it with other pluralities found within science. I conclude by emphasizing the importance of this study for scientific research and pedagogy.

Multiple historical interpretations of duality

Before illustrating the plurality of wave–particle duality in contemporary physics, I want to discuss few interpretations in the history of quantum mechanics that have been considered as proposals of wave–particle duality. This discussion about the prominent views – that of Einstein, de Broglie, Born, Bohr and Heisenberg – sets the primary ground for highlighting the plural interpretations about duality since the contemporary plurality can be considered as the outcome of the multiple sources present in the history.

Most of the historical works on duality and quantum mechanics in general consider the conclusion present in the 1909 paper of Einstein as the first formulation of wave–particle duality[1,8–10]. In this article, Einstein attempts to understand the physical 'constitution' of radiation with Planck's distribution law as the starting point of inquiry. Through the analysis of momentum fluctuation equation, Einstein argues that radiation exhibits two different behaviours: at lower frequency, radiation behaves wave-like and at higher range, radiation acts as if it is constituted by 'very small-sized complexes of energy'[11]. Given the influence this analysis had on the theoretical development in the later decades, it has been labelled as the 'most famous and universal puzzle of radiation theory'[12]. For Einstein, however, radiation showing these two 'structural properties' was not paradoxical and he even comes up with a physical interpretation to accommodate these features, according to which radiation is nothing but energy singularities surrounded by force fields[11].

The above analysis of Einstein was the seed for other wave–particle duality proposals. One such hypothesis was that of Louis de Broglie. Unlike Einstein's inference that was limited to radiation alone, de Broglie made the bold move and raised duality to a fundamental principle. According to this, physical entities – both light and matter – possess the dual characteristics. de Broglie's initial research was historically rooted in the early 20th century challenges concerning the physical structure of X-rays. While working on these questions, de Broglie attempted to bring together two fundamental proposals of Einstein: the theory of relativity and the hypothesis of light-quanta[13]. Through this analysis, he proposed in 1923 that every material particle possesses an internal periodic phenomenon which is in sync with the 'phase waves' surrounding the particle[1,12]. In the following year, de Broglie was able to work out a theory which exhibits the complete equivalence between matter and light[14]. In this matured theory, a particle is guided by the phase waves such that these waves determines the probability of the particle's position by exerting 'quantum force' on it[15]. This theory, which did not receive much attention initially, was rejuvenated through Bohm's contribution in 1952 (refs 16, 17).

The other notable duality interpretation stemming from Einstein's proposal is that of Born's. This theory provides a new perspective about wave aspects of particles. While analysing the diffraction pattern observed in the case of electron collision experiments, Born suggested that the Schrödinger's wave function should be interpreted as providing the probability density for the distribution of electrons on the screen[18–20]. In this formulation, 'probability waves' associated with individual particles give rise to the wave-like pattern observed at the ensemble level. As Born recalls much later, the source for this interpretation was Einstein's physical rendition of light proposed in the 1909 paper[1]. The parallel between these two duality theses can be seen in the following description provided by Born: 'the guiding field which is represented by a scalar function ψ…spreads according to Schrödinger's differential equation. Energy and momentum, however, are transferred as if corpuscles were really flying around'[21]. In this sense, the dual aspects of electrons are distinctly observed in the diffraction experiment: the particle feature is seen when individual electrons are detected and the wave aspect becomes evident at the ensemble level.

Apart from the proposals arising from Einstein's view, there are two other dominant interpretations of wave–particle duality: Bohr's and Heisenberg's. The view held by Bohr is contextualized in the principle of complementarity that he proposed in the 1927 Como lecture. Here, his main concern was about the 'fundamental limitation' of classical physical ideas for atomic phenomena. To highlight this, he shows how in the quantum theory, the 'space-time co-ordination' and the 'claim of causality' turn out to be 'complementary but exclusive features of the description'[22]. Even though in this lecture Bohr does not explicitly discuss about duality, his discussions with other physicists during that time clearly bring his views of wave–particle complementarity. For instance, in his response to Heisenberg's 1927 paper on the uncertainty principle, Bohr uses the gamma-ray thought experiment to illustrate how wave and particle interpretations of light are mutually exclusive[23,24].

Another influential interpretation of duality is that of Heisenberg's, which was first published in his 1929 paper[25]. This view gets further articulated in his book, where he mentions 'the problem of quantum theory centers on the fact that the particle picture and the wave picture are merely two different aspects of one and the same physical reality. Although this is a problem of purely physical nature it is satisfying to find a counterpart to this duality in the mathematical apparatus of the theory…one

and the same set of mathematical equations can be interpreted at will in terms of either picture'[26]. As this quoted passage clarifies, for Heisenberg, duality is a situation where there are two equivalent mathematical formulations for describing the same quantum phenomenon[24]. Similar view about duality was held by Dirac. This view is latently present in his 1927 paper, where he shows that both the physical interpretations of radiation – as a series of waves and as a stream of particles – provide the same expression for the hamiltonian of the system. By demonstrating this, he concludes that 'the wave point of view is thus consistent with the light-quantum point of view'[27,28].

