ularly J. L. Le Sonn who breadboarded all of the radiating elements presented here, and F. Crocq for his helpful previous works on aperture-coupled antennas.

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A Procedure for Synthesizing a Specified Sidelobe Topography Using an Arbitrary Array

S. R. Nagesh and T. S. Vedavathy

Abstract— **In this communication, a simple method to arrive at an optimum set of array excitations to achieve a specified radiation pattern is presented. An auxiliary function is formulated, based on the envelope of the required sidelobe structure and the array factor, in the sidelobe region. This function is minimized, subject to the main lobe constraints and the null steering constraints, to determine the excitations. An iterative procedure is used to generate the desired pattern, with the additional constraints like sidelobe level, symmetric excitation, and so on. The method has been successfully applied to synthesize linear arrays and circular arc arrays and to generate symmetric as well as asymmetric patterns.**

I. INTRODUCTION

Synthesis of antenna arrays to generate specified radiation patterns is of very much importance in all array applications. This involves the determination of excitation to the elements, dimension and position of the elements, and type of the elements. Excitation to the various elements predominantly determines the radiation pattern. Thus determination of excitation to achieve a specified pattern has been considered extensively in the literature [1]–[5]. In this process, it is desirable to include as many details of the array as possible. This led to the inclusion of the element pattern along with the element position in the synthesis procedure [3], [5], [6].

The radiation pattern to be synthesized may have several requirements. The main lobe is to be synthesized with the global maximum along a specified direction. The direction of the nulls may be specified as, for instance, in the case of null steering. It may be required to maximize the main beam efficiency. Also, the specification on the nature of the sidelobe structure may be stringent, as to have low far-out sidelobes [7], [8] or to have sidelobes on only one side of the main lobe. In some other cases it may be required to have a pattern with a highly reduced array response over a range of angles where high levels of interference are known to exist, and the sidelobes could be higher at other angles. In addition, the sidelobe level could be specified. Further, it may be desirable to have symmetric excitation.

In this communication, a procedure to synthesize an arbitrary array is proposed which takes care of all these requirements. Specifically, the nature of the sidelobe structure which may be called sidelobe topography (SLT) is included in the synthesis procedure. The procedure developed is applied to linear arrays as well as circular arc arrays to synthesize different types of specified patterns. A circular arc array of 25 elements, considered in [9] and [10], is synthesized to generate a tapered sidelobe structure. The apparent grating lobe level has been reduced to around -35 dB using this method, as compared to around -20 dB in [9] and [10]. An asymmetric pattern is also synthesized.

This communication is organized as follows: Section II formulates the problem taking into account all the requirements stated above. Section III describes the procedure for the synthesis. Section IV deals with the application of this procedure to several cases.

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II. Formulation of the Problem

The array factor of an antenna array of N directive elements with element pattern $g_k(\theta,\phi)$ located at the position vectors $\underline{r}'_k$ with excitations w_k^* can be represented in terms of the radiation field and radiated power, respectively, as

$$\begin{aligned} f(\theta,\phi) &= \sum_{k=1}^{N} w_k^*[g_k(\theta,\phi)\exp(j\beta\hat{\imath}_r\cdot\underline{r}'_k)] \\ &= \underline{w}^H\underline{v}(\theta,\phi) \end{aligned} \tag{1}$$

and

$$\begin{aligned} s(\theta,\phi) &= f(\theta,\phi)f^*(\theta,\phi) \\ &= \underline{w}^H[\underline{v}(\theta,\phi)\underline{v}^H(\theta,\phi)]\underline{w} \end{aligned} \tag{2}$$

where $j^2=-1$, β is the free-space propagation constant, $\hat{\imath}_r$ is the radial unit vector at the observation point

$$\underline{w} = [w_1, w_2, \cdots, w_N]^T$$

and

$$\underline{v}(\theta,\phi) = [v_1(\theta,\phi), v_2(\theta,\phi), \cdots, v_N(\theta,\phi)]^T$$

with

$$v_k(\theta,\phi) = [g_k(\theta,\phi)\exp(j\beta\hat{\imath}_r\cdot\underline{r}'_k)].$$

k is the element number ranging from one to N. θ and ϕ are the usual spherical coordinates at the observation point. Here the superscripts $^*, T$, and H denote the operators, complex conjugate, transpose, and Hermitian transpose, respectively. Here and throughout, an underlined lower case letter denotes a vector.

