Codes for Optical CDMA

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Abstract. There has been a recent upsurge of interest in applying Code Division Multiple Access (CDMA) techniques to optical networks. Conventional spreading codes for OCDMA, known as optical orthogonal codes (OOC) spread the signal in the time domain only, which often results in the requirement of a large chip rate. By spreading in both time and wavelength using two-dimensional OOCs, the chip rate can be reduced considerably. This paper presents an overview of 1-D and 2-D optical orthogonal codes as well as some new results relating to bounds on code size and code construction.

Keywords: Optical orthogonal codes, OOC, constant weight codes, optical CDMA, OCDMA, 2-D OOC, MWOOC, wavelength time codes.

1 Introduction

Recently there has been an upsurge of interest in applying code division multiple access (CDMA) techniques to optical networks [1], at least in part due to the increase in security afforded by optical CDMA (OCDMA) as measured for instance, by the increased effort needed to intercept an OCDMA signal, and in part due to the flexibility and simplicity of network control afforded by OCDMA.

As in conventional CDMA, each of the users in an optical CDMA system is assigned a unique spreading code that enables the user to distinguish his signal from that of the other users. In optical CDMA (OCDMA), the typical modulation scheme used is On-Off Keying (OOK) and as a result, the spreading codes are binary, with symbols in $\{0, 1\}$. These spreading codes are termed optical orthogonal codes (OOC). Traditionally, as in the case of wireless communication, the spreading has been carried out in time and we will refer to this class of OOC as one-dimensional OOCs (1-D OOCs).

One drawback of one-dimensional (1-D) OOCs is the requirement of a large chip rate. For example, consider the situation when it is desired to assign codes to 8 potential users, each transmitting data at 1 Gbit/sec of which at most 5 users are active at any given time. If one attempts to design a 1-D OOC to meet this requirement, one will end up with a chip-rate on the order of 161 Gchips

G. Gong et al. (Eds.): SETA 2006, LNCS 4086, pp. 34–46, 2006.

per second (Gcps) (See Example 1). By employing two-dimensional (2-D) OOCs that spread in both wavelength and time, it turns out that the above requirement can be met using a chip rate of just 6 Gcps.

Section 2 reviews bounds on the size of 1-D codes as well as some of the better known code construction techniques. 2-D OOCs are introduced in Section 3. Bounds on the size of a 2-D OOC are treated in Section 4, while Section 5 presents a new construction technique based on the use of rational functions.

2 One-Dimensional Optical Orthogonal Codes

An (n, ω, κ) Optical Orthogonal Code (OOC) $\mathcal{C}$ where $1 \leq \kappa \leq \omega \leq n$, is a family of {0,1}-sequences of length n and Hamming weight ω satisfying:

$$\sum_{k=0}^{n-1} x(k)y(k \oplus_n \tau) \leq \kappa \tag{1}$$

for every pair of sequences $\{x, y\}$ in $\mathcal{C}$ whenever either $x \neq y$ or $\tau \neq 0$. We have used $\oplus_n$ to denote addition modulo n. We will refer to κ as the maximum collision parameter(MCP).

For a given set of values of n, ω, κ, let $\Phi(n, \omega, \kappa)$, denote the largest possible cardinality of an (n, ω, κ) OOC code.

2.1 Bounds on the Size of 1-D OOCs

If $\mathcal{C}$ is an (n, ω, κ) 1-D OOC, then by including every cyclic shift of each codeword in $\mathcal{C}$ one can construct a constant weight code with parameters (n, ω, κ) of size $= n \mid \mathcal{C} \mid$. This observation allows us to translate the Johnson bounds A, B, C on constant weight codes [2] [3] as well as the improvement of Johnson bound B due to Agrell et. al. [4] and the improvement of Johnson bound C due to Moreno et. al. [5] into bounds on the cardinality of an OOC. These are reproduced below:

Johnson Bound A:

$$\Phi(n, \omega, \kappa) \leq \left\lfloor \tfrac{1}{\omega} \left\lfloor \tfrac{n-1}{\omega-1} \cdots \left\lfloor \tfrac{n-\kappa}{\omega-\kappa} \right\rfloor \right\rfloor \right\rfloor := J_A(n, \omega, \kappa), \tag{2}$$

first noted by Chung, Salehi, and Wei in [6].