Current plural views about duality

The above views of wave–particle duality are not obsolete. Most of the articulations of duality found in the current scientific literature are endorsements of the above interpretations. However, the present discussions focus on one of the above views and overlook the others. Also, these discussions project different attitudes about the concept: some emphasize the need and relevance of this concept for contemporary physics; others think otherwise. Thus, there is considerable ambivalence surrounding the concept of wave–particle duality at present. This situation becomes apparent when the depiction of duality in the current scientific literature is observed. And it is this that I intend to carry out in this section. However, an attempt to provide a survey – even a non-exhaustive one – of a concept that has percolated deeply within the discipline is fraught with several hurdles. There is an abundant number of texts on this topic, which includes not only scientific ones, but historical and philosophical texts as well. And in this corpus, there are numerous views about duality, apart from those discussed in the previous section. Given these difficulties, since my intention is to just indicate the prevalence of plural views, I will survey few of the prominent scientific pedagogic texts like textbooks and book chapters, and highlight how they differ in their interpretation about duality.

I want to begin the survey by noting that wave–particle duality is not ubiquitously mentioned or discussed in scientific texts. There are several texts that do not invoke duality. For example, there is no mention of duality in the well-known graduate level textbooks on quantum mechanics by Landau and Lifshitz[29] and Sakurai and Napolitano[30]. And there are others, for instance the one by Schiff[31], which mention duality but do not say anything meaningful about it. Apart from this set, there are ample well-known contemporary scientific textbooks that do discuss this concept. Majority of these books formulate duality either within Born's formalism or introduce Bohr's interpretation. Shankar, in his well known textbook on quantum mechanics, defines duality using Born's theoretical vocabulary: 'each particle has associated with it a wave function $\psi(x, t)$, such that $|\psi(x, t)|^2$ give the probability of finding it at a point x at time t. This is called wave–particle duality'[32]. Similarly, the book by Cohen-Tannoudji[33] on quantum mechanics states 'We can summarize [wave–particle duality] schematically as follows: (i) The particle and wave aspects of light are inseparable. Light behaves simultaneously like a wave and like a flux of particles, the wave enabling us to calculate the probability of the manifestation of a particle. (ii) Predictions about the behaviour of a photon can only be probabilistic…'. Even though Feynman does not invoke the phrase 'wave–particle duality' in his lectures on quantum mechanics, he does mention how an electron acts 'sometimes like a particle and sometimes like a wave' in the double-slit thought experiment and ends by providing the probabilistic interpretation of the behaviour of electrons[34]. Ballentine[35], while introducing the structure of his book, mentions how the derivation of the non-commutative aspects of quantum operators based on symmetry principles 'replaces the heuristic but inconclusive arguments based upon analogy and wave–particle duality, which so frustrate the serious student…'. Later, in a chapter while analysing the diffraction experiments of particles using interference of probability densities, he mentions 'The interpretation suggested by this analysis is best described by the phrase wave–particle duality. It suggests that there is a wave associated with a particle, although the nature of the association is not entirely clear…'[35].

The above texts do not discuss Bohr's notion of duality and this is not surprising given that most of them do not mention the complementarity principle either. Cohen-Tannoudji[33] does a cursory mention of complementarity once and Ballentine[35] refers to it negatively in the introduction. However, there are other set of texts that present duality formulated in the context of the principle. In most of these books, complementarity and duality are considered equivalent to each other. For instance, the book by Zettili[36] has a sub-section titled 'Wave–Particle Duality: Complementarity', in which the principle of complementarity is introduced. Apart from these, as mentioned earlier, wave–particle complementarity is still widely used in the experimental domain. For instance, the demonstration of wave–particle complementarity of photons by Grangier *et al.*[37] is a well-known recent instance. Also, the demonstration by Scully and Walther[38] has influenced a series of 'which-path' experiments that highlight the complex relation between duality and measurement. As these usages indicate, due to the close conceptual association between Bohr's proposal and the notion of measurement in quantum mechanics, duality gets articulated through these related concepts as well. To illustrate this, consider the introductory textbook on quantum mechanics by Phillips[39]. Here, there is a sub-section titled 'Measurement and wave–particle duality' in which the author does not define duality, but instead discusses 'how the concepts of

measurement and uncertainty can be used to provide a logical and consistent description of the wave–particle properties of quantum particles'. Even though this discussion describes how the act of measurement decides what is observed in a double-slit experiment, there is no mention of the complementarity principle in the whole book.

In contrast to the above collection, there are other texts that explicitly dismiss the concept of duality solely because they find the complementarity principle either trivial or invalid. For instance, the well-known textbook by Griffiths[40] mentions in a footnote: 'The so-called wave–particle duality, which Niels Bohr elevated to the status of a cosmic principle (complementarity), makes electrons sound like unpredictable adolescents, who sometimes behave like adults, and sometimes, for no particular reason, like children. I prefer to avoid such language'. Another relevant book to discuss here is by Home[15]. Even though the book does mention Einstein's notion of wave–particle duality from the historical perspective, the central discussion about the concept is carried out in the chapter 'Wave particle duality of light and complementarity'. In this chapter, Home discusses the problems pertaining to the complementarity principle and thus in turn about the duality concept based on it. According to Home, 'The Bohrian interpretation of wave–particle dualism stems from the consideration that apart from formal predictions of observed results, some intuitive understanding is also required in terms of classically visualizable pictures of particles and waves' and thus 'if we remain confined within the formalism of quantum theory without demanding a visualisable understanding, the problem of wave–particle duality ceases to be relevant'. Home goes on to point out 'the conceptual as well as empirical inadequacies of the Bohrian perception of wave particle complementarity' and argues how 'a rational synthesis between wave and particle pictures' is possible. Apart from these texts, a strong voice against complementarity and duality was raised by Lamb[41], in a paper which argues for the redundancy of photons in modern physics. He states that duality 'may be necessary for those who are unwilling or unable to acquire an understanding of the theory. However, this concept is even more pointlessly introduced in discussions of problems in the quantum theory or radiation…The "Complementarity Principle" and the notion of wave–particle duality were introduced by N. Bohr in 1927. They reflect the fact that he mostly dealt with theoretical and philosophical concepts, and left the detailed work to post-doctoral assistants. It is very likely that Bohr never, by himself, made a significant quantum-mechanical calculation after the formulation of quantum mechanics in 1925–1926'.