The requirement of achieving the maximum permissible main-beam efficiency is equivalent to minimizing the power in the sidelobe region for a given main-lobe power. The power in the sidelobe region is given by

$$P_{\mathrm{SLR}} = \int_\Omega s(\theta,\phi)\,d\Omega$$

where Ω is the sidelobe region. In general, this is a double integral that can be evaluated by numerical integration as

$$P_{\mathrm{SLR}} = \sum_{l=1}^{L} h_l^L s(\theta_l,\phi_l)$$

where the weighting coefficients h_l^L and the arguments (θ_l,ϕ_l) are determined by the particular cubature or the quadrature formula used as the case demands. Thus $\underline{w}^H\boldsymbol{A}\underline{w}$ is to be minimized, where

$$\boldsymbol{A} = \left[\sum_{l=1}^{L} h_l^L\underline{v}(\theta_l,\phi_l)\underline{v}^H(\theta_l,\phi_l)\right].$$

Here and througout, a bold-face capital letter denotes a matrix.

Another requirement on the pattern is to achieve a specified SLT. This requirement can be represented as a number of constraints on the level of sidelobes in certain specified directions as suggested in [3]. This method was employed to synthesize a tapered SLT using an array with elements on a circular arc and found not to be very effective. This motivated us to include the SLT in the function to be minimized.

Consider the function

$$P_{\nu\,\mathrm{SLR}} = \int_\Omega \frac{s(\theta,\phi)}{\mathrm{SLT}(\theta,\phi)}\,d\Omega.$$

This is a function of the direction related to the sidelobe region and is the integrated function of the actual radiated power represented as $s(\theta,\phi)$, modified by the desired radiated power represented as $\mathrm{SLT}(\theta,\phi)$. It is to be seen that the inclusion of SLT can help in shaping the sidelobe region. It is obvious, however, that the inclusion of SLT brings no difference for the case of pattern synthesis with equal level sidelobes on both sides of the main beam. But use of SLT can help significantly in many cases as discussed further in the Section IV. This function can again be evaluated by numerical integration and the problem of array synthesis involves minimizing $\underline{w}^H\boldsymbol{A}_\nu\underline{w}$ where

$$\boldsymbol{A}_\nu = \left[\sum_{l=1}^{L} h_l^L\frac{\underline{v}(\theta_l,\phi_l)\underline{v}^H(\theta_l,\phi_l)}{\mathrm{SLT}(\theta_l,\phi_l)}\right]. \tag{3}$$

Thus with the main-lobe constraints [11] and the null steering constraints, the problem of array synthesis can be arranged as

$$\min_{\underline{w}} \quad \underline{w}^H\boldsymbol{A}_\nu\underline{w} \tag{4}$$

$$\text{subject to} \quad \underline{v}^H(\theta_s,\phi_s)\underline{w} = 1 \tag{5}$$

$$\mathrm{Re}\,[\underline{v}_\theta^H(\theta_s,\phi_s)\underline{w}] = 0 \tag{6}$$

$$\mathrm{Re}\,[\underline{v}_\phi^H(\theta_s,\phi_s)\underline{w}] = 0 \tag{7}$$

$$\underline{v}^H(\theta_i,\phi_i)\underline{w} = 0, \quad i = 1, 2, \cdots, N_{\mathrm{nulls}}. \tag{8}$$