Improved Johnson Bound B: Provided $\omega^2 > n\kappa$

$$\phi(n, \omega, \kappa) \leq \min(1, \left\lfloor \tfrac{\omega-\kappa}{(\omega^2-n\kappa)} \right\rfloor) := J_B(n, \omega, \kappa). \tag{3}$$

The observation that $\Phi(n, \omega, \kappa) \leq 1$ for $\omega^2 > n\kappa$ first appears in [7], its constant weight code equivalent is proved in [4].

Improved Johnson Bound C:

$$\Phi(n, \omega, \kappa) \leq \left\lfloor \frac{1}{\omega} \left\lfloor \frac{n-1}{\omega-1} \cdots \left\lfloor \frac{n-(\ell-1)}{\omega-(\ell-1)} h \right\rfloor \cdots \right\rfloor \right\rfloor \;:=\; J_C(n, \omega, \kappa), \tag{4}$$

$$\text{where } h = \min(n-\ell, \left\lfloor \frac{(n-\ell)(\omega-\kappa)}{(\omega-\ell)^2-(n-\ell)(\kappa-\ell)} \right\rfloor),$$

and, where ℓ is any integer, $1 \le \ell \le \kappa - 1$, such that $(\omega - \ell)^2 > (n - \ell)(\kappa - \ell)$. The Improved Johnson Bound B and C as applied to OOC appeared for the first time, to the best of our knowledge in [5]. However the observation implicit in the Improved Johnson Bound B that $\Phi(n, \omega, \kappa) \le 1$ for $\omega^2 > n\kappa$ may be found in [7].

Since $\Phi(n, \omega, \kappa)$ denotes the largest possible size of a 1-D OOC, an OOC $\mathcal{C}$ of size P is said to be *optimal* when $P = \Phi(n, w, \kappa)$ and *asymptotically optimum* if: $\lim_{n\to\infty} \frac{P}{\Phi(n,w,\kappa)} = 1$.

2.2 Constructions

There is a large literature on constructions of optical orthogonal code, see for instance [6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,5]. Algebraic constructions for families of OOCs can be found in [6,7,8,9,12,20,28, 29,5]. Recursive constructions appear in [10,13,19,22,27]. Constructions specific to a particular choice of weight parameter ω can be found in [16,15,17,18,21,23, 24,25,26]. Optimum constructions are known only for $\kappa = 1$ [6,29,16,15,17,18, 21,23,24,25] and $\kappa = 2$ [7,26]. Constructions that are asymptotically optimum can be found in [8,9,28].

Most papers on 1-D OOC only make use of Johnson Bound A. There are examples of 1-D OOCs which do not achieve Johnson Bound A with equality, which however, are optimal with respect to Johnson Bound B, see [5].

From the point of view of application to fiber-optic communications, a principal drawback of one-dimensional (1-D) OOCs is the requirement of a large chip rate. We illustrate this with an example.

Example 1. Consider the situation where it is desired to assign codes to 8 potential users of which at most 5 users are active at any given time. We assume that the data rate of each user is set to 1 Gbit/sec. A natural attempt at meeting this requirement might be to set the maximum-collision parameter (MCP) κ equal to 1 and set $\omega = 5 > (5-1)\kappa$ where $(5-1)\kappa$, represents the maximum possible interference presented by the 4 other active users under a uniform power assumption. When $\omega = 5$ and $\kappa = 1$, the Johnson bound A (Equation (2)) yields

$$\Phi(n, 5, 1) \le \left\lfloor \frac{1}{\omega} \left\lfloor \frac{n-1}{\omega - 1} \right\rfloor \right\rfloor \le \frac{n-1}{\omega(\omega-1)},$$

from which it follows that

$$n - 1 \ge 8 \times 5 \times 4$$

i.e., $n \ge 161$. Thus in this case, the chip-rate must necessarily equal or exceed 161 G chips per second (Gcps) which is currently infeasible to implement. Even if it were feasible to implement, the chip-rate is still large in relation to the data rate supplied to each user. As we shall see, by spreading in both wavelength and time, this chip-rate requirement can be reduced substantially.

A tabular listing of algebraically constructed OOCs appears in Table 1 of Appendix A. The codes appearing in this table can be used in the construction of 2-D OOCs which are based on 1-D OOCs (see [33]).

3 Two-Dimensional Optical Orthogonal Codes

The advent of Wavelength-Division-Multiplexing (WDM) and dense-WDM (D-WDM) technology has made it possible to spread in both wavelength and time [34]. The corresponding codes, are variously called, wavelength-time hopping codes, and multiple-wavelength codes. Here we will refer to these codes as two-dimensional OOCs (2-D OOCs).

