Gravitational Phase Transition in Neutron Stars

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Abstract

The possibility of a gravitational phase transition, especially with respect to neutron stars is investigated. First, a semiclassical treatment is given, predicting a gravitational London penetration depth of 12km for neutron stars. Second, the problem is considered from a Ginzburg-Landau point of view. A gravitational Meissner effect, a gravitational Aharanov-Bohm type effect and a gravitational ferromagnetic type phase are predicted. Finally, a field theoretic consideration predicts a mass term for the graviton below a certain critical temperature.

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1 Introduction

In the post-Newtonian approximation, gravity looks like an effective U(1) theory, closely resembling the theory of electrodynamics. In this work we consider effects arising from the breaking of this U(1) symmetry. Basically, we consider the possibility of a macroscopic quantum gravity effect in analogy to the effect of superconductivity in the electromagnetic theory.

Using a semiclassical approach in close analogy to London's original approach, we find that for neutron stars, macroscopic quantum gravity effects seem to become significant. We predict a gravitational Meissner effect with a gravitational London penetration depth for neutron stars of about 12km, which is somewhat smaller than the neutron star's radius.

A Ginzburg-Landau treatment leads to the same result. We use the close analogy between electromagnetism and the post-Newtonian approximation and predict again the Meissner effect, a gravitational Aharanov-Bohm effect and a gravitational ferromagnetic type phase, where all the spin angular momenta of the neutron star become aligned.

Finally we look at the graviton. We use the gravitational wave expansion from general relativity, treating the graviton as a spin two field. Adding the Ginzburg-Landau terms through the energy momentum tensor, we observe that the theory undergoes a phase transition, with the graviton becoming massive below a certain critical temperature. Again, this predicts a Meissner effect, justifying the semi-classical treatment.

2 Motivation

Neutron stars are extraordinarily fascinating objects: Not only are they our preferred laboratory to test general relativity, but they also seem to exhibit macroscopic quantum effects, i.e., it is believed that they are superfluid and maybe superconducting [24]. Although these two issues seem to be separate, we will consider here the possibility of a macroscopic gravitational quantum effect, in close analogy to the case of superconductivity.

Neutron stars are made mostly out of neutrons. They have usual radii of about 15 km and masses of about 1.4 times the mass of the sun. Theoretical calculations indicate that for low enough temperatures ($< 10^9$K) the neutrons are expected to pair up [30], similar to nucleons forming pairs inside nuclei in the Interacting Boson Model (IBM) [12, 21]. The calculations indicate that in the crust region the singlet ${}^1S_0$ state should be the preferred one, however, for the majority the triplet ${}^3P_2$ state should be the preferred state [1, 3, 32]. These neutron pairs are bosons by statistics and bose condensation is expected.

Interesting enough, there is experimental evidence for the pairing: one, it comes from the theory of cooling of neutron stars, where the reduction of the specific heat due to extensive pairing is essential [23], and two, evidence for superfluidity can be found in the glitches in the timing history of pulsars [24, 30].

Therefore, we may assume that there exists a bose condensate inside a neutron star. This bose condensation then is responsible for the breaking of the translational symmetry leading to superfluidity [18] and possibly the electromagnetic U(1) symmetry leading to superconductivity [19, 30]. Since gravity in the post-Newtonian approximation is an effective U(1) theory, we may wonder what would happen if in this approximation, this gravitational U(1) symmetry were to be broken.

3 Semiclassical Treatment

For weak gravitational fields and low velocities, Einstein's field equations can be written in the post-Newtonian approximation [20, 37]. We are mostly interested in magnetic-type gravity, and therefore we shall use the truncated and rewritten version of the parametrized-post-Newtonian (PPN) formalism given by Braginsky et. al. [2]. In this formalism, Einstein's field equations can be rewritten in a form very much resembling that of Maxwell's equations

$$\nabla \cdot \mathbf{g} = -4\pi\rho_0 + \mathcal{O}\left(c^{-2}\right), \qquad \nabla \times \mathbf{g} = -\frac{1}{\mathrm{c}}\frac{\partial \mathbf{H}}{\partial \mathrm{t}} \tag{1}$$

$$\nabla \cdot \mathbf{H} = 0, \qquad \nabla \times \mathbf{H} = 4\left(-4\pi\rho_0\frac{\mathbf{v}}{\mathrm{c}} + \frac{1}{\mathrm{c}}\frac{\partial \mathbf{g}}{\partial \mathrm{t}}\right), \tag{2}$$

where ρ_0 is the density of rest mass in the local rest frame of the matter and $\mathbf{v}$ is the ordinary velocity of the rest mass relative to the PPN coordinate frame. The equations of motion of an uncharged particle are identical to the electromagnetic Lorentz force law [2, 33],

$$\frac{d\mathbf{v}}{dt} = \mathbf{g} + \frac{1}{c}\mathbf{v} \times \mathbf{H} + \mathcal{O}\left(c^{-2}\right). \tag{3}$$

In this weak field, slow motion expansion of general relativity, $\mathbf{g}$ contains mostly first order corrections to flat space-time, and $\mathbf{H}$ contains second order corrections. For the solar system, $\mathbf{g}$ is just the normal Newtonian gravitational acceleration, whereas $\mathbf{H}$ is related to angular momentum interactions and effects due to $\mathbf{H}$ are about 10^{12} times smaller than those due to $\mathbf{g}$ [33].

