The evolution of correlation functions in the Zel'dovich approximation and its implications for the validity of perturbation theory.

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ABSTRACT

We investigate whether it is possible to study perturbatively the transition in cosmological clustering between a single streamed flow to a multi streamed flow. We do this by considereing a system whose dynamics is governed by the Zel'dovich approximation (ZA) and calculating the evolution of the two point correlation function using two methods: 1. Distribution functions 2. Hydrodynamic equations without pressure and vorticity. The latter method breaks down once multistreaming occurs whereas the former does not. We find that the two methods give the same results to all orders in a perturbative expansion of the two point correlation function. We thus conclude that we cannot study the transition from a single stream flow to a multi-stream flow in a perturbative expansion. We expect this conclusion to hold even if we use the full gravitational dynamics (GD) instead of ZA.

We use ZA to look at the evolution of the two point correlation function at large spatial separations and we find that until the onset of multi-streaming the evolution can be described by a diffusion process where the linear evolution at large scales gets modified by the rearrangement of matter on small scales. We compare these results with the lowest order nonlinear results from GD. We find that the difference is only in the numerical value of the diffusion coefficient and we interpret this physically.

We also use ZA to study the induced three point correlation function. At the lowest order of nonlinearity we find that, as in the case of GD, the three point correlation does not necessarily have the hierarchical form. We also find that at large separations the effect of the higher order terms for the three point

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correlatin function is very similar to that for the the two point correlation and in this case too the evolution can be be described in terms of a diffusion process.

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1. Introduction

The inviscid hydrodynamic equations without pressure and vorticity (referred to as the HD equations in the rest of this paper) are often used to describe the evolution of disturbances in an expanding universe filled with collisionless particles that interact only through Newtonian gravity. The disturbances that are usually considered are such that initially all the particles at any point have the same velocity i.e. it is a single streamed flow. Such a situation is correctly described by the HD equations. As the disturbances evolve the particle trajectories intersect and there are particles with different velocities at the same point i.e. the flow becomes multi-streamed. When this occurs the HD equations are no longer valid. This is because the HD equations neglect the local stress tensor associated with the moments of the velocity about the mean velocity at a point.

The BBGKY hierarchy of equations obeyed by the distribution functions can be used instead of the HD equations. The distribution functions keep track of the position and velocity of the particles and these equations are valid even if multistreaming occurs. The question we would like to address in this paper is whether we can study the effects of multistreaming by using distribution functions perturbatively to follow the evolution of the disturbances.

We look at the perturbative evolution of the density - density two point correlation function for Gaussian initial conditions in a universe with $\Omega = 1$. The perturbative expansion of this function using the HD equations has been studied by many authors (Juskewicz 1981; Vishniac 1983; Fry 1994). In a recent paper (paper II,Bharadwaj 1995) we have calculated the lowest order nonlinear term for the two point correlation function using the moments of the BBGKY hierarchy. These equations are based on the distribution functions and are valid even in the multi-streamed regime. The two different methods of calculation (HD and BBGKY) aro found to give the same result at the lowest order of nonlinearity, and hence, to this order, distribution functions have not been able to capture any effect of multi-streaming. In this paper we investigate whether by going to higher orders of perturbation theory we shall be able to study any effects of multi-streaming or if it is a limitation of perturbation theory that it cannot follow the transition from a single streamed flow to a multi streamed flow.

Because of the difficulty in calculating the higher order terms in a perturbative treatment of gravitational dynamics (GD), we look at a simpler system where we use the Zel'dovich approximation (ZA, Zel'dovich 1970) to determine the motion of the particles. In this situation too the transition from a single streamed flow to a multi-streamed flow occurs and we can analyse it to see if in a perturbative calculation using distribution functions we can include any effects of multi-streaming which would be missed if the HD equations were used instead.

In section 2 we discuss the evolution equations. In section 3 we use distribution functions to calculate the evolution of the two point correlation function. In section 4 we do the same calculation using the HD equations and compare the result with that obtained in section 3.

Bond and Couchman (1988) have studied the evolution of the two point correlation function using ZA and the calculation presented in section 3 is on similar lines. In a more recent paper Schneider and Bartlemann (1995) have studied the evolution of the power spectrum in ZA. For a comprehensive article on various aspects of ZA the reader is referred to a review by Shandarin and Zel'dovich (1989).

In paper II we investigated the lowest order nonlinear correction (using GD) to the two point correlation for initial power spectra of the form $P(k) \propto k^n$ at small k and an exponential or Gaussian cutoff at large k. We found that for $0 < n \leq 3$, the numerical results for the nonlinear correction to the two point correlation function at large x could be fitted by a simple formula . We also interpreted this formula in terms of a simple diffusion process. In section 5 of this paper we investigate the evolution of the two point correlation function at large separations using ZA and compare it with the results from GD.

In section 6 we look at the evolution of the induced three point correlation function using ZA. This was first calculated for GD by Fry (1984) and he concluded that for power law initial conditions, at large separations, the three point correlation function could be described by the hierarchical form where it can be written in terms of products of the two point correlation function evaluated at the separations involved. In an earlier paper (part I, Bharadwaj 1994) we calculated the same quantity and found that these conclusions were not fully correct. We showed that the three point correlation function at some length scale, depends not only on the two point correlation at the same length scales but on all smaller scales also. As a result we found that the hierarchical form is true for only a class of initial conditions and that there was a class for which it did not hold. In this paper we first calculate the three point correlation function at the lowest order of nonlinearity for ZA and compare it to the results from GD. We then go on to study the effect of the higher order nonlinear terms at large separations.

The calculations using ZA are valid for any value of Ω_0 but whenever we make comparisons with GD it is for the specific value $\Omega = 1$.

A similar calculation has been done by Grinstein and Wise (1987) who have studied the evolution of skewness of the density field averaged over a Gaussian ball. Also, Munshi and Starobinsky (1994) have considered the evolution of the skewness of the density field for ZA and various other approximations, and Bernardeau et. al. (1993) have calculated the evolution of the skewness of the density field averaged over top hat filters. All of these calculations have been done at the lowest order of nonlinearity.

In section 7 we present a discussion of the results obtained and the conclusions.

2. Evolution of the distribution fuction

The Zel'dovich approximation defines a map from the initial position af a particle to its position at any later instant. If $x_\mu(t)$ is the comoving coordinate of a particle at any time t, the initial instant being t_0, and $b(t)$ the growing mode in the linear analysis of density perturbations, this map is

$$x_\mu(t) = x_\mu(t_0) + b(t)u_\mu \, . \tag{1}$$

The quantity u_μ is related to the peculiar velocity $\mathrm{v}_\mu(\mathrm{t})$ at any instant by

$$\mathrm{v}_\mu(\mathrm{t}) = \mathrm{a}(\mathrm{t})\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{x}_\mu(\mathrm{t}) = \mathrm{a}(\mathrm{t})\dot{\mathrm{b}}(\mathrm{t})\mathrm{u}_\mu \tag{2}$$

where $a(t)$ is the scale factor.

We consider a system of particles whose motion is governed by this mapping. This can be described by a distribution function $f(x,u,t)$, where $f(x,u,t)d^3x d^3u$ is the number of particles in the volume d^3x around the point x and having a value of u in an interval d^3u around u.

We can see that Liouville theorem is true for the mapping defined in equation (1). Using this we can obtain the equation for the time evolution of the distribution function f,

$$f(x,u,t) = f(x - b(t)u, u, t_0) \, . \tag{3}$$

We can also use equation (1) to obtain a differential equation for the evolution of the distribution function

$$\frac{\partial}{\partial b} f(x,u,b) + u_\mu \frac{\partial}{\partial x_\mu} f(x,u,b) = 0 \, , \tag{4}$$

where we use the growing mode b instead of time as the evolution parameter.

We are interested in the evolution of the statistical properties of an ensemble of such systems.