The subscripts $\theta(\phi)$ denote partial differentiation with respect to $\theta(\phi)$. The constraints (5)–(7) ascertain a unity gain peak along the looking direction (θ_s,ϕ_s) [3], [11] and hence establish the main lobe. The constraints (5)–(8) can be arranged as

$$\boldsymbol{C}^H\underline{w} = \underline{d}. \tag{9}$$

III. Solution of the Problem

The solution of (4) subject to (9) can be obtained by the method of Lagrangian multipliers provided the matrix $\boldsymbol{A}_\nu$ is positive-definite Hermitian and the matrix $\boldsymbol{C}$ is of full rank, as [12]

$$\underline{w} = \boldsymbol{A}_\nu^{-1}\boldsymbol{C}(\boldsymbol{C}^H\boldsymbol{A}_\nu^{-1}\boldsymbol{C})^{-1}\underline{d} \tag{10}$$

or as

$$\begin{bmatrix} \boldsymbol{A}_\nu & -\boldsymbol{C} \\ \boldsymbol{C}^H & \mathbf{0} \end{bmatrix}\begin{bmatrix} \underline{w} \\ \underline{\lambda} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{d} \end{bmatrix} \tag{11}$$

where $\underline{\lambda}$ is the vector indicating the Lagrangian multipliers, $\underline{0}$ is the zero vector, and $\mathbf{0}$ is the null matrix.

Having obtained the solution vector $\underline{w}$, the resulting pattern can be computed and compared with the desired pattern. A little modification, if necessary, can be obtained by following an iterative scheme in which the incremental vector $\Delta\underline{w}$ is obtained as described in [3] but using the matrix $\boldsymbol{A}_\nu$. If necessary, at the end of each iteration, the excitations may be forced to be symmetric by replacing the excitations of the symmetric elements by the average of their computed values.

The procedure for the array synthesis can be summarized in the following steps.

1) Formulate a suitable SLT function based on the radiation pattern to be synthesized.
2) Based on the SLT and the element features, compute the matrix $\boldsymbol{A}_\nu$.
3) Obtain the solution vector $\underline{w}$ from (10) or (11).
4) Make the excitations symmetric (if necessary) and compute the resulting pattern.
5) If the computed pattern is satisfactory, stop; otherwise, formulate a constrained minimization problem for $\Delta\underline{w}$ [3]. ($\boldsymbol{A}_\nu$ has to be used instead of $\boldsymbol{A}$).
6) Obtain the solution vector $\Delta\underline{w}$ and augment with the vector $\underline{w}$.
7) Go to Step 4.

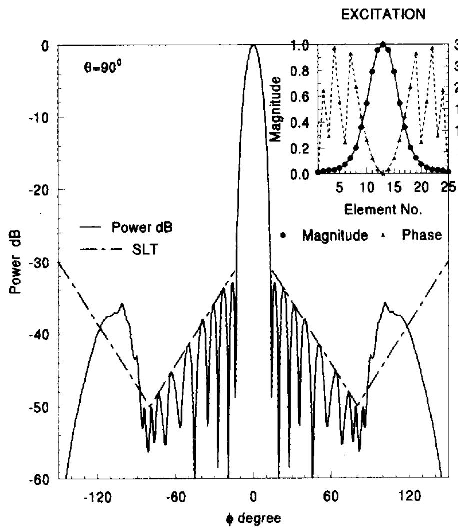

Fig. 1. Computed radiation pattern along with the SLT of a 25-element circular arc array, Array radius 6.72 wavelengths; element spacing 5.625 degrees; element pattern $\cos\phi$ in half-space; element excitations in the inset.

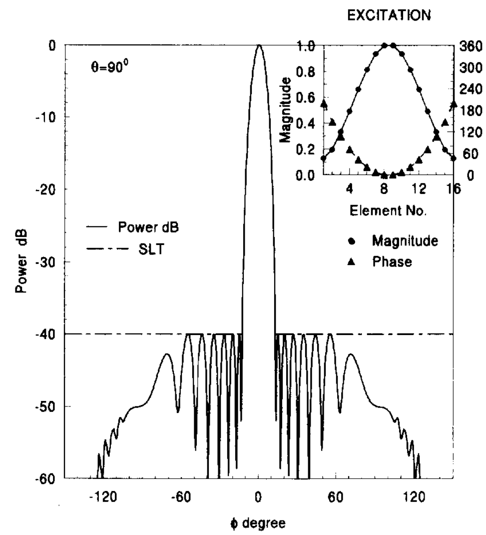

Fig. 2. Computed radiation pattern along with the SLT of a 16-element circular arc array. Array radius: 12.5 wavelengths; element spacing 0.5 wavelength; element pattern $\frac{1}{3}[1 + 2\max(\cos\phi, -\frac{1}{2})]$ in the plane of the array; element excitations in the inset.