With these gravitational Maxwell equations we can derive the equivalent of London's equations [14, 15, 31] for gravity. The derivation is completely analogous to the electromagnetic case [13]. This leads to the second London equation for gravity

$$\nabla \times \mathbf{j} = -\sqrt{G}\frac{\rho_0}{c}\mathbf{H}, \tag{4}$$

which in turn predicts the Meissner-Ochsenfeld effect [16] with a gravitational London penetration depth of

$$\Lambda_L = \left(\frac{c^2}{16\pi G\rho_0}\right)^{1/2} = 5.18 \times 10^{12}\ \rho_0^{-1/2}, \tag{5}$$

where the density ρ_0 is measured in $\mathrm{kg/m^3}$ and Λ_L is given in meters.

Calculating the London penetration depth for neutron stars with approximate densities of about $\rho_{NS} \cong 2 \times 10^{17}\mathrm{kg/m^3}$, we obtain a London penetration depth of 12km, which is slightly smaller than the radius of a neutron star with that density. The London length is inversely proportional to the density, which means increasing density leads to decreasing London penetration depth.

As a result, the onset of a gravitational Meissner effect is predicted. In essence it implies that the gravitational magnetic field caused by the huge angular momentum of the star, will be expelled from the center of the neutron star. This could be accomplished through induced matter-supercurrents in the outer layers of the neutron stars, creating counter-magnetic fields to expel the gravitational magnetic field from its interior, in analogy to the electromagnetic case. The gravitational Meissner effect may lead to significant modifications with respect to the collapse of a neutron star.

For our derivation of the London equations to be valid we must require that the London penetration depth Λ_L is much larger than the coherence length ξ_0 [9]. A simple upper estimate for the coherence length yields $\xi_0 \leq 10^{-13}$m [13], which is larger than the separation between two nucleons, but much smaller than the London penetration depth.

4 Gravitational Ginzburg-Landau Theory

The previous approach was basically classical in nature. For a Ginzburg-Landau treatment [11] of the problem we need a quantum theory. We first show how to modify Schrödinger's equation for the post-Newtonian approximation. Then we describe the neutron pair bose condensate. With this in place a standard Ginzburg-Landau treatment leads to the prediction of Meissner effect and a Aharanov-Bohm type effect. In addition for the triplet state we predict a macroscopic ordering of the spins similar to ferromagnetism.

4.1 Gravitational Schrödinger equation

The first experiment showing the direct influence of gravity on the quantum mechanical phase of a wave function was done a little more than twenty years ago by Colella, Overhauser and Werner, now known as the COW-experiment

[8, 25]. They used thermal neutrons in an interferometer type experiment and showed that rotating the experimental apparatus in the earth's gravitational field gives an interference pattern exactly as predicted by the 'gravitational' Schrödinger equation (see Sakurai [28]),

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + m\phi\right)\psi = i\hbar\frac{\partial\psi}{\partial t}. \tag{6}$$

Since then additional experiments have been done, as for example the influence of the earth's rotation on the phase (Sagnac effect) has been verified experimentally [26]. Recently atomic interferometry has become available, which allows for much higher precision [5]. In the near future this should allow for the measurement of such effects as coupling of a particle's spin to the earth's rotation, as well as second order correction to the Newtonian potential caused by curvature, and the somewhat more distant future, Lense-Thirring effect and gravitational waves.

Although these experiments have not been done yet, we are encouraged to follow the analogy of the electromagnetic treatment and propose the Schrödinger equation for post-Newtonian gravity,

$$\left(-\frac{\hbar^2}{2m}\left(\nabla - \frac{iq}{\hbar c}\mathbf{A}\right)^2 + q\phi\right)\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{7}$$

where we identify the minimal coupling

$$\nabla \to \nabla - \frac{iq}{\hbar c}\mathbf{A}. \tag{8}$$

Here, $q = \sqrt{G}m$, and details for the derivation and units can be found in the appendix. Please note, that equation (7) is invariant under the following gauge transformation:

$$\mathbf{A} \to \mathbf{A} + \nabla\Lambda(x) \qquad \text{and} \qquad \psi \to \mathrm{e}^{\mathrm{iq}\Lambda(\mathrm{x})/\hbar\mathrm{c}}\psi. \tag{9}$$

One major difference to electrodynamics is that $(\phi, \mathbf{A})$ is not a four-vector, but rather transforms as a particular 0 component of a rank two tensor, which becomes clear considering its definition in terms of the metric tensor,

$$g_{00} = 1 - 2\phi, \qquad g_{ij} = (1 - 2\phi)\delta_{ij}, \qquad \text{and} \qquad \mathrm{g_{0i}} = -\mathrm{A_i}. \tag{10}$$

However, in the following construction we never require it to be a four vector.

4.2 Long Range Order

As indicated in the introduction, there is experimental evidence that the neutrons inside a neutron star are superfluid. This means that there exists a bose-condensate already, which can be described by a Ginzburg-Landau type theory. Let us review the basic ideas behind this long-range ordering.

Neutrons are fermions with spin of 1/2. Due to the Pauli exclusion principle, no one-particle level can have a macroscopic occupation number in a fermion system, so long range order in the one-particle density matrix is forbidden. But there can be long range order in the two-particle density matrix [18]:

$$\left\langle r_1 s_1, r_2 s_2 \left| \rho^{(2)} \right| r_1' s_1', r_2' s_2' \right\rangle = \left\langle \Psi^\dagger_{s_1}(r_1) \Psi^\dagger_{s_2}(r_2)\, \Psi_{s_2'}(r_2') \Psi_{s_1'}(r_1') \right\rangle . \tag{11}$$

In the ordered phase, when the pair of points r_1, r_2 is far removed from the pair r_1', r_2', this becomes

$$\left\langle r_1 s_1, r_2 s_2 \left| \rho^{(2)} \right| r_1' s_1', r_2' s_2' \right\rangle \sim \Phi^*_{s_1 s_2}(r_1, r_2)\, \Phi_{s_1' s_2'}(r_1', r_2'), \tag{12}$$

and such ordering is called pairing.