Every member of the ensemble initially has the particles uniformly distributed. Initially each particle can be labeled by its coordinate x_μ. The particles are given velocities $u_\mu(x)$. The velocity field is the gradient of a function $\psi(x)$ which for each system is a different realization of a Gaussian random field. It is assumed that ψ is statistically homogenous and isotropic. The statistical properties of the ensemble are initially fully specified by the two point correlation of ψ which is defined as $\phi(x) =< \psi(0)\psi(x) >$, where the angular brackets $<>$ denote ensemble averageing.

The statistical quanitity whose evolution we shall focus on in this paper is the density two point correlation function $\xi(x,t)$. This is defined by the relation

$$< \rho >^2 (1 + \xi(x)) =< \rho(0)\rho(x) > \ . \tag{5}$$

where $\rho(x)$ is the mass density. This is just the number density of particles multiplied by the mass of each particle which is assumed to be the same for all the particles.

3. The two point correlation using distribution functions

In this section we look at the evolution of the ensemble averaged two point distribution functions ρ_2. This is defined as

$$\rho_2(x^1, x^2, u^1, u^2, t) =< f(x^1, u^1, t) f(x^2, u^2, t) > . \tag{6}$$

From homogeneity and isotropy we can also say that

$$\rho_2(x^1, x^2, u^1, u^2, t) = \rho(x, u^1, u^2, t) \tag{7}$$

where

$$x_\mu = x_\mu^2 - x_\mu^1. \tag{8}$$

The density two point correlation function is related to the zeroth moment of the two point distribution function with respect to u.

$$< \rho >^2 (1 + \xi(x,t)) = \int \rho_2(x, u^1, u^2, t) d^3u^1 d^3u^2. \tag{9}$$

In this paper we normalize $< \rho >= 1$.

The initial two point distribution is a Gaussian in the velocities and hence specified by the covariance matrix

$$T_{\mu\nu}^{ab}(x) =< u_\mu^a u_\nu^b > (x) = \int u_\mu^a u_\nu^b \rho_2(x, u^1, u^2, t_0) d^3u^1 d^3u^2 \tag{10}$$

where a, b take values $1, 2$. The initial two point distribution function then is the Gaussian distribution

$$\rho_2(x, u^1, u^2, t_0) = \frac{1}{(2\pi)^3 \sqrt{\Delta T(x)}} \exp\left[-\frac{1}{2} u^a_\mu u^a_\mu (T^{-1})^{ab}_{\mu\nu}(x)\right] , \tag{11}$$

where $\Delta T(x)$ is the determinant of the covariance matrix. In terms of the potential ϕ we have

$$< u^1_\mu u^2_\nu >= -\partial_\mu \partial_\mu \phi(x) \tag{12}$$

and

$$< u^1_\mu u^1_\nu >= -\frac{1}{3} \nabla^2 \phi(0) \delta_{\mu\nu}. \tag{13}$$

We use equation (3) to obtain the time evolution of ρ_2

$$\rho_2(x, u^1, u^2, t) = \rho_2(x - (u^2 - u^1) b(t), u^1, u^2, t_0). \tag{14}$$

This may also be written as

$$\rho(x, u^1, u^2, t) = \int \delta^3 \left[x^{'} - \left(x - (u^2 - u^1) b\,(t)\right)\right] \rho_2(x^{'}, u^1, u^2, t_0) d^3 x^{'} . \tag{15}$$

Using the Fourier expansion of the Dirac delta function and using equation (11) we have

$$\begin{aligned} \rho(x, u^1, u^2, t) &= \int \left(\frac{1}{2\pi}\right)^3 \exp\left[ik_\mu \left(x^{'}_\mu - x_\mu\right)\right] \exp\left[ik_\mu \left(u^2_\mu - u^1_\mu\right) b\,(t)\right] \\ &\times \frac{1}{(2\pi)^3 \sqrt{\Delta T(x^{'})}} \exp\left[-\frac{1}{2} u^a_\mu u^a_\mu (T^{-1})^{ab}_{\mu\nu}(x^{'})\right] d^3k d^3x^{'} \end{aligned} \tag{16}$$

Using this in equation (9) and doing the u integrals we get

$$1 + \xi(x, t) = \left(\frac{1}{2\pi}\right)^3 \int \exp\left[ik_\mu \left(x^{'}_\mu - x_\mu\right)\right] \exp\left[-\frac{b^2(t)}{2} k_\mu k_\nu F_{\mu\nu}(x^{'})\right] d^3x^{'} d^3k\,, \tag{17}$$

where

$$F(x)_{\mu\nu} = -\frac{2}{3} \nabla^2 \phi(0) \delta_{\mu\nu} + 2\partial_\mu \partial_\mu \phi(x) \tag{18}$$

Doing the k integral we obtain the two point correlation as

$$1 + \xi(x, t) = \frac{1}{(2\pi)^{\frac{3}{2}}\, b^3(t)} \int \frac{1}{\sqrt{\Delta F(x^{'})}} \exp\left[-\frac{1}{2b^2(t)} \left(x^{'}_\mu - x_\mu\right) \left(x^{'}_\nu - x_\nu\right) F^{-1}_{\mu\nu}(x^{'})\right] d^3x^{'} . \tag{19}$$

Instead of integrating equation (17), if we do a Taylor expansion of

$$exp\left[-\frac{b^2(t)}{2} k_\mu k_\nu F_{\mu\nu}(x^{'})\right]$$

and then do the k and the x' integrals, we obtain

$$\begin{aligned} 1+\xi(x,t) &= \sum_{n=0}^{\infty}\frac{b^{2n}}{n!}\partial_{\mu_1}\partial_{\nu_1}...\partial_{\mu_n}\partial_{\mu_n}\left[\left(\partial_{\mu_1}\partial_{\nu_1}\phi(x)-\delta_{\mu_1\nu_1}\frac{\nabla^2\phi(0)}{3}\right)...\right. \\ & \left. ...\left(\partial_{\mu_n}\partial_{\nu_n}\phi(x)-\delta_{\mu_n\nu_n}\frac{\nabla^2\phi(0)}{3}\right)\right]\,. \qquad (20) \end{aligned}$$

Nowhere above has any assumption been made about the number of streams in the flow . Equation (19) obviously has the effects of multistreaming built into it. Equation (20) is what one would obtain if one did a perturbative expansion of the distribution function and calculated the two point correlation function. Whether by doing the perturbative analysis this way (i.e. using distribution functions) we are able to include the effects of multistreaming is what has to be checked.

4. The two point correlation using the hydrodynamic equations

In this section we shall work in the single stream approximation. We cosider any one member of the ensemble described previously. Its evoulution is described by equation (4). If we take the zeroth moment of this equation with respect to u. Using the definitions

$$\rho(x,b)=m\int f(x,u,b)d^3u \qquad (21)$$

and

$$\rho(x,b)v_\mu(x,b)=m\int u_\mu f(x,u,b)d^3u \qquad (22)$$

we have the continuity equation

$$\frac{\partial}{\partial b}\rho(x,b)+\partial_\mu(\rho(x,b)v_\mu(x,b))=0\,. \qquad (23)$$

Next, taking the first moment of equation (4) and using equation (23) we have

$$\begin{aligned} & \rho(x,b)[\frac{\partial}{\partial b}v_\mu(x,b)+v_\nu(x,b)\partial_\nu v_\mu(x,b)]+ \\ + \quad & m\partial_\nu\int(v_\nu(x,b)-u_\nu)(v_\mu(x,b)-u_\mu)f(x,u,b)d^3u=0\,. \qquad (24) \end{aligned}$$

In the single stream approximation the last term in the above equation is dropped, and we have

$$\frac{\partial}{\partial b}v_\mu(x,b)+v_\nu(x,b)\partial_\nu v_\mu(x,b)=0\,. \qquad (25)$$

We shall use use equations (23) and (25) to perturbatively evolve the density and velocity fields of the system. We then take ensemble averages and use these equations to calculate the two point correlation function.