IV. Results and Discussion

The procedure described above is applied to a number of cases of array synthesis. Planar arbitrary arrays are synthesized to generate a radiation pattern specified in the plane of the array. In computing the matrix $\boldsymbol{A}_\nu$, the sidelobe region Ω is considered to exclude the main-beam region from the entire region. Numerical integration is performed by using a six-point adaptive Gaussian quadrature formula.

The first case is the synthesis of a -40-dB equal-level sidelobe pattern from a uniform linear array of 20 half-wavelength spaced isotropic elements along the x-axis with a main lobe along the broadside direction. The sidelobe region used for computing the matrix $\boldsymbol{A}_\nu$ includes the region ($\phi = 0$ degrees, $\phi = 80$ degrees), and ($\phi = 100$ degrees, $\phi = 180$ degrees), in the plane $\theta = 90$ degrees. The resulting pattern compares very well with the pattern obtained using the traditional Dolph–Chebychev procedure. The mean-square error between the excitations obtained by this method and the traditional procedure is found to be less than 10^{-3}. Since two degrees-of-freedom out of the possible 20 are being used in establishing the broadside-looking main lobe, 18 sidelobes aligned precisely at the specified -40-dB level.

While the above result validates the proposed method of pattern synthesis, the procedure is used to synthesize different patterns using circular arc arrays. The elements of the circular arc arrays synthesized are placed symmetrically about $\phi = 0$ degrees in the plane specified by $\theta = 90$ degrees.

The second case is the synthesis of a 25-element circular arc array on a circle of radius 6.72 wavelengths with an inter-element angular difference of 5.625 degrees, having an element pattern of $\cos\phi$ in the respective half-space [10]. This array is synthesized to generate a main lobe along the axis of symmetry with a -30-dB sidelobe level. Computation of the matrix $\boldsymbol{A}_\nu$ involved integration over the range of ϕ from -157.5 degrees to -10 degrees and from 10 degrees to 157.5 degrees in the plane of the array. This is because outside this range in the plane of the array the pattern is identically 0 due to the nature of the element pattern. Initially, the SLT is assumed to be tapering off from the main lobe on both sides. A reasonable pattern is obtained, but the excitations are not totally tapering off. Hence the SLT is taken to be as in Fig. 1. The resulting pattern and the excitations are also shown in this figure. It is to be noted that the apparent grating lobe level is -35 dB. This pattern is obtained within four iterations. At each step, the excitations are made symmetric as explained in Section III.

The third case is the synthesis of a 16-element circular arc array on a circle of radius 12.5 wavelengths with a half-wave length inter-element arcual distance to obtain -40-dB sidelobes. The element pattern of each element is assumed to be

$$g(\theta, \phi) = \tfrac{1}{3}\sin\theta[1 + 2\max(\cos\phi, -\tfrac{1}{2})] \tag{12}$$

in the local coordinate system [5], [6]. Computation of the matrix $\boldsymbol{A}_\nu$ involved integration over the range of ϕ from -90 degrees to -10 degrees and from 10 degrees to 90 degrees in the plane of the array. The resulting pattern is shown in Fig. 2, along with the excitations and the SLT. The excitations are made symmetric at each step in this case as well.

The fourth case is the synthesis of an asymmetric pattern with -60-dB sidelobes on one side of the main lobe and a tapered-sidelobe structure from -30 to -60 dB on the other side. A 25-element circular arc array on a circle of radius 12.5 wavelengths with an inter-element arcual distance of a half wavelength is considered. The element pattern is assumed to be as in (12). Computation of the matrix $\boldsymbol{A}_\nu$ involved integration over the range of ϕ from -90 degrees to -10 degrees and from 10 degrees to 90 degrees in the plane of the array. The resulting pattern is shown in Fig. 3 along with the excitations and the SLT. It can be seen that the specified pattern, and the pattern obtained by this procedure, are in good agreement.