As de Broglie–Bohm theory was sidelined for a long time, it is only in recent times that discussions about this theory and related pedagogic materials are becoming available. Even within this short duration, however, there seem to be several views about duality found within this theory. For instance, consider the following two essays present in an edited volume about Bohemian mechanics. Dewdney and Horton[42] are of the opinion that 'in the pilot-wave theory the long-standing interpretative puzzles, for example the measurement problem and wave–particle duality, are resolved simply by completing the quantum formalism through the postulation of individuals (the particles) which maintain their identity through the continuity of their space-time trajectories'. In contrast to this interpretation, Fine[43] suggests that unlike Bohrian position which 'merely flirts with dualism but avoids commitment, Bohmian mechanics embraces it. Bohmian mechanics requires both wave (= ψ function) and particle (= position coordinates) in order to specify the state of a system'. After this initial suggestion, Fine further goes on to provide an open ended analysis by suggesting that the theory can either be interpreted 'dualistically as involving the same old things, a one-way dualism of waves guiding particles' or 'monistically as involving a new kind of unitary world-stuff'. Compared to these discussions of duality, a recent textbook on de Broglie–Bohm theory by Bricmont[44] does not discuss wave–particle duality.

There has also been another interpretation of duality contextualized in field theories, like QED. One of the open questions about these theories pertains to their ontology: are both field and quanta required or are fields physically fundamental? Given that this theoretical enquiry has overlap with the historical context of wave–particle duality, it is not surprising that the concept of duality was used in this situation as well. For instance, a review paper on the history of photons mentions: 'The duality of light, coupled with the corpuscular photon model, has been given many conflicting interpretations and has promoted almost universal confusion among nonexperts'[45]. In this sense, some physicists consider the 'field–particle duality' is continuation of the older debate about the wave–particle duality[46]. Even though the question about the need of both photons and fields is categorically different from the other notions of duality discussed above, the proponents of field theories largely consider the subsequent development of QED resolved wave–particle duality. According to Bunge[47], 'the wave–particle duality …stimulated the creation of another theory, QED, that did away with that duality …The optical duality is then a relic of the 1905–1927 interregnum'. In another review article on the concept of photons, Scully and Sargent[48] mention 'there is no need to switch from quantum to classical descriptions or to introduce a mysterious wave–particle dualism in order to explain interference and diffraction'. Most of these arguments are based on the success of Dirac's theory to show the equivalence between both the wave and particle formulations of the system. Contrary to these opinions, it is important to note that Dirac 'always kept a relaxed attitude to the wave–particle problem' and in an interview conducted in 1982,

he reiterated the view discussed in the previous section: 'One can treat light as composed of electromagnetic waves, each wave to be treated like an oscillator; alternatively, one can treat light as composed of photons, the photons being bosons and each photon state corresponding to one of the oscillators of the electromagnetic field. One then has the reconciliation of the wave and corpuscular theories of light. They are just two mathematical descriptions of the same physical reality'[49].

Drawbacks of unified narrations

The above discussion illustrates the various ways in which duality is defined at present. Each of these definitions is situated in a unique theoretical context that has a different criterion of what counts as a valid experimental observation of duality. In spite of the obvious distinctions among these views, the plurality associated with duality has been overlooked. Among the many reasons for this, a prominent one is the presumption that this plurality is nothing but the accretion of multiple interpretations, offered at different stages, as the concept underwent gradual maturation. This kind of unified narration provided by several reviews gets certain aspects about the history of the concept wrong.

Among the several historical works on wave–particle duality, there are some that depict a linear narration of wave–particle duality – it undergoing continuous development, from being a tentative proposal to a well-confirmed hypothesis. Consider, for instance, the review of duality provided by Milonni[50]. This essay starts with Einstein's 1909 paper, in which he is supposed to have 'provided the first clear indication of the wave–particle duality of light', subsequently discusses the 1926 paper of Born, Heisenberg and Jordan, and ends with the analysis of Dirac's 1927 contribution by noting that this theory 'incorporates naturally the wave–particle duality of light'. This way of connecting the important junctures of theoretical development portrays the typical progression of a concept through stages like proposal, reinterpretation and resolution. Other surveys construct a similar picture, but with different narrations. To mention one, consider the review provided by Combourieu and Rauch[51]. This work covers, along with the relevant historical events, the modern experimental efforts that examine duality. In the historical summary, the authors start with Einstein's papers and discuss de Broglie's 'dualist principle'. In a section titled '1924–1927: The Multiplicity of Dualist Interpretations…', the authors summarize differing 'views on the way in which waves and particles are articulated in the electromagnetic field' and subsequently moves to Bohr's formulation of complementarity principle, which they label as 'the official version of dualism'. This authoritative characterization of Bohr's formulation is re-emphasized at several places in the paper: they mention how 'the complementarity principle found its mathematical justification …with the Heisenberg indeterminacy relations' and conclude the discussion by stating 'this has been the majority attitude of physicists since that time'. Thus, the authors build a narration where duality concept goes through various formulations before acquiring the matured form.