Using equation (23) we can obtain an equation for the first derivative of the two point correlation function

$$\frac{\partial}{\partial b}[<\rho>^2 (1+\xi(x,b))] = -<\partial^1_\mu(\rho(x^1)v_\mu(x^1))\rho(x^2)> - <\rho(x^1)\partial^2_\mu(\rho(x^2)v_\mu(x^2))> . \quad (26)$$

Using the normalization $<\rho>=1$, the above equation may be written as

$$\frac{\partial}{\partial b}\xi(x,b) = -\partial^{a_1}_{\mu_1} <\rho(1)v^{a_1}_{\mu_1}\rho(2)> . \quad (27)$$

We can use equation (23) and (25) to obtain equations for the higher derivatives of the two point correlation

$$\frac{\partial^n}{\partial b^n}\xi(x,b)) = (-1)^n\partial^{a_1}_{\mu_1}\partial^{a_2}_{\mu_2}...\partial^{a_n}_{\mu_n} <\rho(1)v^{a_1}_{\mu_1}v^{a_2}_{\mu_2}...v^{a_n}_{\mu_n}\rho(2)> . \quad (28)$$

Next we write the two point correlation function as a Taylor series in powers of the growing mode b

$$\xi(x,b) = \sum_{n=1}^{\infty}\frac{b^n}{n!}\frac{\partial^n}{\partial b^n}\xi(x,b)_{b=0} . \quad (29)$$

It should be noted that this allows us to express the two point correlation function at any instant in terms of the derivatives of the two point correlation function at the initial instant. Next, using equation (28) we get

$$\xi(x,b) = \sum_{n=1}^{\infty}\frac{b^n(-1)^n}{n!}\partial^{a_1}_{\mu_1}\partial^{a_2}_{\mu_2}...\partial^{a_n}_{\mu_n} <\rho(1)v^{a_1}_{\mu_1}v^{a_2}_{\mu_2}...v^{a_n}_{\mu_n}\rho(2)>_{b=0} .. \quad (30)$$

Then using the fact that the initial density is uniform, we can write the two point correlation function at any arbitrary time in terms of the initial velocities only i.e.

$$\xi(x,b) = \sum_{n=1}^{\infty}\frac{b^n}{n!}(-1)^n\partial^{a_1}_{\mu_1}\partial^{a_2}_{\mu_2}...\partial^{a_n}_{\mu_n} <v^{a_1}_{\mu_1}v^{a_2}_{\mu_2}...v^{a_n}_{\mu_n}>_{b=0} .. \quad (31)$$

Also the initial velocity field is Gaussian and hence all the odd terms in equation (31) are zero. We can then write this equation as

$$\xi(x,b) = \sum_{n=1}^{\infty}\frac{b^{2n}}{(2n)!}\partial^{a_1}_{\mu_1}\partial^{b_1}_{\nu_1}...\partial^{a_n}_{\mu_n}\partial^{b_n}_{\nu_n} <v^{a_1}_{\mu_1}v^{b_1}_{\nu_1}...v^{a_n}_{\mu_n}v^{b_n}_{\nu_n}>_{b=0} .. \quad (32)$$

For the Gaussian initial velocity field we have

$$<v^{a_1}_{\mu_1}v^{b_1}_{\nu_1}v^{a_2}_{\mu_2}v^{b_2}_{\nu_2}...v^{a_n}_{\mu_n}v^{b_n}_{\nu_n}> = \sum_P <v^{a_1}_{\mu_1}v^{b_1}_{\nu_1}><v^{a_2}_{\mu_2}v^{b_2}_{\nu_2}> ... <v^{a_n}_{\mu_n}v^{b_n}_{\nu_n}> , \quad (33)$$

where the sum is over all possible ways of pairing the u's.

Using this and the fact that the derivatives are symmetric in all the indices involved, we have for the initial velocity field

$$\begin{aligned} & \partial^{a_1}_{\mu_1}\partial^{b_1}_{\nu_1}...\partial^{a_n}_{\mu_n}\partial^{b_n}_{\nu_n} < v^{a_1}_{\mu_1}v^{b_1}_{\nu_1}...v^{a_n}_{\mu_n}v^{b_n}_{\nu_n} > \\ = & \frac{(2n)!}{n!2^n}\partial^{a_1}_{\mu_1}\partial^{b_1}_{\nu_1}...\partial^{a_n}_{\mu_n}\partial^{b_n}_{\nu_n}\left[< v^{a_1}_{\mu_1}v^{b_1}_{\nu_1} > ... < v^{a_n}_{\mu_n}v^{b_n}_{\nu_n} >\right] . \end{aligned} \tag{34}$$

This, when used in equation (32), give us

$$\xi(x,b) == \sum_{n=1}^{\infty}\frac{b^{2n}}{n!2^n}\partial^{a_1}_{\mu_1}\partial^{b_1}_{\nu_1}\partial^{a_2}_{\mu_2}\partial^{b_2}_{\nu_2}...\partial^{a_n}_{\mu_n}\partial^{b_n}_{\nu_n}\left[< v^{a_1}_{\mu_1}v^{b_1}_{\nu_1} >< v^{a_2}_{\mu_2}v^{b_2}_{\nu_2} > ... < v^{a_n}_{\mu_n}v^{b_n}_{\nu_n} >\right]_{b=0} . \tag{35}$$

Summing the superscripts $a_1, b_1, ...a_n, b_n$ over the values 1 and 2 and using the fact that for the initial velocity field

$$< v^a_\mu v^b_\nu >= \frac{-\nabla^2\phi(0)}{3}\delta_{\mu\nu} \text{ if } a = b \tag{36}$$

and

$$< v^a_\mu v^b_\nu >= \partial^a_\mu\partial^b_\nu\phi(x) \text{ if } a \neq b \tag{37}$$

we have

$$\begin{aligned} 1+\xi(x,t) = & \sum_{n=0}^{\infty}\frac{b^{2n}}{n!}\partial_{\mu_1}\partial_{\nu_1}...\partial_{\mu_n}\partial_{\mu_n}\left[\left(\partial_{\mu_1}\partial_{\nu_1}\phi(x) - \delta_{\mu_1\nu_1}\frac{\nabla^2\phi(0)}{3}\right)...\right. \\ & \left. ...\left(\partial_{\mu_n}\partial_{\nu_n}\phi(x) - \delta_{\mu_n\nu_n}\frac{\nabla^2\phi(0)}{3}\right)\right] . \end{aligned} \tag{38}$$

This is the same as equation (20) which was obtained using distribution functions. Thus we see that the perturbative calculation of the two point correlation function using distribution functions has no effects of multistreaming and hence we reach the conclusion that it is not possible to perturbatively follow the transition from a single streamed flow to a multi streamed flow.

5. The two point correlation at large separations.

In this section we investigate the evolution of the two point correlation function in the regime where it can be studied perturbatively and we look at the behaviour at large separations. The initial conditions for the evolution of the cosmological correlations may be expressed in terms of the potential $\phi(x)$ or equivalently in terms of the matter two point correlation in the linear epoch, $\xi^{(1)}(x,t)$. The two are related by the equation

$$\xi^{(1)}(x,t) = b^2(t)\nabla^4\phi(x)\,. \tag{39}$$

Usually the initial conditions are given in terms of the matter two point correlation $\xi^{(1)}(x,t)$ or its Fourier transform $b^2(t)P_1(k)$ which is the power spectrum. One then has to invert equation (39) to obtain the potential $\phi(x)$ and its derivatives. In doing so one has the freedom in choosing boundary conditions and the effect of changing the boundary condition is

$$\nabla^2\phi(x) \to \nabla^2\phi(x) + C_1 \tag{40}$$

and

$$\phi(x) \to \phi(x) + \frac{C_1 x^2}{6} + C_2\,. \tag{41}$$

Equation (20) for the two point correlation function is invariant uder these transformations and we are free to choose any boundary condition. For initial conditions where the integral $\int_0^\infty \xi^{(1)}(x,t)xdx$ (or $\int_0^\infty P_1(k)dk$) is finite we can choose the boundary condition $\text{limit}_{x\to\infty}\nabla^2\phi(x) = 0$. We then have

$$< u^2 >= -\nabla^2\phi(0) = \int_0^\infty \xi^{(1)}(x)xdx\,. \tag{42}$$

In addition, if at large x the function $\partial_\mu\partial_\nu\phi(x)$ is monotonically decreasing and $\partial_\mu\partial_\nu\phi(x) \ll (\delta_{\mu\nu}/3)\nabla^2\phi(0)$, we can then neglect all but one of the $\partial_\mu\partial_\nu\phi(x)$ terms that appear in equation (20). For initial conditions where the power spectrum has the form $P(k) \propto k^n$ at small k and if it has a cutoff at large k, the conditions discussed above are satisfied for $n > -1$. For these cases we obtain for the two point correlation function at large x

$$\xi(x,t) = \sum_{n=0}^{\infty} \frac{b^{2(n+1)}}{n!}\left(\frac{-\nabla^2\phi(0)}{3}\right)^n \left(\nabla^2\right)^n \nabla^4\phi(x)\,. \tag{43}$$

