

Hidden Isometry in a Chiral Gauged WZW Model

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Abstract

It is shown that the asymmetric chiral gauging of the WZW models give rise to consistent string backgrounds. The target space structure of the $[SL(2, \mathbb{R})/SO(1, 1)]_L \otimes [SL(2, \mathbb{R})/U(1)]_R$ model is analyzed and the presence of a hidden isometry in this background is demonstrated. A nonlinear coordinate transformation is obtained which transforms the asymmetric model to the symmetric one, analyzed recently by two of the present authors.

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Various techniques have been recently devised for studying and generating large number of classical solutions of [1, 2, 3] string theory. The solutions obtained by gauging [4, 5, 6] of the WZW models have been quite popular, since they provide an exact string theory generalization of many of the nontrivial solutions of general relativity. An exact conformal field theory describing a two dimensional black hole [7] can be obtained by gauging the axial or vector $U(1)$ subgroup of an $SL(2, R)$ WZW model [4]. Several interesting generalizations of this model have been reported [5, 6].

In general, the resulting background obtained by gauging still has residual isometries. Most often these isometries, like translations, can be obtained by a simple observation of the background configuration. It has been shown that for d -isometries, acting as translations on the coordinates, the string effective action (*SEA*) is invariant under an $O(d, d)$ group [1, 2] of transformations. These transformations are useful in relating those classical solutions with each other which depend on the same number of coordinates. These symmetries manifest themselves in terms of the conserved currents on the string worldsheet.

In this paper, we show that another gauging, namely the chiral gauging [8, 9, 10], can be used in left-right asymmetric manner for the construction of consistent string backgrounds. We explicitly work out the case of the asymmetric chiral gauged $[SL(2, \mathfrak{R})/SO(1, 1)]_L \otimes [SL(2, \mathfrak{R})/U(1)]_R$ model. The resulting target space has an isometry acting as translation. However, unlike

the case of the symmetric chiral gauged, $[SL(2, \mathfrak{R})/U(1)]_L \otimes [SL(2, \mathfrak{R})/U(1)]_R$, model [9], the other translation isometry is absent and the metric cannot be diagonalized by any coordinate independent orthogonal transformation. Since the background in the two cases depend on different number of coordinates, they cannot be transformed into each other by any global transformation like $O(d, d)$ symmetry mentioned above.

We also show the presence of a hidden isometry in the background configuration of the asymmetric chiral gauged model mentioned above by explicitly finding out the Killing vectors. The corresponding coordinate transformations are nonlinear and they give rise to a chiral conserved current on the worldsheet. Then, by using the tensor transformation properties of the Killing vectors, we obtain a set of general coordinate transformations and analytic continuations which transform the background fields of the asymmetric model to the symmetric one [9]. Interestingly, the transformation is much more nontrivial than the case of the vector gauging, where the choice of different gauged subgroups lead to backgrounds which are simply related by an analytic continuation.

We start by writing down the action for a general chiral gauged WZW ($CGWZW$) model [8],

$$S = S^{WZW} + \frac{k}{2\pi} \int d^2z \operatorname{Tr} [A_z^R U^{-1} \bar{\partial} U + A_{\bar{z}}^L \partial U U^{-1} + A_z^R U^{-1} A_{\bar{z}}^L U], \quad (1)$$

where A_z^R ($A_{\bar{z}}^L$) in eq.(1) are the $z(\bar{z})$ components of the gauge fields A_μ^R (A_μ^L). $U(z, \bar{z}) \in G$ and the gauge fields A_z^R ($A_{\bar{z}}^L$) correspond to the generators in

subgroups H_1 (H_2) of the group G . The action (1) has an underlying gauge invariance [8] whose transformation parameters are independent in the left and right-moving sectors. The action (1) gives a Lagrangian representation of the coset conformal field theory $[G/H_1]_L \otimes [G/H_2]_R$ where H_1 and H_2 are two subgroups of G which are not necessarily same and may even have different ranks.