The first thing to note about the above reviews is that none of them defines duality explicitly and they take for granted what this concept means. Moreover, there are several problems with these narrations. First, this kind of portrayal implies that there was this definite concept called wave–particle duality which was present right from the beginning for the physicists to refer and deliberate upon. Also, this narration paints a linear development of the concept, as if there was a single problem right from the beginning that gets resolved eventually. However, when the history of quantum mechanics is closely scrutinized, these presumptions get challenged.

The above discussed reviews, by providing the narration of how the early physicists analysed this concept and how it underwent subsequent changes, presume that there was this specific concept 'wave–particle duality' available during this time. However, as I will show, none of the initial prominent proponents of duality – Einstein, de Broglie, Bohr and Dirac – use phrases like 'wave–particle duality', 'duality' or 'dualism' to refer to their respective initial proposals. In Einstein's 1909 German paper, there is no mention of cognates of 'duality'. Instead, as the English translation of the central idea that Einstein's proposes – 'a theory of light that can be understood as a kind of fusion of the wave and emission theories of light' – indicates, he was suggesting 'fusion' of wave and particle aspects[11]. In fact, according to the historian of science Alexei Kojevnikov, 'Einstein did not use the word "duality" either before or after 1925, nor did he make any clear assertion of the principle of the wave–particle duality'[6]. In the case of de Broglie, his 1924 doctoral thesis also does not contain any direct invocation of 'duality'. The closest reference in this work to duality is de Broglie discussing, under the section *The motion of an atom of light*, how the coincidence of 'light wave' and 'phase wave' for atom 'evokes the *double aspect* of particle and wave'[52]. Similarly, Bohr[22] and Dirac[27] do not mention any of the cognates related to wave–particle duality in their initial papers. The absence of the phrase 'wave–particle duality' and its cognates in the initial set of texts suggests that none of its proponents initially associated or identified their views as *duality* proposals to begin with. This implies that the consideration of these being 'duality' interpretations must have been later ascriptions. Since 'duality' was not used by the initial proponents during the formulation of their proposals, it would be whiggish to consider these as expressions of *wave–particle duality* at a later point of time[53]. Thus, without being careful about historiographic guidelines, the initial views were labelled

as 'duality' interpretations. Nonetheless, the whiggish accusation has limitation since some of the initial proponents – like Bohr, de Broglie, Dirac – subsequently adopted the label of 'wave–particle duality' to talk about their views.

Wave–particle duality and its cognates, like 'dual' and 'dualism', started emerging in English scientific literature only towards the end of 1920s. With respect to the presence of duality in German literature, Kojevnikov[6] observes that 'dualismus' and 'dualistae' – the German language concepts that are cognates of duality – were used in theology and philosophy since 18th century. He adds that 'it is not easy to establish who first brought the term into physics. Neither Einstein nor de Broglie, to my knowledge, had used it, but by 1927, *Dualismus* has already been present in a number of German-language physical papers and typically attributed to Einstein's and de Broglie's views'[6]. Here, it can be asked why 'duality' came into usage and became popular only by 1927 in spite of the availability of Einstein's initial proposal in 1909? A response to this can be attempted by using some of the information that Kojevnikov provides in his analysis. As he notes, even though Einstein had suggested the physical interpretation of radiation having both the wave and particle aspects in 1909, it was not immediately recognized as 'duality' proposal as this fluctuation formula could be interpreted in variety of ways, 'duality being only one of them'. And, since Einstein accepted duality 'only in a negative sense', his attitude might not have been conducive for the rise of duality. Moreover, it appears that the 1909 proposal of Einstein might not be the source for the emergence of duality. Kojevnikov points out that during the early 1920s, the physicists became aware of Einstein's idea of duality through his 1925 paper, where he proposes the Bose-Einstein statistics. Thus, only by accepting Einstein's proposal 'in a positive sense, as one of the most basic principles of the new theory', the physicists of the 'new quantum mechanics' like Schrödinger, Born, Jordan made 'Einstein's reputation as a dualist became solidly established'.

Post 1927, the notion of duality was not confined to Einstein's and de Broglie's views alone. Within a decade, duality is found in several papers having the range of connotations that this concept currently possess. For instance, in his 1929 paper, Compton[54], mentions 'the fundamental things in nature, matter and radiation, present to us a dual aspect. In certain ways they act like particles, in others like waves. The experiments tell us that we must seize both horns of the dilemma'. The phrase 'wave–particle dualism' is found in a 1929 paper where the phrase refers to Compton's effect and the Davisson and Germer experiments[55]. A 1936 paper, notes how after de Broglie's proposal and discovery of Compton's effect, 'much has been written and discussed on the wave–particle duality…by now it has been thoroughly worked into the structure of theoretical physics and has become a commonplace in our everyday discussions of atomic processes'[56]. To cite another instance, a 1938 paper, which reviews a scientific exhibition at Paris, notes how a certain exhibit highlighted 'the wave particle duality of electrons and of radiation based on the immortal experimental findings of Gray, Compton, Davisson and Germer, and G. P. Thomson together with its enunciation in Heisenberg's principle of indetermination…'[57].