In general, in a paired system the order parameter $\Phi_{s_1 s_2}(r_1, r_2)$ has a richer structure: it is a 2×2 matrix in spin space, each element of which is a complex function of the difference variable $r = r_1 - r_2$. Because the fermion field operators anti-commute it follows that the order parameter behaves like a two-particle wave function under interchange of particles $\Phi_{s_1 s_2}(r_1, r_2) = -\Phi_{s_2 s_1}(r_2, r_1)$. It also follows that the order parameter transforms like a two-particle wave function under rotations in spin space, rotations in position space, or Galilean transformations [18].

The order parameter can be expanded in a basis of four spin states consisting of the usual singlet state $|0\rangle$, where the spins of the two neutrons effectively cancel, and three triplet states $|x_\mu\rangle$, where the spins add up to one. The singlet state is invariant under spin rotations and the triplet states transform like the x-, y-, and z-components of a vector. We now can write the order parameter or two-particle wave function as

$$\Phi_{s_1 s_2}(r_1, r_2) = \langle s_1, s_2 \,|\, \Phi(r_1, r_2) \rangle \,, \tag{13}$$

where

$$\Phi(r_1, r_2) = \phi(r_1, r_2)\, |0\rangle + \psi_\mu(r_1, r_2)\, |x_\mu\rangle \tag{14}$$

is a combination of singlet $\phi(r_1, r_2)$ and triplet $\psi_\mu(r_1, r_2)$ order parameters.

4.3 Singlet $^1S_0$ State

In the singlet state, the quantum numbers S and L are zero, meaning that the order parameter $\Phi_{s_1 s_2}(r_1, r_2) = \phi(r_1, r_2)$ is a spin scalar and depends only on the magnitude of r. This is equivalent to the type of ordering in helium-4, and the only degree of freedom of the order parameter is associated with the phase degeneracy.

Since only the two-particle wave function $\phi\phi^*$ has a non-vanishing expectation value, physically measurable properties of a spatially uniform system can depend only on the product $\phi\phi^*$. If, in the spirit of Landau, we expand the free energy of ordering in powers of the order parameter, the first two non-vanishing terms will give a free energy density of the form

$$\mathcal{F}_0 = \mu^2 \left|\phi\right|^2 + \lambda \left|\phi\right|^4 . \tag{15}$$

Slow spatial variations in the order parameter can be treated in the usual gradient expansion. To leading order there is only one term invariant under rotations and gauge transformations and $\mathcal{F}_0$ becomes

$$\mathcal{F} = \frac{1}{2} K \left|\nabla\phi\right|^2 + \mu^2 \left|\phi\right|^2 + \lambda \left|\phi\right|^4 . \tag{16}$$

The coupling to gravity can be accomplished through minimal coupling. Recall equation (8) and the fact that the mass of the neutron pair is $2q$, we find for the free energy

$$\mathcal{F} = \frac{K}{2} \left(\nabla - i\frac{2q}{\hbar c}\mathbf{A}\right) \phi \left(\nabla + i\frac{2q}{\hbar c}\mathbf{A}\right) \phi^* + \mu^2 \phi\phi^* + \lambda \left(\phi\phi^*\right)^2 . \tag{17}$$

From here we can obtain the currents associated with the gauge symmetry (9) through variation, remembering that the mass of the neutron pair is $2q$. Thus we find for the conserved current density

$$\mathbf{j} = \frac{qK}{\hbar} \left(i\phi^*\nabla\phi - i\phi\nabla\phi^* - \frac{4q}{\hbar c}\mathbf{A} \left|\phi\right|^2\right) . \tag{18}$$

Now assume that $\phi = \left|\phi\right| e^{i\varphi}$ has its spatial variation in the phase φ and not in the magnitude $\left|\phi\right|$, then the current simplifies to

$$\mathbf{j} = -\frac{2qK}{\hbar} \left(\nabla\varphi + \frac{2q}{\hbar c}\mathbf{A}\right) \left|\phi\right|^2 . \tag{19}$$

Since the curl of any gradient vanishes, and since $\left|\phi\right|^2$ is essentially constant, we immediately deduce the second London equation

$$\nabla \times \mathbf{j} = -\frac{4q^2 K}{\hbar^2 c} \left|\phi\right|^2 \, \nabla \times \mathbf{A}. \tag{20}$$