A 2-D $(\Lambda \times T, \omega, \kappa)$ OOC $\mathcal{C}$ is a family of $\{0,1\}$ $(\Lambda \times T)$ arrays of constant weight ω. Every pair $\{A, B\}$ of arrays in $\mathcal{C}$ is required to satisfy:

$$\sum_{\lambda=1}^{\Lambda} \sum_{t=0}^{T-1} A(\lambda, t) B(\lambda, (t \oplus_T \tau)) \leq \kappa \tag{5}$$

where either $A \neq B$ or $\tau \neq 0$. We will refer to κ as the maximum collision parameter(MCP). Note that asynchronism is present only along the time axis.

It can be shown that it is possible to construct a 2-D $(\Lambda \times T = 6 \times 6,\ \omega = 5,\ \kappa = 1)$ OOC of size 8. Figure 1 shows such a 2-D OOC. Thus in comparison with the earlier 1-D OOC of Example 1 which required a chip-rate in excess of 161 Gcps, with this 2-D code, one can accommodate the same number of users with a chip rate of 6 Gcps.

Practical considerations often place restrictions on the placement of pulses within an array. With this in mind, we introduce the following terminology:

- arrays with one-pulse per wavelength (OPPW): each row of every $(\Lambda \times T)$ code array in $\mathcal{C}$ is required to have Hamming weight $= 1$
- arrays with at most one-pulse per wavelength (AM-OPPW): here each row of any $(\Lambda \times T)$ code in $\mathcal{C}$ is required to have Hamming weight ≤ 1
- arrays with one-pulse per time slot (OPPTS): here each column of every $(\Lambda \times T)$ code array in $\mathcal{C}$ is required to have Hamming weight $= 1$
- arrays with at most one-pulse per time slot (AM-OPPTS): here each column of any $(\Lambda \times T)$ array in $\mathcal{C}$ is required to have Hamming weight ≤ 1.

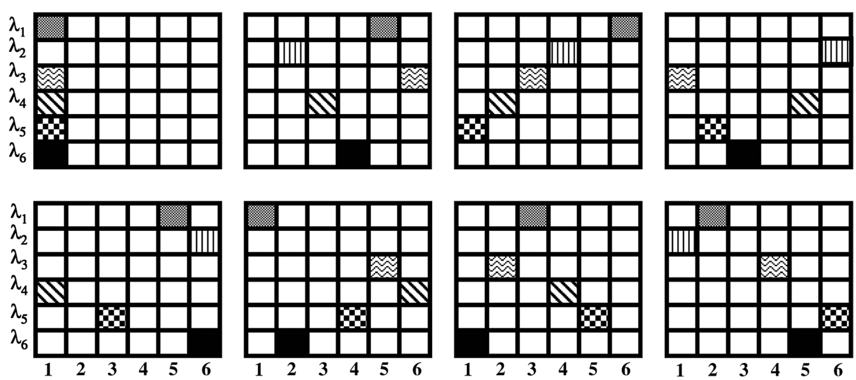

Fig. 1. A (6 × 6,5,1) 2-D OOC. In this figure each row shows a different wavelength, and each column is a different chip time.