Using this we obtain the lowest order nonlinear correction to the two point correlation function at large x,

$$\xi^{(2)}(x,t) = \frac{b^2}{3} < u^2 > \nabla^2\xi^{(1)}(x,t) \tag{44}$$

In paper II we have calculated the same quantity using GD and we found that for $0 < n \leq 3$ at large x the results can be fitted by the formula

$$\xi^{(2)}(x,t) = .194b^2 < u^2 > \nabla^2\xi^{(1)}(x,t) \tag{45}$$

We find that the two equations are very similar and they differ only in the numerical coefficient. In paper II we also interpret equation (45) in terms of a simple heuristic model based on a diffusion process. We consider a particular member of the enemble and look at the evolution of the density in volume elements located at a separation x from each other. We assume that the density in each volume element grows according to linear theory and the volume elements get rearranged randomly on small scales because of their peculiar velocities. Based on this model we obtained an equation identical to equation (44). Thus we see that this model gives an exact description of what happens in ZA at large scales in the regime when the perturbative treatment is valid. In ZA, like in our heuristic model, the velocity of the particles is fixed whereas in GD the particle velocity changes as evolution proceeds. We believe that this is responsible for the smaller diffusion coefficient for GD compared to ZA.

Going back to equation (43) and writing it in Fourier space we obtain for the power spectrum

$$P(k,t) = \left[\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{-b^2k^2 < u^2 >}{3}\right)^n\right] b^2 P_1(k)\,, \tag{46}$$

Summing up the terms in the square brackets we have

$$P(k,t) = \exp\left(\frac{-b^2k^2 < u^2 >}{3}\right) b^2 P_1(k)\,. \tag{47}$$

which in real space gives us

$$\xi(x,t) = \frac{1}{(\sqrt{\pi}2L(t))^3}\int_0^\infty \exp\left[-\frac{(x-x^{'})^2}{4L(t))^2}\right]\xi^1(x^{'},t)d^3x^{'}\,, \tag{48}$$

where

$$L^2(t) = \frac{1}{3}b^2(t) < u^2 > \ . \tag{49}$$

The length scale $L(t)$ is the r.m.s. deviation of the particles from their Lagrangian (or initial) positions at any time t. We see that the nonlinear evolution of the two point correlation function at large x corresponds to a convolution of the linear two point correlation with a Gaussian whose width is proportional to $L(t)$. This is consistent with our interpretation of the evolution in terms of a diffusion process.

For the case when the initial power spectrum has the form

$$P_1(k) = Ae^{-\alpha^2k^2}k^n\,, \tag{50}$$

using equation (43) at small k, we have for the nonlinear power spectrum at small k

$$P_1(k) = Ae^{-(\alpha^2+L^2(t))k^2}k^n\,,\ . \tag{51}$$

Using equation (50) and (51), and using the fact that

$$\int e^{ikx} e^{-\beta^2\alpha^2 k^2} P_1(k) d^3k = \frac{1}{\beta^{3+n}} \int_{-\infty}^{\infty} e^{ik\frac{x}{\beta}} e^{-\alpha^2 k^2} P_1(k) d^3k \tag{52}$$

we obtain for the nonlinear two point correlation function at large x

$$\xi_1(x,t) = \left[1 + \left(\frac{L(t)}{\alpha}\right)^2\right]^{-\frac{3+n}{2}} \xi_2^{(1)}\left(\frac{x}{\sqrt{1+\left(\frac{L(t)}{\alpha}\right)^2}}, t\right). \tag{53}$$

This formula relates the nonlinear two point correlation at some separation x at a time t to the linear two point correlation at a smaller separation at the same time. Thus, at large x, for small values of the two point correlation, we have information being transferred out from the smaller scales to the larger scales.

We next numerically investigate the evolution of the two point correlation function at large separations for the initial power spectrum $P_1(k) = .5e^{-k^2}k$. Figure 1 shows the function $\xi^{(1)}(x)$ as a function of x. This function multiplied by the square of the scale factor gives the linear two point correlation $\xi^{(1)}(x,t)$. At large x the function $\xi^{(1)}(x)$ has a negative sign and a power law behaviour x^{-4}. We investigate the evolution of the two point correlation function at the large separation $x = 10$. We do this using four different approximations which we list below:
(a). linear perturbation theory
(b). linear theory + the lowest order nonlinear correction using GD (paper II).
(c). the result obtained from summing the whole perturbation series for the ZA with the extra assumptions about the evolution at large separations made in this section i.e. equation (53)
(d). the non-perturbative two point correlation calculated using ZA (19)

This exercise allows us to investigate two different issues. The first thing that we can check is how well ZA approximates GD. This can be done by comparing (b) with (c) and (d). In this section we have made some assumptions about the large x behaviour of the two point ocrrelation function and arrived at the diffusion picture for the evolution. We can put these assumptions to test by comparing (c) with (d). The results are shown in figure 2. We find all the four approximations match in the early stages of the evolution. The two point correlation at this separation is initially negative and this value evolves according to linear theory where it gets multiplied by b^2. The different approximations start to differ as the evolution proceeds. The first thing to note is that they start to differ much before $\xi(x,t) \sim 1$ when one would naively expect the perturbation series to break down. This is a consequence of the non-local nature of the nonlinear terms for the two point correlation. As discussed in paper II, this can be understood using equation (42)

$$< u^2 >= \int_0^\infty \xi^{(1)}(x) x dx$$

which shows that the nonlinear correction depends on the linear two point correlation condition at all the scales and the major contribution to this integral comes from the small scales. The small scales become strongly nonlinear very early in the evolution and it is because of this that the nonlinear term starts contributing at large x even when $\xi(x,t) \ll 1$. In all the approximations (i.e. (b),(c) and (d)) the effect of the initial deviation from the linear theory is to make the growth rate faster than $b^2(t)$. In the initial stages approximations (b), (c) and (d) exhibit qualitatively similar behaviour but as the evolution proceeds we find that (d) starts showing a behaviour completely different from (b) and (c). We find that the rapidly decreasing function (d) slows down its decrease and then starts to increase. This is quite different from the behaviour of (b) and (c) which continue to decrease. This difference is because of the effects of multi-streaming. In ZA the correlations get washed out after multi streaming occurs. Until the onset of multistreaming the diffusion picture (c) matches quite well with the full ZA i.e. (d). A comparison of (b),(c) and (d) shows that ZA qualitatively predicts the same behaviour as GD and the quantitative difference may be attributed to the difference in the diffusion coefficients. In the case of the actual gravitational dynamics (non-perturbative) we expect that the results may be different because there the particles will get 'stuck' in bound objects once multistreaming occurs (e.g. the adhesion model; Gurbatov, Saichev and Shandarin 1989) As a result of this the mean square displacement of the particles will be much less than in ZA or in perturbative GD. Although we expect this diffusion picture to hold for the actual evolution of the two point correlation function at large x, the perturbative treatment of GD and also calculations using ZA may overestimate what would be obtained in N-body simulations. Incidentally, the regime treated here would be difficult to study using such simulations since it involves the low amplitude tail of the two point correlation function which would be limited by the size of the box and it would require a large dynamical range.