We work out the case when the group G in eq.(1) is $SL(2, R)$, $H_1 = SO(1, 1)$ and $H_2 = U(1)$. For our example, therefore, the full nontriviality of the chiral gauging is not incorporated, since both H_1 and H_2 have the same rank. This however allows us to illustrate another interesting aspect, namely that they are related to the symmetric chiral gauging, i.e., $H_1 = H_2 = U(1)$ by a nonlinear coordinate transformation and analytic continuation.

For the asymmetric chiral gauged model, the currents corresponding to the gauged subgroups are $J_{\bar{z}}^L \equiv -\frac{i}{2} \text{Tr}(\sigma_1 U^{-1} \partial_{\bar{z}} U)$ in the left and $J_z^R \equiv \frac{1}{2} \text{Tr}(\sigma_2 \partial_z U U^{-1})$ in the right. We represent the gauge fields as $A_{\bar{z}}^L \equiv (\frac{1}{2} \mathcal{A}_{\bar{z}}^L \sigma_1)$, $A_z^R \equiv (-\frac{i}{2} \mathcal{A}_z^R \sigma_2)$ and parametrize the group manifold as $U \equiv \exp(\frac{i}{2} \phi_L \sigma_2) \exp(\frac{r}{2} \sigma_1) \exp(\frac{i}{2} \phi_R \sigma_2)$. Then, by integrating out the gauge fields [4] from the action (1), we get

$$S = \frac{k}{8\pi} \int d^2 z [\partial r \bar{\partial} r - \partial \phi_L \bar{\partial} \phi_L + \partial \phi_R \bar{\partial} \phi_R + 2 \coth r \cot \phi_L \partial r \bar{\partial} \phi_L + 2 \frac{\cot \phi_L}{\sinh r} \partial r \bar{\partial} \phi_R + R^{(2)}(\ln \sin \phi_L \sinh r + \text{const.})]. \quad (2)$$

The background fields are therefore found to consist of the metric $(G_{\mu\nu})$,

antisymmetric tensor ($B_{\mu\nu}$) and dilaton (Φ):

$$(G + B) = \begin{pmatrix} 1 & 2 \coth r \cot \phi_L & 2 \cot \phi_L / \sinh r \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3)$$

and $\Phi = -\ln \sin \phi_L \sinh r + const.$. Here we have modded out a constant ($k/4$).

We observe that the above background fields depend on two coordinates (r and ϕ_L). We have explicitly verified that the above background configuration satisfy the field equations of the three dimensional *SEA* for the cosmological constant $V = -1$. The scalar curvature is $R = -[7/(2 \sin^2 \phi_L \sinh^2 r)]$. This shows that the target space-time contains a line singularity along $\phi_L = 0$ and π for all r . However unlike the case of the three dimensional black string [9], the translation symmetry along the singularity direction is lost. In the asymptotic limit, the metric becomes flat and the torsion vanishes. We have also checked that the underlying $O(1, 1)$ symmetry of the *SEA* for the background (3), cannot generate any inequivalent solution.

We now examine the symmetries of the background configuration (2). In this connection, it has been shown that there is a one-to-one correspondence between the chiral conserved currents on the worldsheet and the set of Killing vectors (labeled by an index i) satisfying the equations [11],

$$K_i^\mu \nabla_\mu \Phi = 0 \quad (4)$$

$$\partial_\mu K_{i\nu} - \Gamma^\lambda_{\mu\nu} K_{i\lambda} \pm \frac{1}{2} H^\lambda_{\mu\nu} K_{i\lambda} = 0 \quad (5)$$

where the plus (minus) signs before the torsion term in eq.(5) correspond to the left (right)-chiral currents *i.e.* $\partial\bar{J} = 0$ ($\bar{\partial}J = 0$). The isometry direction is given by the coordinate transformation, $\delta x^\mu = \epsilon K_i^\mu$.