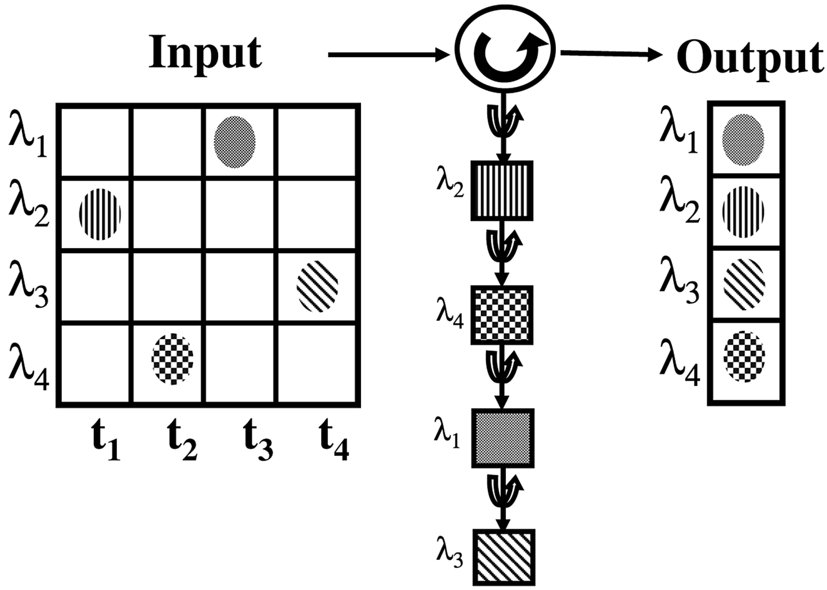

Fig. 2. All optical correlator

A simple means of implementing an optical correlator appears in Fig. 2. Each rectangular box represents an optical filter implemented using for example either a fiber-Bragg grating or an optical micro-resonator. This filter reflects light of the wavelength shown alongside the box and allows light of all other wavelengths to pass through. Filters placed further along the reflection path will suffer an increased delay and in this manner, the placement of the filters can be adjusted to bring the pulses of all the different wavelengths in the desired code matrix into time alignment at the output of the correlator. Implicit in this implementation, is the assumption that the desired code matrix satisfies both the AM-OPPW and AM-OPPTS restrictions.

Constructions for frequency-hopping spreading codes [35,36,37,38,39,40] can often be used to provide 2-D OOCs that satisfy the OPPTS or AM-OPPTS restriction. Papers in the literature dealing with the design of 2-D OOCs include [33,41,42,43,44,45,34,46,47,48,49,50,51,52].

4 Bounds on the Size of a 2-D OCDMA Code

For a given set of values of $\Lambda, T, \omega, \kappa$, let $\Phi(\Lambda \times T, \omega, \kappa)$, denote the largest possible cardinality of a $(\Lambda \times T, \omega, \kappa)$ 2-D OOC code. We define optimal and asymptotically optimum 2-D OOCs as was done in the case of 1-D OOCs in Section 2.

4.1 Johnson Bound

If $\mathcal{C}$ is a $(\Lambda \times T, \omega, \kappa)$ 2-D OOC, then by including every column-cyclic shift of each codeword in $\mathcal{C}$ one can construct a constant weight code using any mapping that reorders the elements of a $\Lambda \times T$ array to form a 1-D string of length ΛT. The resultant constant weight code has parameters $(\Lambda T, \omega, \kappa)$ and size $= T \mid \mathcal{C} \mid$. This observation allows us to translate bounds on constant weight codes to bounds on 2-D OOCs as it is done in Section 2:

We shall refer to these bounds as Johnson bounds for unrestricted 2-D OOCs. They are given by:

Johnson Bound A:

$$\Phi(\Lambda \times T, \omega, \kappa) \leq= \left\lfloor \frac{\Lambda}{\omega} \left\lfloor \frac{\Lambda T - 1}{\omega - 1} \cdots \left\lfloor \frac{\Lambda T - \kappa}{\omega - \kappa} \right\rfloor \right\rfloor \right\rfloor := J_A(\Lambda \times T, \omega, \kappa) \quad (6)$$

This bound was first pointed out by Yang, and Kwong in [34].

Improved Johnson Bound B: Provided $\omega^2 > \Lambda T \kappa$

$$\Phi(\Lambda \times T, \omega, \kappa) \leq= \min(\Lambda, \left\lfloor \frac{\Lambda(\omega - \kappa)}{(\omega^2 - n\kappa)} \right\rfloor) := J_B(\Lambda \times T, \omega, \kappa). \quad (7)$$

Improved Johnson Bound C:

$$\Phi(\Lambda \times T, \omega, \kappa) \leq= \left\lfloor \tfrac{\Lambda}{\omega} \left\lfloor \tfrac{\Lambda T - 1}{\omega - 1} \cdots \left\lfloor \tfrac{\Lambda T - (\ell - 1)}{\omega - (\ell - 1)} h \right\rfloor \cdots \right\rfloor \right\rfloor := J_C(\Lambda \times T, \omega, \kappa), \quad (8)$$

$$\text{where } h = \min(\Lambda T - \ell, \left\lfloor \tfrac{(\Lambda T - \ell)(\omega - \kappa)}{(\omega - \ell)^2 - (\Lambda T - \ell)(\kappa - \ell)} \right\rfloor)$$

and where ℓ is any integer, $1 \leq \ell \leq \kappa - 1$, such that $(\omega - \ell)^2 > (\Lambda T - \ell)(\kappa - \ell)$.