Comparing this with our result from the previous chapter, equation (4), we can use this equation to relate macroscopic to microscopic quantities, that is, we can identify the density

$$\rho_0 = \frac{4q^2 K}{\sqrt{G}\hbar^2} \left|\phi\right|^2 . \tag{21}$$

Notice that by redefining $\phi \to \sqrt{K}\phi$ in equation (16), we can absorb the K into the $\left|\phi\right|^2$. Simultaneously we will also redefine $\mu \to \sqrt{K}\mu$ and $\lambda \to K^2\lambda$. This then allows us to express $\left|\phi\right|^2$ in terms of known quantities, i.e.,

$$\left|\phi\right|^2 = \frac{\sqrt{G}\hbar^2}{4q^2} \rho_0. \tag{22}$$

4.3.1 Meissner Effect

One peculiar feature of the Ginzburg-Landau free energy is that its minimum depends on temperature. For temperatures T above a certain critical temperature T_c, the minimum of the potential will simply be at $|\phi|^2 = 0$. When the temperature T is below the critical temperature T_c, i.e., $T < T_c$, then the minimum is at

$$|\phi|^2 = -\frac{\mu^2}{2\lambda} > 0. \tag{23}$$

Furthermore, we assume that ϕ varies only very slightly over the sample, i.e., $|\phi|^2$ is essentially constant,

$$\nabla \times \mathbf{j} = -\frac{4q^2}{\hbar^2 c} |\phi|^2 \, \nabla \times \mathbf{A}, \tag{24}$$

and taking the curl of Ampere's equation (2)

$$\nabla \times \mathbf{H} = -\frac{16\pi}{c}\mathbf{j}$$

we find

$$\nabla^2 \mathbf{H} = \pi \left(\frac{8q}{\hbar c}\right)^2 |\phi|^2 \, \mathbf{H} = k^2 \mathbf{H}, \tag{25}$$

where we defined

$$k^2 = \pi \left(\frac{8q}{\hbar c}\right)^2 |\phi|^2 \tag{26}$$

For a neutron star of density $2 \times 10^{17}\mathrm{kg/m^3}$ we can calculate k using the fact that $k = 1/\Lambda_L = 7.4 \times 10^{-9} 1/m^2$.

4.3.2 Flux Quantization: Gravitational Aharanov-Bohm Effect

As a result of the Meissner effect, appreciable currents can only flow near the surface of a gravitational superconductor. As a consequence, if we integrate the current over a closed ring inside the superconductor, we find that

$$0 = \oint \mathbf{j} \cdot \mathbf{dl} = -\frac{2q}{\hbar} |\phi|^2 \oint \left(\nabla\varphi + \frac{2q}{\hbar c}\mathbf{A}\right) \cdot \mathbf{dl}. \tag{27}$$

Then Stokes theorem gives us

$$\oint \mathbf{A} \cdot \mathbf{dl} = \int \nabla \times \mathbf{A} \cdot \mathbf{dS} = \int \mathbf{H} \cdot \mathbf{dS} = \Phi_H, \tag{28}$$

where Φ_H is the flux enclosed in the ring. Since the order parameter Φ is single-valued, its phase must be an integer multiple of 2π, and therefore

$$\oint \nabla\varphi \cdot \mathbf{dl} = 2\pi n = \frac{2q}{\hbar c}\Phi_H = \frac{2q}{\hbar c} \oint \mathbf{A} \cdot \mathbf{dl} \tag{29}$$

or

$$\Phi_H = \frac{hc}{2q} n = \Phi_0 n \tag{30}$$

with the quantity $\Phi_0 = hc/2q = 59.8[m^3/s^2\sqrt{G}]$ being the fluxoid or flux quantum.

4.4 Triplet $^3P_2$ State

Only neutrons in the outer layers of a neutron star will be in the singlet state. Numerical simulations suggest that most of the neutrons will be found in the triplet, the $^3P_2$ state [1, 3, 32]. Therefore, we will briefly consider the $^3P_2$ state with quantum numbers $S = 1$, $L = 1$ and $J = 2$. In the triplet state, ϕ vanishes in equation (14) and the vector ψ_μ is of the form

$$\psi_\mu(r) = A_{\mu\nu}\, \widehat{r}_\nu\, \chi\left(|r|\right), \tag{31}$$

where $A_{\mu\nu}$ is a complex 3×3 matrix. Under rotations of the orbital coordinates $A_{\mu\nu}$ transforms as a vector with respect to index ν, under rotations of the spin coordinates, $A_{\mu\nu}$ transforms as a vector with respect to the index μ. In general, $A_{\mu\nu}$ represents all $l = 1$ order parameters. To stay in the $^3P_2$ subspace we must restrict the matrix A to be traceless and symmetric. Then the most general $^3P_2$ Ginzburg-Landau functional to order four, is [22, 29, 30, 36]

$$\mathcal{F}_0 = \frac{1}{3}\alpha\ \mathrm{TrAA}^\dagger + \beta_1 \left|\mathrm{TrA}^2\right|^2 + \beta_2 \left(\mathrm{TrAA}^*\right)^2 + \beta_3 \left(\mathrm{TrA}^2\mathrm{A}^{*2}\right). \tag{32}$$

This equation represents the most general mean field theory of $^3P_2$ pairing near T_c consistent with the overall rotational and gauge invariance.

The gradient term for the triplet state looks somewhat more complicated,

$$\mathcal{F}_{grad} = \frac{K_1}{2}\left(\nabla_i A^*_{\mu\, i}\right)\left(\nabla_i A_{\mu\, i}\right) + \frac{K_2}{2}\left(\nabla_i A^*_{\mu\, j}\right)\left(\nabla_j A_{\mu\, i}\right) + \frac{K_3}{2}\left(\nabla_i A^*_{\mu\, i}\right)\left(\nabla_j A_{\mu\, j}\right), \tag{33}$$

but it gives a current similar to (18), predicting a Meissner effect. Notice, that for example for helium-3 the three coefficients are nearly the same $K_1 = K_2 = K_3$ [18].

What changes significantly in the triplet state are the minima of the free energy. Again, above a certain critical temperature, the minimum will be for $\mathcal{F}_0 = 0$. However, below this temperature there are several different configurations.