The translation in ϕ_R is obviously a symmetry of the background in eq.(3) and gives rise to the right-chiral current, $J = (\partial\phi_R + \frac{\cot\phi_L}{\sinh r}\partial r)$. This corresponds to the Killing vector, $K_1^\mu \equiv (K_1^r, K_1^{\phi_L}, K_1^{\phi_R}) = -(0, 0, 1)$.

We now show the presence of a hidden isometry for the background in eq.(3). This isometry acts nonlinearly on the space-time coordinates and gives a new chiral conserved current on the worldsheet. We will show that, in fact these are the only isometries of the background in eqs.(2)-(3) which can give chiral conserved currents.

To find the new hidden isometry, we start by writing the most general candidate for the left-chiral current as,

$$\bar{J} = f\bar{\partial}r + g\bar{\partial}\phi_L + h\bar{\partial}\phi_R \quad (6)$$

where f , g and h are apriori arbitrary functions of r , ϕ_L and ϕ_R . We then impose the condition that

$$\begin{aligned} \partial\bar{J} = 0 &= -(\sinh r \sin^2 \phi_L) ([E_{\phi_R}] g, _r + [E_{\phi_L}] h, _r + [E_r] h, _{\phi_L}) \\ &+ f, _{\phi_L} \partial\phi_L \bar{\partial}r + f, _{\phi_R} \partial\phi_R \bar{\partial}r + g, _{\phi_R} \partial\phi_R \bar{\partial}\phi_L + h, _{\phi_R} \partial\phi_R \bar{\partial}\phi_R \\ &+ [A] \partial r \bar{\partial}r + [B] \partial \bar{\partial}r + [C] \partial\phi_L \bar{\partial}\phi_L + [D] \partial \bar{\partial}\phi_L + [F] \partial \bar{\partial}\phi_R , \end{aligned} \quad (7)$$

where $E_\mu = \delta S/\delta x^\mu$, ($x^\mu = r, \phi_L, \phi_R$) are the coordinate variations of the action in eq.(2) and vanish due to the field equations. Explicit form for the

quantities, [A], [B], [C], [D] and [F] are given as,

$$\begin{aligned}
[A] &= f_{,r} - g_{,r} \sin \phi_L \cos \phi_L \coth r + h_{,r} (\sin \phi_L \cos \phi_L / \sinh r) \\
[B] &= f + g_{,r} \sin \phi_L \cos \phi_L - h_{,r} \sin \phi_L \cos \phi_L \cosh r + h_{,\phi_L} \sin^2 \phi_L \sinh r \\
[C] &= g_{,\phi_L} - h_{,\phi_L} \cosh r \\
[D] &= g + h_{,r} \sin^2 \phi_L \sinh r + h_{,\phi_L} \cos \phi_L \sin \phi_L \cosh r \\
[F] &= h + g_{,r} \sin^2 \phi_L \sinh r + h_{,\phi_L} \sin \phi_L \cos \phi_L . \tag{8}
\end{aligned}$$

Using the equations of motion, $E_\mu = 0$, we now conclude that $\partial \bar{J} = 0$ gives the conditions $f_{,\phi_L} = f_{,\phi_R} = g_{,\phi_R} = h_{,\phi_R} = 0$ and $[A] = [B] = [C] = [D] = [F] = 0$. This implies a set of five first order partial differential equations. After some manipulations, it can be shown that they have a unique solution given by $f(r) = 0$, $g(r, \phi_L) = (C_0 \coth r / \sin \phi_L)$ and $h(r, \phi_L) = (C_0 / \sin \phi_L \sinh r)$, where C_0 is a constant. Therefore the expression for the only left-chiral current is given as,

$$\bar{J} = (C_0 / \sin \phi_L \sinh r) (\cosh r \bar{\partial} \phi_L + \bar{\partial} \phi_R). \tag{9}$$

The corresponding Killing vector is written as,

$$K_2^\mu \equiv (K_2^r, K_2^{\phi_L}, K_2^{\phi_R}) = i [\cos \phi_L, (-\sin \phi_L \coth r), (\sin \phi_L / \sinh r)] \tag{10}$$

and it can be checked that K_2^μ satisfies the Killing eqs.(4) and (5) for the background in eqs.(2)-(3).