Theorem 1. *When $n = \Lambda T$ the bound J_i, $i \in \{A, B, C\}$ on the size of a $(\Lambda \times T, \omega, \kappa)$ 2-D OOC satisfies the following inequality compared to the bound on the size of $(\Lambda T, \omega, \kappa)$ 1-D OOC:*

$$\Lambda J_i(\Lambda T, \omega, \kappa) \leq J_i(\Lambda \times T, \omega, \kappa) \leq \Lambda J_i(\Lambda T, \omega, \kappa) + (\Lambda - 1)$$

where $J_i(\Lambda T, \omega, \kappa)$ denotes the upper bound for 1-D OOC stated in equations (2),(3),(4)*, and $J_i(\Lambda \times T, \omega, \kappa)$ denoted the upper bound for 2-D OOC stated in equations* (6),(7),(8)*.*

Roughly speaking the theorem suggests that by going to 2-D case, we gain an increase in family size by a factor of Λ.

4.2 Bounds on 2-D AM-OPPW OOCs

Theorem 2. *For any maximally one pulse per wavelength OOC $\mathcal{C}$:*

$$\Phi_{AM-OPPW}(\Lambda \times T, \omega, \kappa) \leq \left\lfloor \frac{\Lambda}{\omega} \left\lfloor \frac{T(\Lambda - 1)}{\omega - 1} \cdots \left\lfloor \frac{T(\Lambda - \kappa)}{\omega - \kappa} \right\rfloor \right\rfloor \right\rfloor$$

For the special case, $\Lambda = \omega$, i.e., for the case when there is exactly one pulse per wavelength, the bound in the Theorem above reduces to

$$\Phi(\Lambda \times T, \omega, \kappa) \ \leq \ T^{\kappa}$$

which is the Singleton bound.

5 2-D OOC Code Construction Using Rational Functions

A 2-D OOC can be regarded as the graph of a function $\lambda = f(t)$, $0 \leq t \leq T-1$, $0 \leq \lambda \leq \Lambda - 1$ mapping time into wavelength or vice-versa, $t = f(\lambda)$. We define an operator $\mathcal{S}_\tau$ such that $\mathcal{S}_\tau(\cdot, f(\cdot))$ is a τ unit cyclic shifted version of the graph, $(\cdot, f(\cdot))$, along the time axis, t. By this definition the construction is a valid OOC with MCP equal to κ iff the equation $\mathcal{S}_\tau(x, f(x)) = (x, g(x))$ has maximally κ solutions for any f and g when either $f \neq g$ or $\tau \neq 0$. Both polynomial and rational functions can be used as the functions $f(\cdot)$. Constructions employing polynomials were reported in [33]. Here we report on rational function constructions.

5.1 Preliminaries

A function of the form $\frac{f(x)}{g(x)}$ in which both $f(x)$ and $g(x)$ are polynomial functions over $GF(q)$ is called a rational function over $GF(q)$. We assume that f and g are relatively prime and that both numerator and denominator are not simultaneously equal to 0 for any value of x.

Since both $f(x)$ and $g(x)$ can take any value in $GF(q)$ (apart from $f(x) = g(x) = 0$) the value of the rational function will be of the form $\frac{a}{b}$ with $a, b \in GF(q)$, either $a \neq 0$ or $b \neq 0$. The fraction $\frac{a}{0}$ is permissible and we define this fraction to equal the symbol ∞. It is shown in [9] that:

Lemma 1. *The number of rational functions* $\frac{f(x)}{g(x)}$ *satisfying:*

- f *and* g *are both nonzero and of degree* $\leq t$,
- f *and* g *are relatively prime,*
- $\frac{f(x)}{g(x)} \neq a$, *for any* a *and*
- f *is monic.*

is given by:

$$c(t) = \begin{cases} q^{2t+1} - q, & t = 1,2,3,4,5,6 \\ \geq q^{2t+1} - q^{2t-6}/7, & t \geq 7 \end{cases}$$

It will be found convenient to identify the pair of symbols $(f(x), g(x))$ for given $x \in GF(q)$ with the element $[f(x), g(x)]^t$ in two-dimensional projective geometry $\mathbb{P}^1(GF(q))$ over $GF(q)$. We have that

$$\mathbb{P}^1(GF(q)) = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} | a, b \in GF(q) \quad \text{and} \quad \begin{bmatrix} a \\ b \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

Two elements $\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right], \left[\begin{smallmatrix} c \\ d \end{smallmatrix}\right] \in \mathbb{P}^1(GF(q))$ are equal provided that there exists an element $\eta \in GF(q)$ such that $\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right] = \eta \left[\begin{smallmatrix} c \\ d \end{smallmatrix}\right]$.