It is to be noted that the inclusion of SLT has significant effect on pattern synthesis. Even though it may be very difficult to establish the effect of SLT by rigorous mathematical analysis, it is very clear that its inclusion can help to shape the sidelobe region. To emphasize this, an attempt is made to synthesize the asymmetric pattern above without including SLT in the computation of the matrix $\boldsymbol{A}_\nu$. The resulting pattern and excitations are shown in the Fig. 4. It can be seen very clearly that the pattern of Fig. 3 is closer to the desired pattern than that of Fig. 4, especially in the range of ϕ far away from the main lobe. Further, the excitations are also much better in the sense that the ratio of the excitation maximum to the minimum is

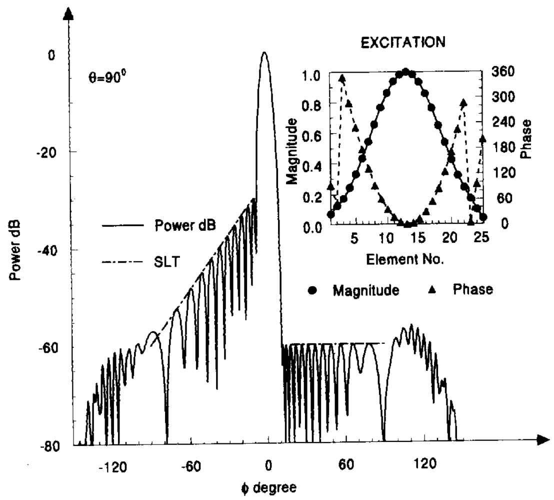

Fig. 3. Computed radiation pattern along with the SLT of a 25-element circular arc array, Array radius 12.5 wavelengths; element spacing 0.5 wavelength; element pattern $\frac{1}{3}[1 + 2\max(\cos\phi, -\frac{1}{2})]$ in the plane of the array; element excitations in the inset; SLT included in computing A_ν.

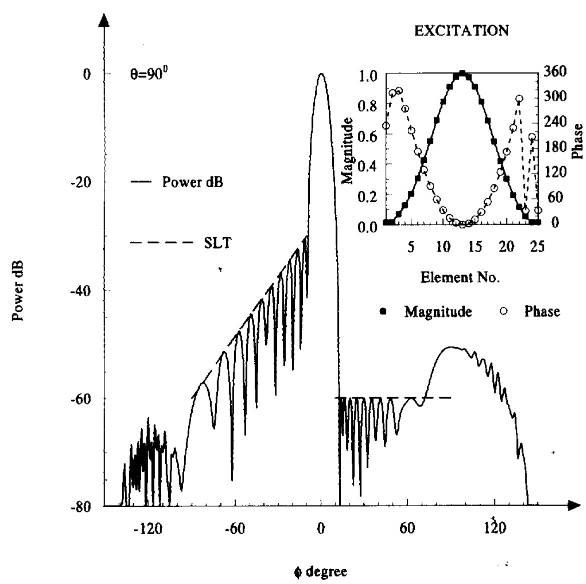

Fig. 4. Computed radiation pattern along with the SLT of a 25-element circular arc array, Array radius 12.5 wavelengths; element spacing 0.5 wavelength; element pattern $\frac{1}{3}[1 + 2\max(\cos\phi, -\frac{1}{2})]$ in the plane of the array; element excitations in the inset; SLT **not** included in computing A_ν.

less in the case where SLT is included. In addition, it was observed that the synthesis of the pattern in Fig. 3 required four iterations in contrast to the 10 iterations required for the case of Fig. 4.

In general, it can be seen that the inclusion of SLT can lead to some of the following benefits:

- It can direct towards the specified pattern.
- It can reduce the ratio of maximum to minimum excitation.
- It can reduce the required number of iterations.

One or more of these benefits may be realized in a particular case of pattern synthesis. It is observed that inclusion of SLT brings much more benefits for the case of arrays that are not linear. It is to be noted that in the specific case where the SLT has been set to a constant none of the benefits can be achieved. It is felt that further investigations are necessary to generalize the effects of SLT on pattern synthesis.

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