6. The 3 point correlation function.

We use ZA to follow the evolution of the N point correlation function. It is possible to do this nonperturbatively by following a line of reasoning very similar to that in section 3. However since ZA is a good substitute for the gravitational dynamics only in the weakly nonlinear regime we prefer to carry out the investigation perturbatively.

We first consider the evolution of the ensemble averaged N point distribution function $\rho_N(x^a, u^a, t)$. This is a generalization of the ensemble averaged two point distribution function introduced in section 3 and the superscript a refers to the various points i.e.1,2... N in phase space which are arguments of this function. Using equation (3) we obtain for the time evolution of this function

$$\rho_{\rm N}(x^a, u^a, t) = \rho_{\rm N}(x^a - b(t)u^a, u^a, t_0)\,. \tag{54}$$

Expanding this in a perturbative series and using $a_1, a_2 \ldots a_n$ for n indices that independently take values between 1 and N, and using $\mu_1, \mu_2 \ldots \mu_n$ for n corresponding Cartesian components, we have

$$\rho_{\rm N}(x^a, u^a, t) = \sum_{n=0}^{\infty} \frac{(-b)^n}{n!} u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n}\, \partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_n}_{\mu_n}\, \rho_{\rm N}(x^a, u^a, t_0)\,. \tag{55}$$

For both the kinds of indices the Einstein summation convention holds and all the a^is have to summed over the range 1 to N whenever they appear twice and the μ_is have to be summed over the three cartesian components whenever the indices are repeated.

To calculate the N point correlation function we take velocity moments of the N point distribution function

$$\int_0^{\infty} \rho_{\rm N}(x^a, u^a, t) d^{3N}u = \sum_{n=0}^{\infty} \frac{(-b)^n}{n!} \partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_n}_{\mu_n} < u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n} > \,. \tag{56}$$

All the terms where n is odd are zero and only the terms with even n contribute. We also have

$$< u^{a_1}_{\mu_1} u^{a_2}_{\mu_2} \ldots u^{a_n}_{\mu_n} u^{b_1}_{\nu_1} u^{b_2}_{\nu_2} \ldots u^{b_n}_{\nu_n} > = < u^{a_1}_{\mu_1} u^{b_1}_{\nu_1 >} \ldots < u^{a_n}_{\mu_n} u^{b_n}_{\nu_n} > + \text{permutations}\,. \tag{57}$$

Using the fact that $\partial^{a_1}_{\mu_1} \partial^{a_2}_{\mu_2} \ldots \partial^{a_{2n}}_{\mu_{2n}}$ is symmetric in all the indices, we can add up all the permutations to obtain for the terms with even n

$$\begin{aligned} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \quad & < u^{a_1}_{\mu_1} u^{b_1}_{\nu_1} \ldots u^{a_n}_{\mu_n} u^{b_n}_{\nu_n} > = \\ & \frac{(2n)!}{2^n n!} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \left[T^{a_1 b_1}_{\mu_1 \nu_1} \ldots T^{a_n b_n}_{\mu_n \nu_n} \right]\,. \end{aligned} \tag{58}$$

where $T^{ab}_{\mu\nu} = < u^a_\mu u^b_\nu >$ is the covariance matrix introduced in section 3 generalized for the N point distribution function.

Using this in equation (55), we have

$$\int_0^{\infty} \rho_{\rm N}(x^a, u^a, t) d^{3N}u = \sum_{n=0}^{\infty} \frac{(b^2)^n}{2^n n!} \partial^{a_1}_{\mu_1} \partial^{b_1}_{\nu_1} \ldots \partial^{a_n}_{\mu_n} \partial^{b_n}_{\nu_n} \left[T^{a_1 b_1}_{\mu_1 \nu_1} \ldots T^{a_n b_n}_{\mu_n \nu_n} \right] \tag{59}$$

In the above equation, for a fixed value of n, there will be a term with n pairs $(a_1b_1),(a_1b_2)...(a_nb_n)$ where each index is independently summed over values 1 to N. Thus, for a fixed value of n, the total contribution is a sum of N^{2n} terms each corresponding to a different set of values for the position indices. In any one of these N^{2n} terms there can be two kinds of pairs
A. if $a_i = b_i$, then $T^{a_ib_i}_{\mu_i\nu_i} = -\frac{1}{3}\delta_{\mu_i\nu_i}\nabla^2\phi(0)$ is a constant
B. if $a_i \neq b_i$ then $T^{a_ib_i}_{\mu_i\nu_i} = \partial^{a_i}_{\mu_i}\partial^{a_i}_{\nu_i}\phi(a_i,b_i)$ is a function of the separation between these two points.
Any of the terms can be representd by a directed graph with N vertices and n edges. The pairs of the kind A correspond to an edge connecting a vertex to itself and a pair of the kind B corresponds to an edge connecting two different vertices (figure(3)). The integral $\int_0^\infty \rho_{\mathrm{N}}(x^a,u^a,t)d^{3N}u$ then corresponds to a sum of graphs with N vertices and the number of edges going from 0 to infinity.

The quantity $\int_0^\infty \rho_{\mathrm{N}}(x^a,u^a,t)d^{3N}u\, d^3x^1 d^3x^2 .. d^3x^{\mathrm{N}}$ is the mean number of particles we expect to find in the volume d^3x^1 at x^1 and d^3x^2 at x^2 and ... d^3x^{N} around x^{N}. This has contribution from the lower (i.e N-1,...1 point) correlation functions also. The residue when the contributions from the lower correlation functions have been removed, is called the reduced N point correlation function. Henceforth we shall refer to the reduced N point correlation function as the N point correlation function. The graphs that do not connect all the N points correspond to functions that do not refer to all the N points and these are the contributions from the lower correlations. The reduced N point correlation can be calulated by considering only the connected graphs with N vertices. The lowest order contribution to the N point correlation corresponds to the connected graphs with the least number of edges. These graphs are the tree graphs and they have N-1 edges. The other terms that contribute to the N point correlation can be generated by adding more edges to the tree graphs.

We use equation (59) to calculate the three point correlation function. The lowest order at which the three point correlation develops is $n=2$ and this can be written as

$$\zeta^{(1)}(1,2,3,t) = \frac{b^4}{2}\partial^{a_1'}_{\mu_1}\partial^{a_2'}_{\mu_2}\partial^{a_2'}_{\mu_3}\partial^{a_3'}_{\mu_4}\left[T^{a_1'a_2'}_{\mu_1\mu_2}T^{a_2'a_3'}_{\mu_3\mu_4}\right] \tag{60}$$

where a_1', a_2' and a_3' are to be summed over all possible permutations of $1,2$ and 3. Equation (60) correponds to the only possible tree graph with three vertices a_1', a_2' and a_3', and two edges (a_1', a_2') and (a_2', a_3') (figure(4)).