The above inferences should be considered as preliminary findings and further extensive historical surveys need to be carried out. Nevertheless, the brief review of the papers from the initial phase of development illustrates an important point: the plurality associated with wave–particle duality is not a recent phenomenon. Within a short span of time since its emergence, wave–particle duality had become a concept that is applicable to diverse theoretical and experimental interpretations. The early onset of plurality counters the narrations found in the reviews discussed above. The tendency to depict duality as a specific concept which reaches its final state has been noted by some historians. For instance, the historian and philosopher of science Mara Beller[58] mentions 'the story of the development of quantum mechanics builds dramatically around the wave–particle dilemma. We follow the piling up of contradictions between the corpuscular and wave aspects of matter and radiation, one after another, until the resolution of the ultimately unbearable conceptual tension by Bohr's principle of "wave–particle complementarity" '. Against this type of unified narration, she argues that 'when we read papers dealing with the wave–particle issue before the rise of the Copenhagen philosophy, we hardly find feelings of desperation or distress. A patient suspense of final judgement seems to be a more fitting characterization of the attitude of physicists'. She further adds that not all scientists of that time felt that the situation demanded the need of two models. Some scientists thought that these two sets of observations were reconcilable; others felt this is just a temporary hurdle till a proper theoretical resolution arrives.

Nature of the plurality

In the previous section, one way through which the plurality of duality gets misinterpreted was considered and critiqued. Apart from that, there are other factors because of which the plurality has not been sufficiently recognized. This is also because the nature of the plurality found in the context of wave–particle duality has not been positively dealt with. Given this, in this section, I want to first conceptually analyse the different notions of duality mentioned earlier and establish how their distinctness ground the plurality. After that, I will characterize this plurality by considering different types of pluralities usually observed in science.

Distinction among the views of duality

Some of the prominent views that are identified as wave–particle duality proposals have been discussed earlier in the text. Historically, these views did not develop in isolation and as several historical works have shown, some of these proposals were formulated as a response to others. Having acknowledged that these are not independent views, I want to emphasize on their distinctness here. To begin with, the contexts of enquiry in the discussed proposals – for which each of these views were supposed to be the solution – do not overlap with one another. This dissimilarity can be seen by comparing the dominant duality views: Einstein's, Bohr's and Heisenberg's. In his 1909 paper, and even later on, Einstein struggled with the physical structure of radiation. The way he tried to answer this, at least in the 1909 paper, is by providing a novel physical interpretation of radiation. The problem Heisenberg was concerned about was the presence of two completely different yet equivalent theoretical formulations. In contrast to these, Bohr's preoccupation was regarding the larger conceptual vocabulary used to describe atomic phenomena. Thus, there was no common problem and each of the proponents was engaged in unique enquiries.

The other crucial difference among the three views is with regard to the meaning of duality. In these cases, even though the solution offered was called 'wave–particle duality', the meaning of the concept is different in each context. When the three proposals are compared, the meaning of the terms 'wave', 'particle' and 'duality' are found to be distinct. In Einstein's 1909 proposal, 'wave' and 'particle' are two different behaviours exhibited by radiation at different frequency ranges. However, in the context of the physical interpretation that he provides towards the end of the paper, 'wave' and 'particle' are physical features of radiation. If 'duality' needs to be understood within this formulation, then it refers to the presence of two constituents of radiation. Bohr considered 'wave' and 'particle' as classical pictures or models that needs to be used for making sense of experimental observations. And 'duality', in his framework of complementarity, acquires the meaning of 'mutually exclusive, but jointly complete'[59]. In the case of Heisenberg, 'wave' and 'particle' refer to theoretical models, each of which belongs to completely different theories. Moreover, given the theoretical equivalence between them, he did not consider that both these pictures are essential for understanding quantum mechanics; either of the approaches are sufficient[24]. As can be seen, in each of these views, duality is a unique concept that has nothing in common with the other definitions. Nevertheless, the above differences between these proposals should not come as a surprise given that each of the physicists belonged to different worldviews. Einstein was greatly influenced from the rivalry between mechanical and electromagnetic worldviews prevalent during his time[60,61]. Bohr, Heisenberg and others belong to a complete different worldview which eschewed mechanical concepts and embraced mathematical, abstract thinking[62,63].

Characteristics of the plurality

After having established the distinctness of the multiple views of wave–particle duality, I want to analyse the characteristics of the plurality that they constitute. This is of interest because not all instances of plurality found in science are of the same kind[64]. In this section, I will briefly introduce few kinds of pluralities through relevant examples and subsequently highlight some aspects of the plurality concerning wave–particle duality.

In science, the common instances of plurality are that of general and fundamental scientific concepts. The basic concepts like scientific method, theory, observation, etc., have unique interpretations in each of the sub-discipline of science. Apart from these general notions, fundamental concepts of every sub-discipline – like for instance the physical notions like mass[65], force[66] and space[67] – also exhibit plurality. Even though recognizing plurality is straightforward, in some cases establishing what makes the seemingly divergent notions 'plural views' of the same concept can be an arduous task. Consider, for example, the plurality of mass. The notion of mass found in theory of relativity corresponds to Newtonian mass when velocity is not close to that of light. On the other hand, electromagnetic mass is categorically different and has nothing in common with the other two notions. Nevertheless, there are cases of plurality where the relation among the plural views is more evident. This can be illustrated through the plurality usually associated with scientific objects. Consider the case of a positron for instance. As Norwood Hanson[68] mentions, 'the discovery of the positive electron was a discovery of three different particles…(1) The Anderson Particle, (2) The Dirac Particle and (3) The Blackett-Occhialini Particle'. Hanson argues that it is better 'to put the matter this way, rather than to remark three different discoveries of the same particle, because the conceptual backgrounds within which the work of Dirac, Anderson, and Blackett took place were so disparate as to leave it unclear until almost 1934 whether their findings had anything in common'[68]. Similar kind of plurality has been shown in the case of electron[69,70] and photons[71]. As all these case-studies highlight, holding onto the presence of a single unobservable entity across theory change and different experimental scenarios invariably gives rise to plural views of that entity. However, what holds all these views together is the presumption that all these still refer to the same entity.