Stability requires that the fourth-order term in the Ginzburg-Landau functional be positive, which implies that $\beta_2 > 0$, and restricts the $\beta_1 - \beta_3$ plane to the region $\beta_3 > -3\beta_2$ and $\beta_3 + 2\beta_1 > -2\beta_2$. Minimizing the free energy leads to the following three subregions [17, 29]:

Region I: $\beta_3 > |\beta_1| - \beta_1$. In this region the order parameter is unique, except for a constant phase factor and a rotation. At the minimum, the free energy is given through

$$\mathcal{F}_{\min} = -\frac{\alpha^2}{4\beta_2}.$$

Region II: $0 > \beta_3 > -6\beta_1$. Except for a constant phase factor and a rotation the solution is again unique and at the minimum

$$\mathcal{F}_{\min} = -\frac{\alpha^2}{4\left(\beta_2 + \frac{1}{3}\beta_3\right)}.$$

Region III: $\beta_3 < -4\beta_1 - 2\,|\beta_1|$. Here the minimum is given by any A that is real, traceless and symmetric, and at the minimum

$$\mathcal{F}_{\min} = -\frac{\alpha^2}{4\left(\beta_1 + \beta_2 + \frac{1}{2}\beta_3\right)}.$$

4.4.1 Macroscopic Quantum Coherence

While the expectation value of the orbital angular momentum along any axis in region II and III vanishes, for the solution in region I it will be maximally aligned [17]. This will lead to an effect analogous to ferromagnetism or the alignment of spins in helium-3 [35]. Assuming all the orbital angular momenta of the ${}^3P_2$ state are aligned, the neutron star will have a total, macroscopic spin angular momentum of

$$S = N \cdot \hbar \sim \frac{M_{NS}}{m_N} \cdot \hbar = 1.8 \cdot 10^{23}\,\text{Js}. \tag{34}$$

Comparing this to the approximate $S = 2 \cdot 10^{40}$ Js of the Crab pulsar due to rotation, we find that the pulsar's period would have to slow to a period of a few hundred million years, for this contribution to become significant.

Furthermore, one would expect gravitational infinite conductivity to occur. For example Weinberg shows that no energy gap is necessary, since infinite conductivity depends only on the spontaneous breakdown of gauge invariance [38].

5 The Graviton becomes Massive

The graviton is a spin two field. This may be the main objection to the previous section. Here we derive the Meissner effect for a spin two graviton. We start with the Lagrangian for a scalar field and add a Ginzburg-Landau potential. After we have the Lagrangian identified, we proceed to calculate the energy-momentum tensor. Expanding the gravitational field in the gravitational wave expansion, we find that the graviton becomes massive below a certain critical temperature, i.e., a phase transition has occurred and we expect the Meissner effect as a consequence. We then repeat the calculations for a vector field instead of a scalar, and arrive at very similar results. We end with a prediction of the graviton's mass.

5.1 Scalar Field

Let us consider the Lagrangian of a complex scalar field ϕ, with terms at most quadratic in the fields, (and we will leave tadpole like terms out of consideration). A good choice is found in Veltman [34] or Feynman [10], and also Callan et. al. [6]:

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \frac{1}{2}\left(\mu^2 - 2aR\right)\phi\phi^* - \lambda\left(\phi\phi^*\right)^2 + bR^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^*\right) \tag{35}$$

We will not consider the b term and also higher terms, since it will imply more than two derivatives. In addition, we can redefine $\mu^2 \to \mu^2 - 2aR$ to simplify the above Lagrangian. We find,

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi^* - \frac{1}{2}\mu^2\phi\phi^* - \lambda\left(\phi\phi^*\right)^2\right). \tag{36}$$

Through variation with respect to the metric we will obtain the energy-momentum tensor for the above Lagrangian, i.e.,

$$T_{\kappa\lambda} = \frac{1}{\sqrt{g}}\frac{\delta\mathcal{L}}{\delta g^{\kappa\lambda}}, \tag{37}$$

and we find that the energy momentum tensor is given through

$$T_{\kappa\lambda} = \frac{1}{2}\left[\partial_\kappa\phi\partial_\lambda\phi^* - \, g_{\kappa\lambda}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi^* - \frac{1}{2}\mu^2\phi\phi^* - \lambda\left(\phi\phi^*\right)^2\right)\right]. \tag{38}$$

We are especially interested in the weak field approximation used for gravitational waves, i.e., $g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu}$. Using this expansion, we find for the energy momentum tensor

$$\begin{aligned} T_{\kappa\lambda} \;=\; & \frac{1}{2}\partial_\kappa\phi\partial_\lambda\phi^* \\ & +\frac{1}{4}\left(-\eta_{\kappa\lambda}\,\eta^{\alpha\beta} + \kappa\left(\eta_{\kappa\lambda}\,h^{\alpha\beta} - h_{\kappa\lambda}\,\eta^{\alpha\beta}\right) + \,\kappa^2 h_{\kappa\lambda}\,h^{\alpha\beta}\right)\partial_\alpha\phi\partial_\beta\phi^* \\ & +\frac{1}{2}\,\eta_{\kappa\lambda}\left(\frac{1}{2}\mu^2\phi\phi^* + \lambda\left(\phi\phi^*\right)^2\right) + \frac{\kappa}{2}h_{\kappa\lambda}\left(\frac{1}{2}\mu^2\phi\phi^* + \lambda\left(\phi\phi^*\right)^2\right). \end{aligned} \tag{39}$$

