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Erratum: 'Continuous gravitational wave from magnetized white dwarfs and neutron stars: possible missions for LISA, DECIGO, BBO, ET detectors'

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This erratum is corresponding to the paper MNRAS 490, 2692-2705 (2019). While calculating the luminosity due to continuous gravitational wave (GW) from an isolated white dwarf in equations (13) and (14) of the original paper (Kalita & Mukhopadhyay 2019), we mistakenly used the formula for the binaries (Ryder 2009). Moreover, this formula does not compute GW luminosity rigorously, e.g. it does not include the information of eccentricity of the source. In this erratum, we provide the correct formula as well as the correct GW luminosities for isolated rotating magnetized white dwarfs. The new formula, we provide here, has the correct information of the ellipticity as well as the angle between the magnetic and rotation axes, which are unavoidable in order to compute GW luminosity. This however does not affect the main conclusion of the work.

LUMINOSITY DUE TO GRAVITATIONAL WAVE

The luminosity due to gravitational radiation is given by

$$L_{\rm GW} = \frac{G}{c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle \tag{1}$$

$$= \frac{G\Omega^6}{5c^5}\epsilon^2 I_{xx}^2 \sin^2\chi(2\cos^2\chi - \sin^2\chi)^2 \times\left\{\frac{1}{4}\cos^2\chi\sin^2 i(1+\cos^2 i) + \sin^2\chi(1+6\cos^2 i+\cos^4 i)\right\}, \tag{2}$$

where G is the Newton's gravitational constant, c is the speed of light, Q_{ij} is the quadrupolar moment of the body, I_{xx} is the moment of inertia about x −axis, ϵ is the ellipticity of the body, Ω is the rotational frequency, χ is the angle between the magnetic field and rotation axes and i is the inclination angle of the rotation axis with respect to our line of sight. The detailed derivation of this formula is given in Appendix A. It is evident from this formula that $L_{\rm GW}$ is directly proportional to $\sin^2\chi$, which also verifies that there will be no gravitational radiation if the magnetic and rotation axes are aligned. Table 1 (Table 8 in the original paper) shows the corrected $L_{\rm GW}$ for a few typical cases for white dwarfs. Even though the GW luminosity decreases compared to previously reported values, it is still larger compared to electromagnetic (EM) spin-down luminosity, and hence the inferences, we have made earlier, do not alter. Moreover the thermal time scale (also known as the Kelvin-Helmholtz time scale) is defined as

$$\tau_{KH} = \frac{GM^2}{RL}, \tag{3}$$

where M, R and L are respectively the mass, radius and luminosity of the body. Substituting the values of $L_{\rm GW}$ from Table 1, we obtain that $\tau_{KH} \sim 10^{7-8}$ years.

Table 1. $L_{\rm GW}$ and $L_{\rm EM}$ for white dwarfs considering $\dot{P} = 10^{-15}$ Hz s^{-1} and $\chi = 3^\circ$. B_s is the surface magnetic field at the pole.

M ($M_\odot$)	R (km)	$I_{z'z'}$ (g cm^2)	P (s)	B_s (G)	$L_{\rm GW}$ (ergs s^{-1})	$L_{\rm EM}$ (ergs s^{-1})
1.420	1718.8	5.17×10^{48}	1.5	6.12×10^{8}	2.91×10^{35}	5.50×10^{34}
1.640	1120.7	6.13×10^{48}	2.0	2.78×10^{9}	3.46×10^{36}	3.03×10^{34}
1.702	1027.2	1.92×10^{48}	3.1	4.52×10^{9}	3.17×10^{35}	8.23×10^{33}

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APPENDIX A: DERIVATION OF THE FORMULA FOR LUMINOSITY DUE TO GRAVITATIONAL WAVE

The relation between the quadrupolar moment and gravitational wave strength is given by

$$h_{ij} = \frac{2G}{c^4 d}\ddot{Q}_{ij}, \tag{A1}$$

where d is the distance of the source from the detector. Moreover, the relation between GW luminosity and quadrupolar moment is (Ryder 2009)

$$L_{\rm GW} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle . \tag{A2}$$

Combining these two equations (A1) and (A2), we obtain

$$L_{\rm GW} = \frac{c^3 d^2}{20G}\left\langle \dot{h}_{ij}\dot{h}_{ij}\right\rangle . \tag{A3}$$

Moreover, using the relation $\left\langle \dot{h}_{ij}\dot{h}_{ij}\right\rangle = 2\left[\left\langle \dot{h}_+^2\right\rangle + \left\langle \dot{h}_\times^2\right\rangle\right]$ with h_+ and $h_\times$ being the two polarizations of GW, equation (A3) reduces to

$$L_{\rm GW} = \frac{c^3 d^2}{10G}\left[\left\langle \dot{h}_+^2\right\rangle + \left\langle \dot{h}_\times^2\right\rangle\right] . \tag{A4}$$

Now the polarizations of GW are given by (Kalita & Mukhopadhyay 2019)

$$\begin{aligned} h_+ &= h_0 \sin\chi \left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi \cos 2\Omega t\right], \\ h_\times &= h_0 \sin\chi \left[\frac{1}{2}\sin i \cos\chi \sin\Omega t - \cos i \sin\chi \sin 2\Omega t\right], \end{aligned} \tag{A5}$$

with the amplitude given by (Kalita & Mukhopadhyay 2019)

$$h_0 = \frac{2G}{c^4}\frac{\Omega^2 \epsilon I_{xx}}{d}(2\cos^2\chi - \sin^2\chi). \tag{A6}$$

Therefore the time derivatives of the above polarizations are given by

$$\begin{aligned} \dot{h}_+ &= h_0 \sin\chi \left[-\frac{\Omega}{2}\cos i \sin i \cos\chi \sin\Omega t - (1+\cos^2 i)\Omega \sin\chi \sin 2\Omega t\right], \\ \dot{h}_\times &= h_0 \sin\chi \left[\frac{\Omega}{2}\sin i \cos\chi \cos\Omega t - 2\Omega \cos i \sin\chi \cos 2\Omega t\right]. \end{aligned} \tag{A7}$$

Hence the average values of $\dot{h}_+^2$ and $\dot{h}_\times^2$ are given by

$$\begin{aligned} \left\langle \dot{h}_+^2\right\rangle &= h_0^2 \sin^2\chi\ \Omega^2 \left[\frac{1}{4}\cos^2 i \sin^2 i \cos^2\chi \frac{1}{2} + (1+\cos^2 i)^2 \sin^2\chi \frac{1}{2}\right], \\ \left\langle \dot{h}_\times^2\right\rangle &= h_0^2 \sin^2\chi\ \Omega^2 \left[\frac{1}{4}\sin^2 i \cos^2\chi \frac{1}{2} + 4\cos^2 i \sin^2\chi \frac{1}{2}\right]. \end{aligned} \tag{A8}$$

Substituting expressions from equations (A8) in equation (A4), we obtain

$$\begin{aligned} L_{\rm GW} = {} & \frac{G\Omega^6}{5c^5}\epsilon^2 I_{xx}^2 \sin^2\chi(2\cos^2\chi - \sin^2\chi)^2 \\ & \times \left\{\frac{1}{4}\cos^2\chi \sin^2 i(1+\cos^2 i) + \sin^2\chi(1+6\cos^2 i + \cos^4 i)\right\}. \end{aligned} \tag{A9}$$

This is the exact expression for the gravitational wave luminosity of an isolated rotating white dwarf.

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