Theorem 3. *If* $f(x) = x^2 + a_1 x + a_0$ *with* $a_1, a_0 \in GF(q)$ *is a primitive polynomial over* $GF(q)$, *then*

$$\Gamma = \begin{bmatrix} 0 & -a_0 \\ 1 & -a_1 \end{bmatrix}$$

is a matrix having the property that the smallest exponent i for which $\Gamma^i[a,b]^t = \eta[a,b]^t$ for some $\eta \in GF(q)$, is $i = (q+1)$.

It follows that the elements of $\mathbb{P}^1(GF(q))$ can be arranged so as to form an orbit of size $(q+1)$:

$$\mathbb{P}^1(GF(q)) = \left\{ \Gamma^i \begin{bmatrix} a \\ b \end{bmatrix} | i = 0, 1, \cdots, q; \begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{P}^1(GF(q)) \right\}$$

5.2 Constructions

All of the constructions below employ rational functions $\frac{f(x)}{g(x)}$ satisfying the conditions of Lemma 1 with both $f(x)$ and $g(x)$ of degree $\leq \kappa'$. In these constructions, we have $\kappa = 2\kappa'$.

Mapping Wavelength to Time, $T = q+1$, q a Power of a Prime: Let $1 \leq \Lambda \leq q$, and $\lambda \in$ some subset of $GF(q)$ of size Λ. Here we consider rational functions $\frac{f(\lambda)}{g(\lambda}$ mapping wavelength into time. Associate to each time slot t, the tth element of a cyclic representation of $\mathbb{P}^1(GF(q))$. Let us define:

$$\Gamma\left(\frac{f(x)}{g(x)}\right) = \mathcal{N}\left(\frac{-a_0 g(x)}{f(x) - a_1 g(x)}\right)$$

where, given a rational function, the operator $\mathcal{N}$ divides out the common factors between numerator and denominator and in addition, scales the two so as to make the numerator monic [9].

Considering f and g are relatively prime, and $a_0 \neq 0$, it is obvious that the operator $\mathcal{N}$ results in some rational function which satisfies the conditions of Lemma 1.

We need to discard all rational functions which are of the form:

$$\frac{f(x)}{g(x)} = \Gamma^k\left(\frac{f(x)}{g(x)}\right) \quad 0 < k \leq q$$

We note that such a rational function doesn't exist since it means that for a certain x_0, $[f(x_0), g(x_0)]^t = \Gamma^k[f(x_0), g(x_0)]^t$ for some $0 < k \leq q$ which is impossible by Theorem 3. Amongst the functions satisfying the conditions of Lemma 1, we have discarded all constant functions, but as this step is unnecessary here, we can add them back.

For any $0 \leq k \leq q$, we declare two rational functions $\frac{f(x)}{g(x)}, \Gamma^k \frac{f(x)}{g(x)}$ to be equivalent. Then the different code matrices correspond to choosing precisely one polynomial from each equivalence class. For each polynomial $f(\cdot)$ the $(\Lambda \times T)$ code array C is given by $C(\lambda, t) = 1$ iff $\frac{f(\lambda)}{g(\lambda)} = t$. This results in a $(\Lambda \times (q+1), \Lambda, 2\kappa')$ 2-D OOC with size $\frac{c(\kappa')}{q+1} + 1$, and $\kappa = 2\kappa' \leq \Lambda \leq q$.

Mapping Time to Wavelength, $\Lambda = q + 1$, q a Power of a Prime: Let $T \mid (q-1)$, and $\beta \in GF(q)$ have multiplicative order T. Take the wavelengths as $\mathbb{P}^1(GF(q))$(the order of elements doesn't matter). Here we consider rational functions mapping time into wavelength. Let us associate to time slot t, the element β^t. We define two rational functions $\frac{f_1(x)}{g_1(x)}, \frac{f_2(x)}{g_2(x)}$ in $\mathcal{F}_\kappa$ to be equivalent if $\frac{f_1(\beta^i x)}{g_1(\beta^i x)} = \frac{f_2(x)}{g_2(x)}$ for some $i \in \mathbb{Z}_T$. First discard all rational functions $\frac{f(x)}{g(x)}$, which satisfy $\frac{f(\beta^i x)}{g(\beta^i x)} = \frac{f(x)}{g(x)}$ for $i \neq 0$. The number of remaining rational functions is computed in [9] as: $\sum_{i|(q-1)} \mu(i) c\left(\lfloor \frac{t}{i} \rfloor\right)$.