Notice that for both possible minimal values of $|\phi|^2$, that is, $|\phi|^2 = 0$ above, and $|\phi|^2 = -\mu^2/2\lambda$ below the transition temperature, the vacuum expectation value of the last two terms vanishes. Next for simplicity neglect terms in κ^2 and furthermore assume that all the space-time dependance is in the phase, i.e.,

$$\partial_\kappa \phi = \partial_\kappa \left(|\phi| \, e^{i\varphi} \right) = i\phi \partial_\kappa \varphi. \tag{40}$$

We then obtain the following form for the energy momentum tensor

$$T_{\kappa\lambda} = \frac{1}{2} \left[-\partial_\kappa \varphi \partial_\lambda \varphi + \frac{1}{2} \eta_{\kappa\lambda} \, \partial_\alpha \varphi \partial^\alpha \varphi - \frac{\kappa}{2} \eta_{\kappa\lambda} \, h^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - \frac{\kappa}{2} h_{\kappa\lambda} \partial_\alpha \varphi \partial^\alpha \varphi \right] |\phi|^2 . \tag{41}$$

This gives the left hand side of Einstein's equations,

$$R_{\alpha\beta} + \frac{1}{2} \, \eta_{\alpha\beta} R = -8\pi G \, T_{\alpha\beta}. \tag{42}$$

For the right hand side we use the Ricci tensor in the gravitational wave expansion [37]

$$R_{\alpha\beta} = \kappa \frac{1}{2} \left(\partial_\lambda \partial^\lambda h_{\alpha\beta} + \partial_\alpha \partial_\beta h^\lambda_{\ \lambda} \right) , \tag{43}$$

where we made use of the gauge condition $\partial_\lambda h^\lambda_{\ \alpha} = 0$. With this we find the following wave equation for the graviton

$$\begin{aligned} &\partial_\alpha \partial^\alpha h_{\kappa\lambda} + \left(\partial_\kappa \partial_\lambda + \, \eta_{\kappa\lambda} \partial_\mu \partial^\mu \right) h^\alpha_{\ \alpha} = \\ &-8\pi G \kappa \left[-\partial_\kappa \varphi \partial_\lambda \varphi - \frac{\kappa}{2} \eta_{\kappa\lambda} \, h^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + \frac{1}{2} \left(\eta_{\kappa\lambda} \, - \kappa h_{\kappa\lambda} \right) \partial_\alpha \varphi \partial^\alpha \varphi \right] |\phi|^2 \end{aligned} \tag{44}$$

Now the vacuum expectation value of $|\phi|^2$ above and below phase transition is given by

$$|\phi|^2 \quad = \quad 0 \qquad \text{for} \qquad \mathrm{T} > \mathrm{T_C} \tag{45}$$

$$|\phi|^2 \quad = \quad -\frac{\mu^2}{2\lambda} \qquad \text{for} \qquad \mathrm{T} < \mathrm{T_C}, \tag{46}$$

and we see that above the critical temperature, the graviton is massless, but below T_C the graviton has acquired a mass.

Clearly this is a case of spontaneous symmetry breaking. Analogous to the Higgs mechanism we expect the graviton to acquire three new degrees of freedom, since it is a spin two field. Those degrees can only come from the scalar field, which means we need a scalar triplet or three scalars to conspire together to give one graviton its mass. This seems somewhat awkward and artificial, but still could be done. However, if the symmetry breaking field is a vector field instead of a scalar, this scenario becomes much more natural: One vector field has three degrees of freedom, and hence one graviton would eat up one vector field to acquire its mass, instead of three scalar fields. Thus we will consider the vector field next.

5.2 Vector Field

We have two reasons to consider the vector field: First, most of the neutron pairs inside a neutron star are in the triplet state. But second, since the graviton needs three degrees of freedom to become heavy, it would be more natural, if the symmetry breaking were done via one vector field instead of three scalar fields.

The Lagrangian for a spin 1 field ψ_α which has both transversal and longitudinal degrees of freedom is given through [4]:

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{2}D_\alpha\psi_\beta D^\alpha\psi^\beta + \frac{c}{2}D_\alpha\psi^\alpha D_\beta\psi^\beta - \frac{1}{2}\left(\mu^2 - 2aR\right)\psi_\alpha\psi^\alpha - \lambda\left(\psi_\alpha\psi^\alpha\right)^2\right), \tag{47}$$

where D_α is the covariant derivative,

$$D_\alpha\psi_\beta = \partial_\alpha\psi_\beta - \Gamma^\gamma_{\alpha\beta}\,\psi_\gamma. \tag{48}$$

As before, we will redefine $\mu^2 \to \mu^2 - 2aR$ to simplify the Lagrangian,

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{2}D_\alpha\psi_\beta D^\alpha\psi^\beta + \frac{c}{2}D_\alpha\psi^\alpha D_\beta\psi^\beta - \frac{1}{2}\mu^2\psi_\alpha\psi^\alpha - \lambda\left(\psi_\alpha\psi^\alpha\right)^2\right). \tag{49}$$