Using

$$\partial_\mu\nabla^2\phi(x) = \frac{x_\mu}{x^3}\int_0^\infty \xi^{(1)}(y)y^2dy = \frac{1}{3}x_\mu\overline{\xi^{(1)}}(x) \tag{61}$$

we have

$$
\begin{aligned}
\zeta^{(1)}(1,2,3,t) &= \frac{b^4}{2}\Big[(1+\cos^2\theta_{xy})\xi^{(1)}(x,t)\xi^{(1)}(y,t) \\
+ & \cos\theta_{xy}\frac{2}{3}\frac{d}{dx}\xi^{(1)}(x,t)y\overline{\xi^{(1)}}(y,t) + \frac{2}{3}(1-3\cos^2\theta_{xy})\xi^{(1)}(x,t)\overline{\xi^{(1)}}(y,t) \\
- & \frac{1}{3}(1-3\cos^2\theta_{xy})\overline{\xi^{(1)}}(x,t)\overline{\xi^{(1)}}(y,t)\Big]
\end{aligned}
\tag{62}
$$

where

$$
x = x^{a_2^{'}} - x^{a_3^{'}} \quad , \; y = x^{a_2} - x^{a_3}
$$

and

$$
\theta_{xy} = \frac{x_\mu y_\mu}{xy} \tag{63}
$$

This explicitly exhibits the dependence of the lowest order induced three point correlation function on the initial two point correlation function. We see that the three point correlation depends on both $\xi^{(1)}(x,t)$ and $\overline{\xi^{(1)}}(x,t)$. Thus we see that the small scales can influence the three point correlation at large scales through the quantity $\overline{\xi^{(1)}}(x,t)$. The lowest order induced three point correlation function calculated using ZA is very similar to that calculated by studying gravitational dynamics perturbatively at the lowest order beyond the linear theory (paper I) and the difference is only in the numerical factors .

We next calculate the higher order terms that contribute to the three point correlation function. These are generated by adding more edges to the tree graphs. Figures 5 and 6 illustrates the simplest cases where we add only one edge to the tree graph. Next consider any of the graphs with $n > 2$ edges. In this graph the tree graph can be embedded in C_2^n ways. Using this in equation (59) we have

$$
\begin{aligned}
\zeta(1,2,3,t) = \sum_{n=0}^{\infty}\frac{b^{2(n+2)}}{2^{n+1}n!}\partial^{a_1}_{\mu_1}\partial^{b_1}_{\nu_1}\,.\quad.\quad.\,\partial^{a_n}_{\mu_n}\partial^{b_n}_{\nu_n}\partial^{a_1^{'}}_{\alpha_1}\partial^{a_2^{'}}_{\alpha_2}\partial^{a_2^{'}}_{\alpha_3}\partial^{a_4^{'}}_{\alpha_4}\Big[T^{a_1b_1}_{\mu_1\nu_1}\,.. \\
.\quad ....\,T^{a_nb_n}_{\mu_n\nu_n}T^{a_1^{'}a_2^{'}}_{\alpha_1\alpha_2}T^{a_2^{'}a_3^{'}}_{\alpha_3\alpha_4}\Big]
\end{aligned}
\tag{64}
$$

As discussed in the previous section, at large x the contributions from the terms with $a_i = b_i$ will dominate. i.e. at the lowest order, graphs of the kind shown in figure 5 . Thus, at large x the three point correlation function may be written as

$$
\zeta(1,2,3,t) = \sum_{n=0}^{\infty}\frac{b^{2n}}{2^n n!}(-\frac{1}{3}\nabla^2\phi(0))^n(\nabla^{a_1})^2(\nabla^{a_1})^2\,..\,(\nabla^{a_n})^2\xi_3^{(1)}(1,2,3,t) \tag{65}
$$

where the index a_i indicates at which point the Laplacian acts, and it is to be summed over the values $1, 2$ and 3. In Fourier space we have

$$F_3(k^1, k^2, k^3, t) = \sum_{n=0}^{\infty} \frac{b^{2n}}{2^n n!} (\frac{1}{3} \nabla^2 \phi(0))^n ((k^{a_1})^2 + (k^{a_2})^2 \ldots (k^{a_n})^2) F_3^{(1)}(k^1, k^2, k^3, t) \tag{66}$$

where F_3 is the Fourier transform of the three point correlation and $F_3^{(1)}$ is the Fourier transform of the lowest order three point correlation. The terms can be summed up to obtain

$$F_3(k^1, k^2, k^3, t) = \exp\left[-\frac{1}{2}\frac{b^2 < u^2 >}{3}((k^1)^2 + (k^2)^2 + (k^3)^2)\right] F_3^{(1)}(k^1, k^2, k^3, t) \tag{67}$$

which gives us in real space

$$\zeta(x^1, x^2, x^3, t) = \frac{1}{(\sqrt{2\pi} L(t))^9} \int \exp\left[-\frac{(x^a - y^a)^2}{2L^2(t)}\right] \zeta^{(l)}(y^1, y^2, y^3, t) d^9 y \,. \tag{68}$$

Thus, at large separation, the effect of including the higher order terms for the three point correlation function is to convolve the lowest order induced three point correlation with a Gaussian of width $L(t)$. As with the two point correlation function, this too can be interpreted in terms of a diffusion process.

7. Discussion and Conclusions.

We find that when we calculate the two point correlation function as a series in powers of the growing mode, we get the same answer if we do the calculation using distribution functions or if we do it in the single stream approximation. Since the first method is valid even after multi streaming occurs and the second method breaks down once multistreaming occurs, once multi streaming has occured we would expect to get different answers using the two different methods. But the two results match to all orders in the expansion parameter. We therefore conclude that even though these equations are valid in the multi streamed epoch, if we start from single streamed initial conditions we cannot perturbatively calculate any effect due to multistreaming e.g. vorticity, pressure. This limitation arises from the fact that the full two point correlation function for ZA, which includes the effects of multistreaming, is an exponential in $\frac{1}{b^2}$. All the derivatives of the function $\frac{1}{b}e^{-\frac{A}{b^2}}$ vanish at $b = 0$. As a result, if we try to expand this function in a series in powers of b around $b = 0$, we find that coefficients of all the powers of b are zero. If one considers the power spectrum instead, it is of the form $e^{-\alpha k^2 b^2}$. This function can be expressed as a power series in b^2 and one might that it is possible to perturbatively study the effects of multi-streaming by working in Fourier space instead of real space. Such a conclusion would be erronous as none of the terms in this expansion would have the effects of multi-streaming. It would be possible to study the effects of multi-streaming only if it were possible to sum the whole series. This point is further illustrated in an appendix where we consider a simpler example where a similar situation occurs.

Shandarin and Zel'dovich (1989) present a formula for N, the mean number of streams at any point, in a situation where the particles are moving in one dimension under ZA. At small b this formula is of the form $N = 1 + e^{\frac{-A}{b^2}}$ where A is a constant characterizing the initial conditions. If we expand this in powers of b, the coefficients for all the terms are zero and we find that the mean number of streams is one. This confirms that the effects of multi streaming cannot be studied perturbatively. Although in this analysis we used ZA, we expect this to hold for the full gravitational dynamics too, as derived at the lowest order of nonlinearity in paper II.

In our comparison of the two point correlation function at large separations we find that the results obtained using ZA are quite similar to the lowest order nonlinear results obtained using GD and both of them can be interpreted in terms of a diffusion process where the rearrangement of matter on small scales affects the two point correlation at large scales. In ZA, for an initial power spectrum with $n > -1$, the mean square displacement of the particles from their original positions is $L^2(t) = b^2(t) < u^2 >$ and this makes its appearance in the formula for the nonlinear corrections to the two point correlation function obtained using ZA. Interpreting the results from GD in a similar fashion, for an initial power spectrum with $n > 0$, we have $L^2(t) \sim .58 b^2(t) < u^2 >$. In paper II we also considered the case with $n = 0$ and for this case we found $L^2(t) \sim 1.49 b^2(t) < u^2 >$. The differences can be understood in terms of the fact that in ZA the particles move along trajectories calculated using linear GD, whereas when we take into account nonlinear corrections, the trajectories get modified by the tidal forces. In the equations for the evolution of the two point correlation function the tidal force acts through the three point correlation function. The tidal force of the third particle (in the three point correlation), will cause the other two particles to move towards or away from one another. This effect will be strongly dependent on the spatial behaviour of the three point correlation function. For the cases with $n > 0$ the induced three point correlation has the hierarchical form at large x whereas s for the case with $n = 0$ the induced three point correlation does not have this form. We propose that it is because of this that the effect of the tidal forces is different in these two cases and in the former case the effect of the tidal forces is to reduce the mean square diplacements relative to ZA whereas in the latter case it increases it. Thus indirectly, it is a diagnostic of the effect of the backreaction of the three point correlation function on the pair velocity which in turn effects the two point correlation.