Choosing one function $\frac{f(\cdot)}{g(\cdot)}$ from each of the remaining equivalence classes and associating to it, the $(\Lambda \times T)$ code array C by letting $C(\lambda, t) = 1$ iff $\frac{f(\beta^t)}{g(\beta^t)} = \lambda$ where $t \in \mathbb{Z}_T$ and $\lambda \in \mathbb{P}^1(GF(q))$ results in a $((q+1) \times T, T, 2\kappa')$ 2-D OOC of size $\frac{1}{T} \sum_{i|(q-1)} \mu(i) c\left(\left\lfloor \frac{\kappa'}{i} \right\rfloor\right)$.

Note 1. In the function plot constructions if one maps wavelength into time, the resulting 2-D OOC will be of maximally OPPW-type.

Theorem 4. *All of the above constructions are asymptotically optimal with respect to the Johnson bound.*

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Appendix A: Constructions of 1-D OOCs

Table 1. Different Known Constructions of 1-D Optical Orthogonal Codes, here p denotes a prime and q denotes a power of a prime

Construction Name	Parameters	Code Size
Singer* [6] [30]	$(q^2+q+1, q+1, 1)$	$\mid \mathcal{C} \mid = 1$
Projective Geometry* [6]	$(\frac{q^{d+1}-1}{q-1}$,q+1,1)	$\mid \mathcal{C} \mid = \begin{cases} \frac{q^d-1}{q^2-1}, & d \ even, \\ \frac{q^d-q}{q^2-1}, & d \ odd. \end{cases}$
Combinatorial Method* [6]	$(n, 3, 1)$, $n \neq 2 \pmod 6$	$\mid \mathcal{C} \mid = \lfloor \frac{n-1}{6} \rfloor$
Chung-Kumar* [7]	$(p^{2m}-1, p^m+1, 2)$	$\mid \mathcal{C} \mid = p^m - 2$
Chung-Kumar* [7] (via Wilson difference sets)	$(p, \omega, 1)$, $p = \omega(\omega-1)r+1$ $\omega = 2m+1$, or $\omega = 2m$	$\mid \mathcal{C} \mid = r$
MZKZ Family $\mathcal{A}$ [9]	(pm, m, t) $m \mid (p-1)$,$1 \leq t \leq m$	$\mid \mathcal{C} \mid$ $= \frac{1}{mp} \sum_{d \mid (p-1)} p^{\lceil (t+1)/d \rceil} \mu(d)$
MZKZ Family $\mathcal{B}$ [9]	$((q-1)p, (p-t), t)$ $1 \leq t \leq (p-t)$	$\mid \mathcal{C} \mid = \frac{q}{p} \left(\frac{q^t-1}{q-1} \right)$
MZKZ Family $\mathcal{C}$ [9]	$(m(q+1), m, 2t)$ $m \mid (q-1)$, $(m, q+1) = 1$ $1 \leq t \leq m/2$	$\mid \mathcal{C} \mid =$ $\frac{1}{(q+1)m} \sum_{d \mid (q-1)} \mu(d) c([t/d])$ $c(t)$ as defined in Lemma 1
Bose-Chowla* Construction [29] [31] [32]	$(q^2-1, q, 1)$	$\mid \mathcal{C} \mid = 1$
Generalized* [29] Bose-Chowla (lines)	$(q^a-1, q, 1)$	$\mid \mathcal{C} \mid = q^{a-2} + q^{a-3} + \cdots + 1$
Generalized* [5] Bose-Chowla (hyperplanes)	$(q^a-1, q^{a-1}, q^{a-2})$	$\mid \mathcal{C} \mid = 1$
Conics on [28] Finite Projective Plane	$(q^3+q^2+q+1, q+1, 2)$	$\mid \mathcal{C} \mid = q^3 - q^2 + q$

An * in the table indicates that the corresponding construction is optimal with respect to some bound.