Again by variation with respect to the metric we obtain the following energy-momentum tensor

$$\begin{aligned} T_{\kappa\lambda} \;=\;& \frac{1}{2}\left[g^{\mu\nu}D_\kappa\psi_\mu D_\lambda\psi_\nu + g^{\alpha\beta}D_\alpha\psi_\kappa D_\beta\psi_\lambda - \frac{1}{2}\,g_{\kappa\lambda}\,g^{\alpha\beta}g^{\mu\nu}D_\alpha\psi_\mu D_\beta\psi_\nu\right] \\ & + c\left(D_\kappa\psi_\lambda - \frac{1}{4}\,g_{\kappa\lambda}\,D_\beta\psi^\beta\right)D_\alpha\psi^\alpha - \left(\frac{1}{2}\mu^2 + 2\lambda\psi_\alpha\psi^\alpha\right)\psi_\kappa\psi_\lambda \\ & + \frac{1}{2}\,g_{\kappa\lambda}\,\left(\frac{1}{2}\mu^2 + \lambda\psi_\alpha\psi^\alpha\right)\psi_\alpha\psi^\alpha. \end{aligned} \tag{50}$$

This looks a little complicated and a few simplifications are in order not to loose sight of what we are looking for. The first thing to notice is that the vacuum expectation value of the last term will always be zero, since above T_C, $|\psi|^2 = 0$, and below T_C, $|\psi|^2 = -\mu^2/2\lambda$. Next we observe that the Christoffel symbols in the gravitational wave expansion [37],

$$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2}\eta^{\gamma\rho}\left(\partial_\alpha h_{\rho\beta} + \partial_\beta h_{\alpha\rho} - \partial_\rho h_{\alpha\beta}\right) + \mathcal{O}\left(h^2\right), \tag{51}$$

are proportional to $\partial^\gamma h_{\lambda\nu}$, and thus we cannot expect them to give us a Meissner effect. Therefore, we will concentrate only on

$$\begin{aligned} T_{\kappa\lambda} \;=\;& \frac{1}{2}\left[g^{\mu\nu}\partial_\kappa\psi_\mu\partial_\lambda\psi_\nu + g^{\alpha\beta}\partial_\alpha\psi_\kappa\partial_\beta\psi_\lambda - \frac{1}{2}\,g_{\kappa\lambda}\,g^{\alpha\beta}g^{\mu\nu}\partial_\alpha\psi_\mu\partial_\beta\psi_\nu\right] \\ & + c\left(\partial_\kappa\psi_\lambda - \frac{1}{4}\,g_{\kappa\lambda}\,\partial_\beta\psi^\beta\right)\partial_\alpha\psi^\alpha - \left(\frac{1}{2}\mu^2 + 2\lambda\psi_\alpha\psi^\alpha\right)\psi_\kappa\psi_\lambda. \end{aligned} \tag{52}$$

We expand the metric $g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu}$ and neglect terms in κ^2 and κ^3. Furthermore assume that all the space-time dependance is in the phase, i.e.,

$$\partial_\kappa \psi_\mu = \partial_\kappa \left(|\psi_\mu| \, e^{i\varphi} \right) = i \psi_\mu \partial_\kappa \varphi. \tag{53}$$

Sorting with respect to $\eta^{\mu\nu}\psi_\mu\psi_\nu$, we arrive at

$$\begin{aligned} T_{\kappa\lambda} = & \\ & \frac{1}{4} \left(\left((\eta_{\kappa\lambda} + \kappa h_{\kappa\lambda}) \eta^{\alpha\beta} - \kappa \eta_{\kappa\lambda} h^{\alpha\beta} \right) \partial_\alpha \varphi \partial_\beta \varphi - 2 \partial_\kappa \varphi \partial_\lambda \varphi \right) \eta^{\mu\nu} \psi_\mu \psi_\nu \\ & + \left(\frac{1}{2} \left(\kappa h^{\alpha\beta} - \eta^{\alpha\beta} \right) \partial_\alpha \varphi \partial_\beta \varphi - \frac{1}{2} \mu^2 + 2\lambda \left(\kappa h^{\alpha\beta} - \eta^{\alpha\beta} \right) \psi_\alpha \psi_\beta \right) \psi_\kappa \psi_\lambda \qquad (54) \\ & + \frac{\kappa}{2} \left(\partial_\kappa \varphi \partial_\lambda \varphi - \frac{1}{2} \eta_{\kappa\lambda} \eta^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) h^{\mu\nu} \psi_\mu \psi_\nu + c \left(\kappa h^{\mu\nu} - \eta^{\mu\nu} \right) \partial_\mu \varphi \partial_\kappa \varphi \psi_\nu \psi_\lambda \\ & + \frac{c}{4} \left(\eta^{\mu\nu} \eta_{\kappa\lambda} \eta^{\alpha\beta} - \kappa \left(\eta^{\mu\nu} \eta_{\kappa\lambda} h^{\alpha\beta} - \eta^{\mu\nu} h_{\kappa\lambda} \eta^{\alpha\beta} + \, h^{\mu\nu} \eta_{\kappa\lambda} \eta^{\alpha\beta} \right) \right) \partial_\mu \varphi \partial_\alpha \varphi \psi_\nu \psi_\beta. \end{aligned}$$

As in the scalar case, the above is going to be on the right-hand side of Einstein's equations (43), and thus in the gravitational wave expansion

$$\partial_\alpha \partial^\alpha h_{\kappa\lambda} + \left(\partial_\kappa \partial_\lambda + \, \eta_{\kappa\lambda} \partial_\mu \partial^\mu \right) h^\alpha_{\ \alpha} = -8\pi G \, T_{\kappa\lambda}.$$