Contrary to the above one, another kind of plurality can be illustrated through the multiple definitions

associated with the notion of a gene. Right from its introduction, biologists have disagreed with one another about the meaning and function of gene. For some, it referred to an entity; and for others, gene served merely as a heuristic purpose in making sense of the observations. Given this, Raphael Falk identifies several notions of gene found in its historical development: instrumental gene, material gene, holistic gene and DNA gene[72]. Subsequently, Falk goes on to argue the difference between the plurality found in the case of genes and the one that is observed in the context of other entities like electron. He observes that unlike the concept 'electron', which 'changes its meaning but not its reference, the term "gene" has changed both its meaning and its reference'. Thus, this is an ideal example of non-simple instances of plurality where the commonality among the different views is not directly given and needs to be worked out.

The above discussion sets the ground for identifying few important characteristics of the plurality observed in the case of wave–particle duality. Since wave–particle duality is not an essential theoretical concept of quantum mechanics – as some views argue for doing away with this concept – its plurality is not as rich and complex as that of other fundamental concepts like mass and space. At the same time, the plurality of duality is also not straightforward as the one observed in the case of positron. This is because the multiple views of duality are distinct and have no obvious common aspect. This plurality seems to be similar to the one observed in the case of gene: the multiple views of duality are not only distinct in their meaning, but also differ in what they pertain to. Given this, as in the case of gene and other interesting instances of plurality, the plurality of duality needs to be further analysed to understand the relation among the plural views.

Conclusion

In this article, I have highlighted the plurality of wave–particle duality and have brought forward few of its characteristics. The claim of plurality, when taken seriously, can not only provide better understanding of the concept, but also meaningfully guide the scientific research in this domain. For instance, few recent works have clarified crucial questions regarding duality by recognizing the difference between various interpretations of duality The historical and philosophical works by Beller[58] and Camilleri[24,73] are worth mentioning here since they have argued against the common belief that Heisenberg and Bohr held similar views about duality. According to them, even though Heisenberg hesitantly agreed with Bohr during 1926–27, theoretical proposals of Jordan, Klein and Wigner in 1927–28 eventually changed his views to conclude that 'wave' and 'particle' are equivalent formulations. This kind of meticulous historical and philosophical efforts are necessary given the complexity of the concept at hand. Overlooking the differences between the views can lead to problematic conflations like 'Einstein may be considered as the godfather of complementarity'[10]. Another important contribution to note here is by Kojevnikov[6]. Apart from the historical clarifications which are already mentioned in the present article, Kojevnikov also provides an interesting classification of the different views of wave–particle duality. According to him, these views belong to either of the two categories: one set of views – which he calls 'dualism' – interprets 'wave' and 'particle' statements at the ontological level; the other position – which he recognizes as 'duality' – subscribes to 'opportunistic freedom' to choose between different 'languages'.

In spite of these efforts to disentangle the complex narrations, duality is still being largely discussed without due care. An apt instance of misuse that still prevails is regarding wave–particle complementarity, which still enjoys considerable support within physics community. The current discussion on this view does not take into account the historical revisions made to this concept. During 1927, Bohr indeed interpreted the wave–particle problem using the complementarity principle. But post 1935, Bohr no more considered wave–particle issue to be a valid complementarity situation[59]. The main hurdle for Bohr was to articulate the mutual exclusivity of wave and particle observations. Also, it is not clear in what sense wave and particle pictures complement each other[74]. Few physicists too have experimentally demonstrated the weakness of complementarity. Ghose and Home[75], for instance, show how both 'wave' and 'particle' features can be observed within the same experimental set-up, much against the common belief about their complementarity. However, these critiques are yet to make a meaningful impact on the current scientific research in this area.

To conclude, it cannot be denied that the present situation about duality is largely due to the inadequate way in which this concept is dealt in the pedagogy of quantum mechanics. There have already been many reports about this state of affairs[76,77]. The survey of duality within the scientific literature and the preliminary characterization of the plurality carried out in this article positively contribute towards addressing some of the concerns. As others have already opined[78], the current situation can also be alleviated by actively incorporating historical and philosophical perspectives in science pedagogy.

1. Jammer, M., *The Conceptual Development of Quantum Mechanics*, McGraw-Hill Book Company, 1966.
2. Ekspong, G., The dual nature of light, as reflected in the Nobel Archive. *Proc. Am. Philos. Soc.*, 1999, **143**, 42–49.
3. Cushing, J. T., *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony*, University of Chicago Press, 1994.
4. Wang, K., Xu, Q., Zhu, S. and Ma, X., Quantum wave–particle superposition in a delayed-choice experiment. *Nature Photon.*, 2019, 1–6.