We find that for ZA, at large x, we can sum up all terms in the perturbative series and the nonlinear two point correlation function is related to the linear two point correlation by a convolution with a Gaussian of width $\propto L(t)$. We also find that for special initial conditions where the power spectrum has a Gaussian cutoff at large k, the evolution at large x can be described by a simple scaling relation according to which the information propagates outward.

We also find that this picture based on diffusion gives a good description of the evolution under ZA until the onset of multistreaming. Based on this we suggest that the evolution of of the two point correlation function in GD can also be described by a diffusion process until the onset of multistreaming.

We have calculated the lowest order induced three point correlation function using ZA and we find that it is very similar to the result obtained using GD and the two differ only in the numerical factors. We also investigate the effect of the higher order nonlinear terms and we find that at large x we can sum the whole perturbation series. We find that the expression obtained after taking into account the nonlinear corrections is related to to the lowest order three point correlation function by a convolution with a Gaussian of width $\propto L(t)$. This is very similar to the evolution of the two point correlation function at large separations.

It can be shown that a similar relation holds for the higher correlation functions also but we do not pursue this matter in this paper.

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A. Appendix

Consider a Gaussian function of the variable x with standard deviation σ. We are interested in the power series expansion of this function in σ around $\sigma = 0$. We can do this expansion by taking the Fourier transform of the Gaussian i.e.

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{2\pi} \int e^{ikx} e^{-\frac{1}{2}k^2\sigma^2} dk \qquad \text{(A1)}$$

and then doing a Taylor expansion (convergent) of $e^{-\frac{1}{2}k^2\sigma^2}$. We then get

$$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{2\pi}\int e^{ikx}\sum_{n=0}^{\infty}\left(-\frac{1}{2}k^2\sigma^2\right)^n\frac{1}{n!}dk \tag{A2}$$

which gives us

$$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}} = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{2}\sigma^2\frac{d^2}{dx^2}\right)^n\delta(x)\,. \tag{A3}$$

Equation (A3) can also be derived if we take the Gaussian function and directly do a Taylor expansion in σ i.e. without going to Fourier space.

We see that the series expansion is entirely made up of Dirac delta functions and its derivatives and hence it has nonzero value only when $x = 0$. This should be compared with the original Gaussian function which has nonzero value even if $x \neq 0$. We see that in this case the Taylor expansion fails to capture an important aspect of the original function and we can attribute this to the fact that we are doing the Taylor expansion of a function which is an exponential in $\frac{1}{\sigma^2}$.

If instead of working in real space we work in Fourier space, we find that we have to deal with the Taylor expansion of a function which is an exponential in σ^2 instead of $\frac{1}{\sigma^2}$. There is no problem in expanding this function in a Taylor series and on might be led to think that the limitation of the Taylor expansion in real space can be overcome by going to Fourier space. But this turns out to be wrong. On comparing equations (A2) and (A3) we see that each term in the expansion in Fourier space corresponds to some derivatives of a Dirac delta function and hence it cannot capture any of the effects missed out if the analysis is done in real space. These effects can be included only if one is able to sum the series in Fourier space and then do the Fourier transform.

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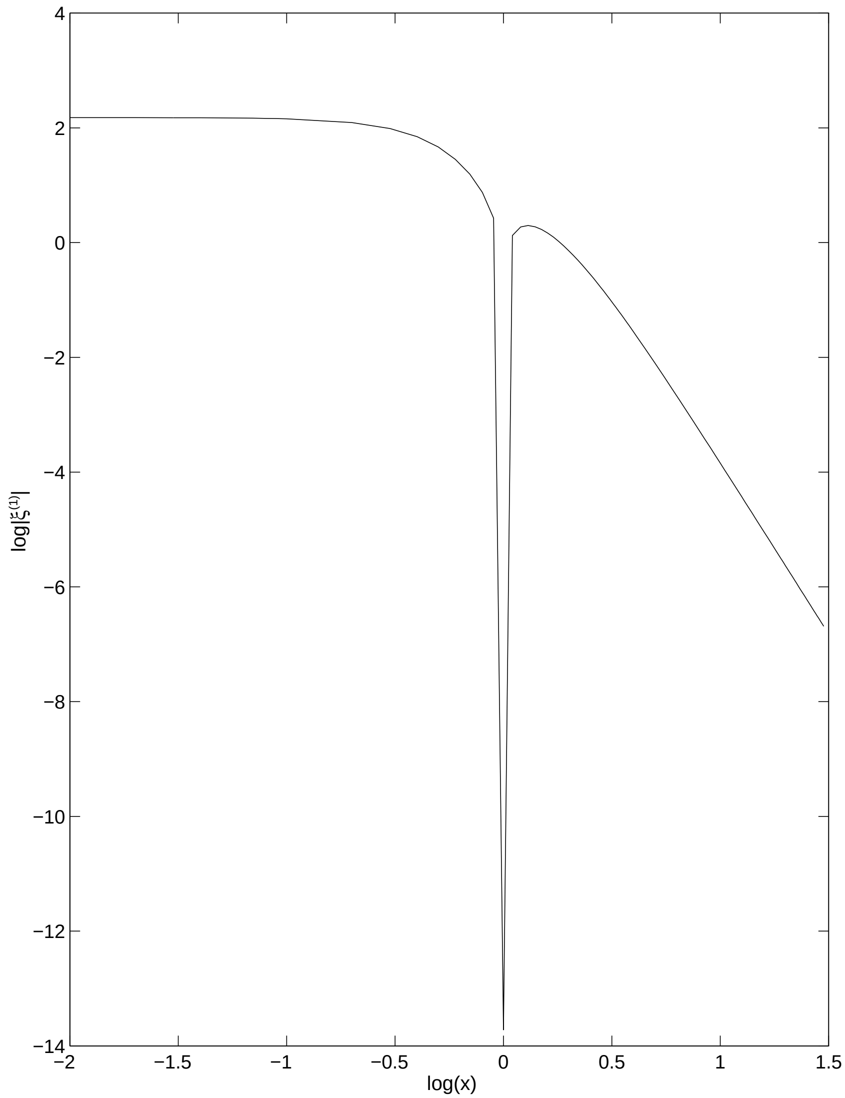

Fig. 1.— The initial two point correlation as a function of the separation for the power spectrum $P(k) = .5e^{-k^2}k$.