By similar technique, one can also show that our model has J as the only right-chiral conserved current. The presence of the extra Killing vector K_2^μ

implies a larger symmetry in the target space. We believe that such hidden isometries may be present in much wider class of classical string solutions. It will be interesting to investigate the presence of these isometries in other backgrounds so as to understand their possible moduli deformations. For example, in this paper we now show the connection of our solution to another one, namely the symmetric chiral gauged model [9], which has the same number of left and right-chiral currents.

The symmetric chiral gauged $[SL(2, \mathfrak{R})/U(1)]_L \otimes [SL(2, \mathfrak{R})/U(1)]_R$ model is analyzed in ref.[9]. It is represented by the nonlinear sigma model action,

$$S^S = \frac{k}{8\pi} \int d^2z [\partial\theta\bar{\partial}\theta + \partial\phi\bar{\partial}\phi + \partial\rho\bar{\partial}\rho + \frac{2}{\cosh\rho}\partial\theta\bar{\partial}\phi + R^{S(2)}(\ln \cosh\rho + Const.)] \quad (11)$$

where ρ , θ and ϕ are the target space coordinates. This action describes the three dimensional static charged black string with scalar curvature $R^S = (7/2\cosh^2\rho)$ when the same factor ($k/4$) as in eq.(3) is once again modded out. The only chiral conserved currents for the symmetric case are, $\bar{J}^s = (\bar{\partial}\theta + \frac{1}{\cosh\rho}\bar{\partial}\phi)$ and $J^s = (\partial\phi + \frac{1}{\cosh\rho}\partial\theta)$. They correspond to the isometries acting as translation in θ and ϕ respectively. The Killing vectors associated with those isometries are simply $\tilde{K}_1^\mu = (0, 1, 0)$ and $\tilde{K}_2^\mu = (0, 0, 1)$.

In order to examine whether the backgrounds in eqs.(2) and (11) are related by a coordinate transformation, we find it useful to use the tensor transformation property of a Killing vector $K_{i\mu} = (\partial\tilde{x}^\nu/\partial x^\mu)\tilde{K}_{i\nu}^i$, ($i = 1, 2$), where $K_{i\mu}$, $\tilde{K}_{i\mu}$ are the covariant components of the Killing vectors. Then

by writing these equations explicitly in the components of $K_{i\mu}$, we obtain a set of linear algebraic equations in $(\theta_{,r}, \phi_{,r})$, $(\theta_{,\phi_L}, \phi_{,\phi_L})$ and $(\theta_{,\phi_R}, \phi_{,\phi_R})$. We also use the invariance of the scalar curvature which implies, $\cosh \rho = a_0 \sinh r \sin \phi_L$, for some constant a_0 . The above set of algebraic equations then have a unique solution, which can be integrated to give,

$$\begin{aligned}\rho &= \cosh^{-1}(i \sin \phi_L \sinh r) \\ \theta &= i \tanh^{-1}(\cos \phi_L \tanh r) \\ \phi &= \tan^{-1}(\tan \phi_L \cosh r) + \phi_R.\end{aligned}\tag{12}$$

We have explicitly verified that the background metric in the two cases, namely the asymmetric and the symmetric models are related by the coordinate transformations (12). For comparison, we would like to mention that for the vector or axial gauging[4], choice of gauging $SO(1, 1)$ or $U(1)$ is related simply by an analytic continuation of the time coordinate. Further, unlike our case, the vector or axial models do not allow independent left-right gauging.

We expect that the presence of the new isometry may give rise to a coordinate dependent generalization of the global $O(d, d)$ symmetry of the string effective action, since it acts more nontrivially than just the translation. Similar techniques may be employed to search for higher spin currents on the worldsheet and they may have interesting target space interpretation.

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