5. Kolesov, R. *et al.*, Wave–particle duality of single surface plasmon polaritons. *Nature Phys.*, 2009, **5**, 470–474.
6. Kojevnikov, A., Einstein's fluctuation formula and the wave–particle duality. In *Einstein Studies in Russia* (eds Balashov, Y., and Vizgin, V.), Birkhäuser, Boston, 2002, pp. 181–228.
7. Bhatta, V. and Sarukkai, S., Duality in science. *Curr. Sci.*, 2020, **118**, 705–713.
8. Klein, M. J., Einstein and the wave–particle duality. *Natural Philos.*, 1964, **3**, 3–49.
9. Hendry, J., The development of attitudes to the wave–particle duality of light and quantum theory, 1900–1920. *Ann. Sci.*, 1980, **37**, 59–79.
10. Pais, A., *Inward Bound: Of Matter and Forces in the Physical World*, Oxford University Press, 1986.
11. Einstein, A., On the development of our views concerning the nature and constitution of radiation. In The Collected Papers of Albert Einstein: The Swiss Years Writings, 1900–1909 Princeton University Press, New Jersey, 1989, pp. 379–394.
12. Darrigol, O., The origin of quantized matter waves. *Hist. Stud. Phys. Biol. Sci.*, 1986, **16**, 197–253.
13. Wheaton, B. R., *The Tiger and the Shark: Empirical Roots of Wave–Particle Dualism*, Cambridge University Press, Cambridge, 1991.
14. de Broglie, L., *Recherches Sur La Theorie Des Quanta*, Masson, Paris, 1963.
15. Home, D., *Conceptual Foundations of Quantum Physics: An Overview from Modern Perspectives*, Springer Science & Business Media, New York, 1997.
16. Bohm, D., A Suggested interpretation of the quantum theory in terms of 'Hidden' variables. I. *Phys. Rev.*, 1952, **85**, 166–179.
17. Cushing, J. T., The causal quantum theory program. In *Bohmian Mechanics and Quantum Theory: An Appraisal* (eds Cushing, J. T., Fine, A. and Goldstein, S.), Springer Science & Business Media, Dordrecht, 1996, pp. 1–20.
18. Born, M., Zur Quantenmechanik der Stoßvorgänge. *Zeit. Phys.*, 1926, **37**, 863–867.
19. Born, M., Quantenmechanik der Stoßvorgänge. *Zeit. Phys.*, 1926, **38**, 803–827.
20. Born, M., On the quantum mechanics of collisions. In *Quantum Theory and Measurement* (eds. Wheeler, J. A. and Zurek, W. H.), Princeton University Press, Princeton, 1983, pp. 52–55.
21. Falkenburg, B., *Particle Metaphysics: A Critical Account of Subatomic Reality*, Springer, 2007.
22. Bohr, N., The quantum postulate and the recent development of atomic theory. *Nature*, 1928, **121**, 580–590.
23. Beller, M., The birth of Bohr's complementarity: the context and the dialogues. *Stud. Hist. Philos. Sci. Part A*, 1992, **23**, 147–180.
24. Camilleri, K., Heisenberg and the wave–particle duality. *Stud. Hist. Philos. Sci. Part B: Stud. Hist. Philos. Modern Phys.*, 2006, **37**, 298–315.
25. Heisenberg, W., Die entwicklung der quantentheorie 1918–1928. *Naturwissenschaften*, 1929, **17**, 490–496.
26. Heisenberg, W., *The Physical Principles of the Quantum Theory*, Dover Publications, Toronto, 1949.
27. Dirac, P. A. M., The quantum theory of the emission and absorption of radiation. *Proc. R. Soc. London. Series A*, 1927, **114**, 243–265.
28. Bromberg, J., Dirac's quantum electrodynamics and the wave–Particle equivalence. In *History of Twentieth Century Physics* (ed. Weiner, C.), Academic Press, New York, 1977, pp. 147–157.
29. Landau, L. D. and Lifshitz, E. M., *Quantum Mechanics: Non-relativistic Theory*, Pergamon Press, Oxford, 1977, 3rd edn.
30. Sakurai, J. J. and Napolitano, J., *Modern Quantum Mechanics*, Addison-Wesley, San Francisco, 2011, 2nd edn.
31. Schiff, L. I., *Quantum Mechanics*, McGraw-Hill Book Company, 1955.
32. Shankar, R., *Principles of Quantum Mechanics*, Plenum Press, 1994.
33. Cohen-Tannoudji, C., *Quantum Mechanics*, Wiley, 1991.
34. Feynman, R. P., Leighton, R. B. and Sands, M., *The Feynman Lectures on Physics*, Basic Books, New York, 2010.
35. Ballentine, L. E., *Quantum Mechanics: A Modern Development*, World Scientific Publishing Company, 1998.
36. Zettili, N., *Quantum Mechanics: Concepts and Applications*, John Wiley & Sons, 2009.
37. Grangier, P., Roger, G. and Aspect, A., Experimental evidence for a photon anticorrelation effect on a beam splitter: A new light on single-photon interferences. *Euro. Lett.*, 1986, **1**, 173–179.
38. Scully, M. O. and Walther, H., Quantum optical test of observation and complementarity in quantum mechanics. *Phys. Rev. A*, 1989, **39**, 5229–5236.
39. Phillips, A. C., *Introduction to Quantum Mechanics*, John Wiley & Sons, 2013.
40. Griffiths, D. J., *Introduction to Quantum Mechanics*, Pearson Education International, New Jersey, 2005, 2nd edn.
41. Lamb, W. E., Anti-photon. *Appl. Phys. B*, 1995, **60**, 77–84.
42. Dewdney, C. and Horton, G., De Broglie, Bohm and the Boson. In *Bohmian Mechanics and Quantum Theory: An Appraisal* (eds Cushing, J. T., Fine, A. and Goldstein, S.), Springer Science & Business Media, Dordrecht, 1996, pp. 169–190.
43. Fine, A., On the Interpretation of Bohmian Mechanics. In *Bohmian Mechanics and Quantum Theory: An Appraisal* (eds Cushing, J. T., Fine, A. and Goldstein, S.), Springer Science & Business Media, Dordrecht, 1996, pp. 231–250.
44. Bricmont, J., *Making Sense of Quantum Mechanics*, Springer International Publishing, Switzerland, 2016.
45. Kidd, R., Ardini J., and Anton, A., Evolution of the modern photon. *Am. J. Phys.*, 1989, **57**, 27–35.
46. Hobson, A., There are no particles, there are only fields. *Am. J. Phys.*, 2013, **81**, 211–223.
47. Bunge, M., Analogy in quantum theory: from insight to nonsense. *Br. J. Philos. Sci.*, 1968, **18**, 265–286.
48. Scully, M. O. and Sargent, M., The concept of the photon. *Phys. Today*, 1972, **25**, 38–47.
49. Kragh, H., *Dirac: A Scientific Biography*, Cambridge University Press, Cambridge, 1990.
50. Milonni, P. W., Wave–particle duality of light: A current perspective. In *The Wave–Particle Dualism* (eds Diner, S. *et al.*), D. Reidel Publishing Company, Dordrecht, 1984, pp. 27–68.
51. Combourieu, M. and Rauch, H., The wave–particle dualism in 1992: A summary. *Found. Phys.*, 1992, **22**, 1403–1434.
52. de Broglie, L., *On the Theory of Quanta*, Foundation Louis de Broglie, Paris, 2004.
53. Brush, S. G., Scientists as historians. *Osiris*, 1995, **10**, 214–231.
54. Compton, A. H., The corpuscular properties of light. *Rev. Modern Phys.*, 1929, **1**, 74–89.
55. Ellett, A., Olson, H. F. and Zahl, H. A., The reflection of atoms from crystals. *Phys. Rev.*, 1929, **34**, 493–501.
56. Hill, E. L., Recent work on the Compton Effect. *Rev. Sci. Instrum.*, 1936, **7**, 225–228.
57. DuMond, J. W. M., The 'Palace of Discovery' at the Paris Exposition of 1937. *J. Appl. Phys.*, 1938, **9**, 289–294.
58. Beller, M., *Quantum Dialogue: The Making of a Revolution*, University of Chicago Press, 2001.
59. Held, C., The meaning of complementarity. *Stud. Hist. Philos. Sci., Part A*, 1994, **25**, 871–893.
60. McCormmach, R., Einstein, Lorentz, and the electron theory. *Hist. Stud. Phys. Sci.*, 1970, **2**, 41–87.
61. McCormmach, R., H. A. Lorentz and the electromagnetic view of nature. *Isis*, 1970, **61**, 459–497.
62. de Regt, H. W., *Understanding Scientific Understanding*, Oxford University Press, 2017.