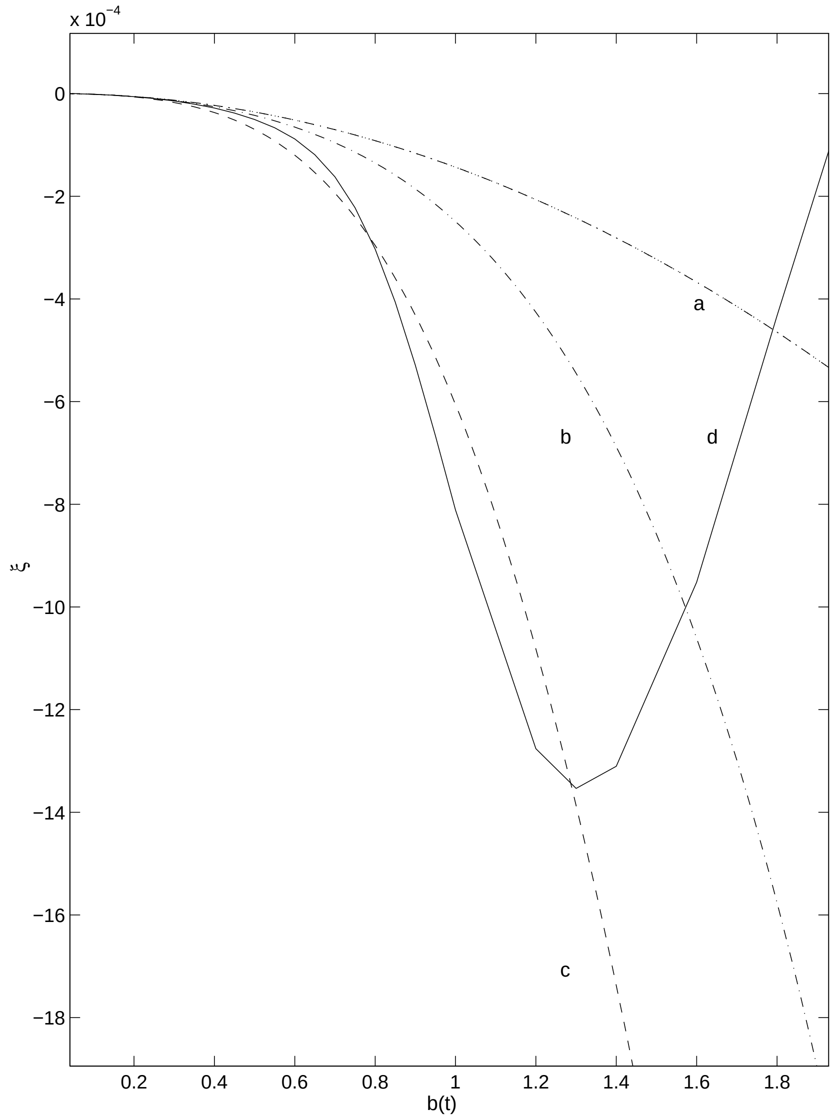

Fig. 2.— The two point correlation at a fixed separation $x = 10$ as a function of the growing mode $b(t)$ for (a) linear theory, (b) linear theory + lowest order nonlinear correction using GD (c) nonlinear evolution using ZA and the assumptions made in section 5 about the large x behaviour, and (d) nonperturbative ZA

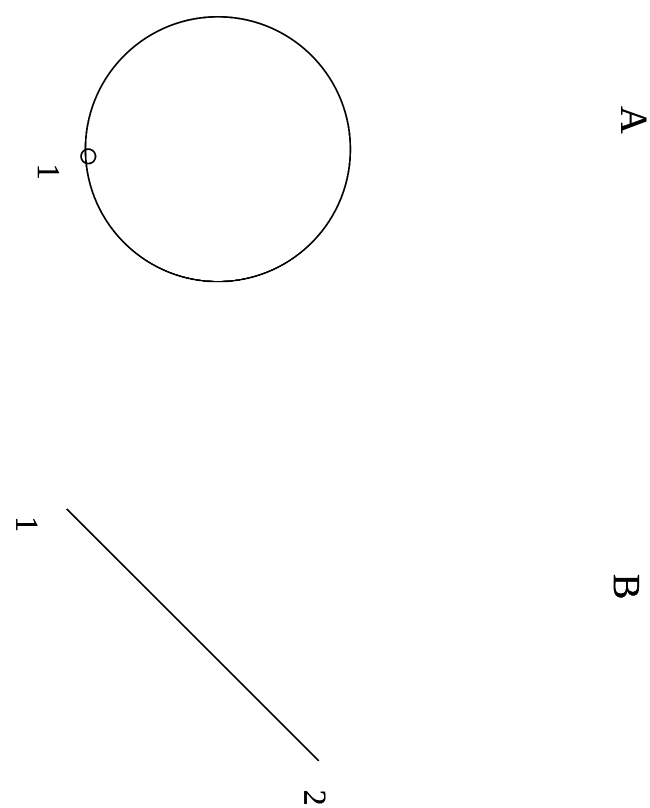

Fig. 3.— This shows the two possible kinds of edges A. connects a vertex to itself B. connects two different vertices.

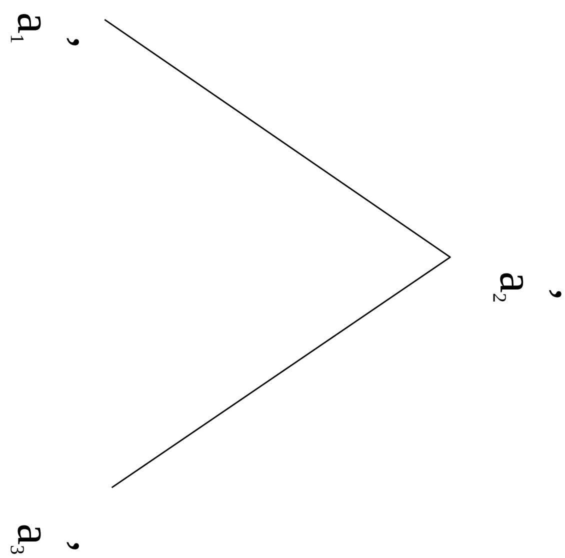

Fig. 4.— This shows the tree graph corresponding to the lowest order induced three point correlation function.

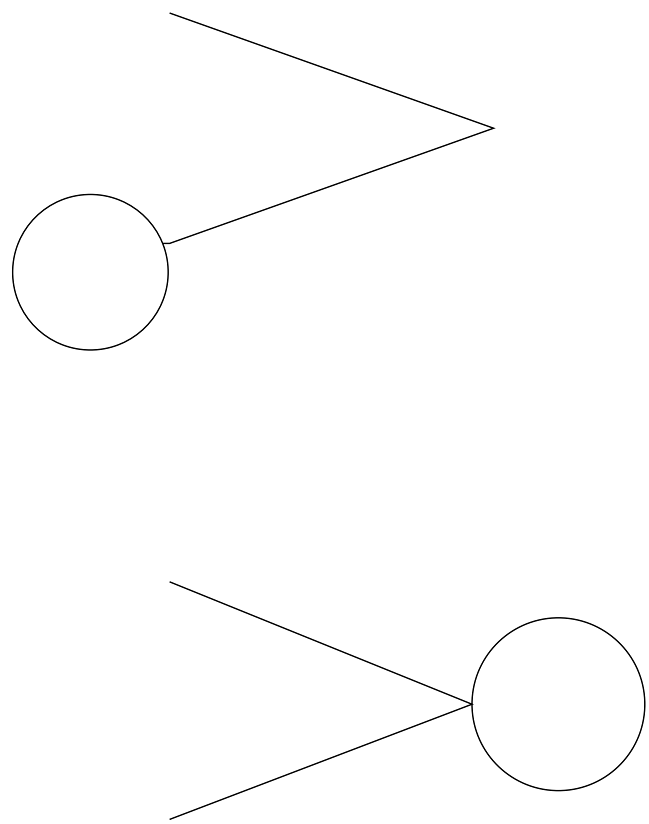

Fig. 5.— This shows some of the graphs corresponding to the contribution to the three point correlation function at one order beyond the lowest. These graphs are all obtained by adding edges to the tree graph. These graphs show those cases where the extra edge connects a vertex to itself.

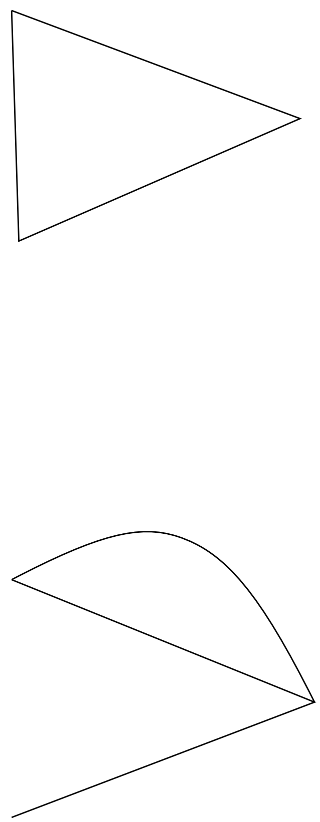

Fig. 6.— This shows some of the graphs corresponding to the contribution to the three point correlation function at one order beyond the lowest. These graphs are all obtained by adding edges to the tree graph. Thee graphs show those cases where the extra edge connects two different vertices.