Not unexpected, the energy-momentum tensor for the vector field is a little more complicated as the one for the scalar field. Direct interaction terms between the graviton and the vector field like $h^{\mu\nu}\psi_\mu\psi_\nu$ appear. However, the first line in equation (54) for the energy momentum tensor, i.e.,

$$\frac{1}{4} \left(\left((\eta_{\kappa\lambda} + \kappa h_{\kappa\lambda}) \eta^{\alpha\beta} - \kappa \eta_{\kappa\lambda} h^{\alpha\beta} \right) \partial_\alpha \varphi \partial_\beta \varphi - 2 \partial_\kappa \varphi \partial_\lambda \varphi \right) \eta^{\mu\nu} \psi_\mu \psi_\nu,$$

will show a phase transition. The vacuum expectation value of this term will be zero above the transition temperature and non-zero below, since

$$|\psi|^2 = 0 \qquad \text{for} \qquad \mathrm{T} > \mathrm{T_C} \tag{55}$$

$$|\psi|^2 = -\frac{\mu^2}{2\lambda} \qquad \text{for} \qquad \mathrm{T} < \mathrm{T_C}. \tag{56}$$

Although a little more complicated than we may have wanted, the vector field too will give us a gravitational Meissner effect. This is very comforting in that it validates the assumptions in the previous sections.

5.3 Discussion

As a consequence of the Meissner effect, we expect the graviton to become massive. We can estimate the mass of the graviton inside a neutron star by taking the Compton wavelength formula

$$m_g = \frac{\hbar}{\lambda_L c} = 2.9 \cdot 10^{-47} \mathrm{kg} = 1.6 \cdot 10^{-11} \mathrm{eV/c^2}, \tag{57}$$

where λ_L is the London penetration depth and we used $\lambda_L = 12$km. As the graviton acquires a mass we then expect that the gravitational Coulomb potential is to be replaced locally by a Yukawa type potential, with possible significant consequences for neutron star collapse, which are currently under investigation.

6 Conclusion

In this paper we tried to shed some light on the problem of breaking the gravitational symmetry. The approach taken is somewhat of a pedestrian in nature. As close a path as possible to observable objects and experiments was taken.

The semiclassical London approach gives us the London penetration length. The number we obtain for neutron stars, $\lambda_L = 12$km, is intriguing and somewhat unexpected, but is an indication that in neutron stars the breaking of the gravitational symmetry may play a significant role.

The strength of the Ginzburg-Landau approach lies in that it is based on laboratory gravitational experiments. The COW experiment was the first one to show the direct influence of gravity on quantum mechanics. We go one step further and propose a minimal coupling scheme. From there everything else falls into place following in close analogy to the electromagnetic case. We predict again the Meissner effect, a gravitational Aharanov-Bohm effect, and for the triplet state a gravitational 'ferromagnetic' type phase.

The last section addresses the problem from the gravitational wave expansion point of view. The spontaneous symmetry breaking character of the phenomenon becomes apparent. Becoming massive, the graviton will acquire three new degrees of freedom in the same way as the photon gets a new degree of freedom in the theory of superconductivity.

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A Appendix

A.1 Remark on Units

The treatment of the Ginzburg-Landau theory is significantly simplified if one follows the electromagnetic example and introduces the equivalent of Gaußian cgs-units. Similarly, as in the electromagnetic case, where the Coulomb force

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \text{becomes} \qquad \mathrm{F} = \frac{\mathrm{e}^2}{\mathrm{r}^2},$$

where the two charges e and q are related through $e = 4.80324 \cdot 10^{-10}$ esu, one can introduce a new gravitational charge q, and

$$F = G\frac{m^2}{r^2} \qquad \text{becomes} \qquad \mathrm{F} = \frac{\mathrm{q}^2}{\mathrm{r}^2}, \tag{58}$$

where, for instance,

$$q_N = \sqrt{G} m_N = 1.35639 \cdot 10^{-32} \mathrm{gsu} \tag{59}$$

for a neutron with $m_N = 1.66 \cdot 10^{-27}$kg. 'gsu' stands for 'gravito-static units' and is gravitational equivalent of 'esu'. Table 1 lists a few units.

	units
q = mass charge	gsu = $\sqrt{G}$ kg
q^2 = (mass charge)^2	(gsu)^2 = N m^2
ρ = density	gsu / m^3
$\mathbf{j}$ = current density	gsu / m^2 s
$\mathbf{F}$ = force	N = kg m / s^2
$\mathbf{g}$ = electric field	N / gsu
$\mathbf{H}$ = magnetic field	N / gsu
ϕ = potential	N m / gsu
$\mathbf{A}$ = vector potential	N m / gsu

Table 1: Gravitostatic units.

A.2 Examples and Numbers

Any gravitating body will be surrounded by a radial gravitoelectric field

$$\mathbf{g} = -\sqrt{G}\frac{M}{r^2}\mathbf{e}_r \tag{60}$$

where the bodies mass M is its gravitoelectric monopole moment.

If the body is rotating, then in addition there will be a gravitomagnetic dipole field of the form

$$\mathbf{H} = \frac{2\sqrt{G}}{c}\left[\frac{\mathbf{S} - 3\left(\mathbf{S}\cdot\mathbf{e}_r\right)\mathbf{e}_r}{r^3}\right] \tag{61}$$

where the gravitomagnetic dipole moment is its spin angular momentum $\mathbf{S}$ [33]. Typical numbers for M and S are given in table 2.

	M	S
Earth	$5.98 \cdot 10^{27}$ g	$7 \cdot 10^{40}$ g cm^2 / s
Sun	$1.99 \cdot 10^{33}$ g	$1.7 \cdot 10^{48}$ g cm^2 / s
Neutron Star	$2.8 \cdot 10^{33}$ g	$2 \cdot 10^{47}$ g cm^2 / s

Table 2: Some typical masses and angular momenti.

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