63. Camilleri, K., Indeterminacy and the limits of classical concepts: the transformation of Heisenberg's thought. *Perspect. Sci.*, 2007, **15**, 178–201.
64. Weiskopf, D., The plurality of concepts. *Synthese*, 2008, **169**, 145.
65. Jammer, M., *Concepts of Mass in Classical and Modern Physics*, Dover Publication, New York, 1997.
66. Jammer, M., *Concepts of Force: A Study in the Foundations of Dynamics*, Dover Publications, 2012.
67. Jammer, M., *Concepts of Space: The History of Theories of Space in Physics*, Dover Publications, 2013.
68. Hanson, N. R., Discovering the positron (i). *Br. J. Philos. Sci.*, 1961, **XII**, 194–214.
69. Falconer, I., Corpuscles, Electrons and Cathode Rays: J. J. Thomson and the 'Discovery of the Electron'. *Br. J. Hist. Sci.*, 1987, **20**, 241–276.
70. Bain, J. and Norton, J., What should philosophers of science learn from the history of the electron. In *Histories of the Electron: The Birth of Microphysics* (eds Buchwald, J. Z. and Warwick, A.), MIT Press, 2004, pp. 451–465.
71. Hentschel, K., *Photons: The History and Mental Models of Light Quanta*, Springer International Publishing AG, 2018.
72. Falk, R., What is a gene? *Stud. Hist. Philos. Sci. Part A*, 1986, **17**, 133–173.
73. Camilleri, K., Bohr, Heisenberg and the divergent views of complementarity. *Stud. Hist. Philos. Sci. Part B: Stud. Hist. Philos. Modern Phys.*, 2007, **38**, 514–528.
74. Bokulich, P., Complementarity, wave–particle duality, and domains of applicability. *Stud. Hist. Philos. Sci. Part B: Stud. Hist. Philos. Modern Phys.*, 2017, **59**, 136–142.
75. Ghose, P. and Home, D., Wave–particle duality of single-photon states. *Found. Phys.*, 1992, **22**, 1435–1447.
76. Cheong, Y. W. and Song, J., Different levels of the meaning of wave–particle duality and a suspensive perspective on the interpretation of quantum theory. *Sci. Educ.*, 2014, **23**, 1011–1030.
77. Greca, I. M. and Freire, O., Meeting the challenge: quantum physics in introductory physics courses. In *International Handbook of Research in History, Philosophy and Science Teaching* (ed. Matthews, M. R.), Springer Netherlands, Dordrecht, 2014, pp. 183–209.
78. Sarukkai, S., Indian experiences with science: considerations for history, philosophy, and science education. In *International Handbook of Research in History, Philosophy and Science Teaching* (ed. Matthews, M. R.), Springer Netherlands, Dordrecht, 2014, pp. 1691–1719.

ACKNOWLEDGEMENTS. I thank Sundar Sarukkai for going through the paper and providing helpful suggestions and the reviewer for providing useful comments. This research was supported by the fellowship provided by Indian Council of Philosophical Research.

Received 6 November 2019; revised accepted 13 February 2020

doi: 10.18520/cs/v118/i9/1365-1374