$qq' \to qq'$: A SECOND LOOK AT THE IR-DIVERGENCES

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Interest in the transport properties of the quark-gluon plasma has arisen mostly keeping in mind two widely differing kinds of systems - that of the high temperature phase created in the Relativistic Heavy Ion Colliders and the effectively zero temperature, Fermi-degenerate phase believed to exist in the cores of the Neutron Stars or in the more exotic Strange Stars. In the first case, the transport coefficients (thermal and electrical conductivities and shear viscosity) determine the nature of the equlibration. Whereas in the case of the astrophysical systems the cooling history, the rotational behaviour and the evolution of the magnetic field depend on such coefficients, and may even provide an answer to the question whether some of the pulsars are indeed Strange Stars [1].

To this end, accurate calculations of the *elastic scattering cross-sections* are required for $qq' \to qq'$ and $q\overline{q}' \to q\overline{q}'$ processes, which could then be used to solve the Boltzman transport equation to yield the required transport coefficients. Such calculations have already been performed at the tree-diagram level where exchange of massless gluons leads to divergent cross sections [2]. This is analogous to Coulomb scattering where transverse photons are exchanged in an electron plasma. For small momentum transfer in qq' scattering, the cross-section is power-divergent just like Rutherford cross-section ($\frac{d\sigma}{d\Omega} \sim \frac{g^4}{q^4}$). Taking the actual amount of momentum transfer into account the cross-section is found to be log-divergent, which is still short of curing the malady.

Debye screening, removes the IR-divergences for the electric or the longitudinal modes by providing an effective length scale to the system. And the current-current interaction term provides a dynamic screening akin to Landau damping for the transverse modes. An effective way to implement this screening is to intoduce a Debye mass as an infrared regulator into the gluon propagator for the thermal gluons. This Debye mass cuts off the magnetic divergence too [3] and the dynamic screening is taken care of.

But, to our dismay, we find that at the two-gluon exchange level such a technique fail to remove the divergences. This constitutes the main result of our work that we would like to report here. In all the previous work, though a thermal mass has been introduced for the gluon propagator to take care of the in-medium effects, but the external fermions (the quarks) are still taken to be massless, in the high-temperature as well as in the effective zero-temperature phase.

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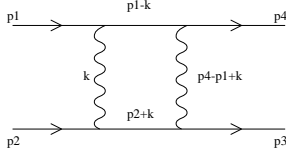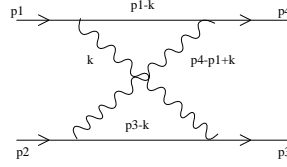
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fig. 1

fig.2

To demonstrate the divergent behaviour in the fermion-fermion scattering cross section with two gluon exchange, we take the external quark lines to be massless and use the following forms for the propagators and the vertex structures [4],

i) $gf^{abc}[(p-q)_\nu g_{\lambda\mu} + (q-r)_\lambda g_{\mu\nu} + (r-p)_\mu g_{\nu\lambda}]$ - three gluon vertex,

ii) $ig\gamma^\mu T_{ij}^a$ - the one gluon - two quark vertex,

iii) $\frac{1}{\not{p}+i\epsilon}$ - the fermion propagator,

iv) $D_{\mu\nu}^{ab}(k) = g_{\mu\nu}\delta^{ab}[\frac{i}{k^2-m_g^2+i\epsilon} + 2\pi\delta(k^2-m_g^2)n(k)]$ - the thermal gluon propagator, where, $n(k)$ is the Bose-Einstien distribution function and the thermal gluon mass is given by $m_g^2 = \frac{1}{6}C_A g^2 T^2 + \frac{1}{12}C_F g^2(T^2 + 3\frac{\mu^2}{\pi^2})$, C_F, C_A being Casimir invariants of the symmetry group in the fundamental representation and the adjoint representation. μ is the chemical potential of the quarks and we work in the regime where $T << \mu$.

To show the IR-divergence of the scattering cross-section in the above-mentioned situation it is enough to consider just the t-channel process. For such a process the diagrams could be classified into three categories, (i) the exchange type similar to those arising in QED, (ii) diagrams where self-energy corrections are included on the external legs, (iii) diagrams with self-energy corrections on the exchanged gluons and (iv) the vertex correction types. It is only the diagrams of type (i) that suffer from the infrared divergences. Therefore, here we concentrate on the diagrams of type (i) particularly those shown in fig.1 and fig.2, since the other diagrams of this variety are suppressed by factors of s (the square of the sum of initial momenta).

At the two-gluon exchange level, the leading contribution to the amplitude, in the limit $t \to 0$ (t is the square of the momentum transfer) comes from the thermal part of the two gluon exchange lines, particularly from the delta functions. So retaining the leading term in $\frac{1}{t}$, the matrix element for these diagrams is given by,

$$
\begin{aligned}
< |M_{qq'}|^2 > &= \frac{1}{9}\Sigma_c(2\pi g^2)^2\frac{1}{4}\Sigma_{\text{spin}}|fig.2a + fig.2c|^2 \\
&= \frac{1}{9}\Sigma_c(2\pi g^2)^2\frac{1}{4}\Sigma_{\text{spin}}|(\Gamma_4^\mu\gamma^\alpha\gamma^\nu\Sigma_{13}\gamma_\mu\gamma^\beta\Gamma_{2\nu})\frac{T}{\sqrt{s}(2\pi)^3(2m_g^2+s)t} \\
&\quad (p_{1\alpha}(p_1-p_2)_\beta - p_{4\alpha}(p_4-p_3)_\beta)
\end{aligned}
$$

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$$+(\Gamma_4^\mu \gamma^\alpha \gamma^\nu \Sigma_{13} \gamma_\nu \gamma^\beta \Gamma_{2\mu}) \frac{T}{\sqrt{s}(2\pi)^3(2m_g^2 + u)t}$$

$$(p_{1\alpha}(p_1 + p_3)_\beta - p_{4\alpha}(p_4 + p_2)_\beta)|^2 \tag{1}$$

where, p_1, p_2 are the incoming momenta, p_3, p_4 are the outgoing momenta with,

$$s = (p_1 + p_2)^2, u = (p_1 - p_3)^2, t = (p_1 - p_4)^2, \tag{2}$$

$$\Gamma_4^\mu = \overline{U}(p_4)\gamma^\mu, \Sigma_{13} = U(p_1)\overline{U}(p_3) \tag{3}$$

and T is the temperature. Where we have, $s + u + t = 0$, as s, u, t are the Mandelstam variables. The matrix element is obtained by summing over the final states and averaging over the initial states. The finite temperature phase space measures are given by,

$$d\phi_4 = \frac{d^4 p_4}{(2\pi)^4} \delta(p_4^2 - m_0^2)[\Theta(p_{40}) - N(|p_{40}|)] \tag{4}$$

$$d\phi_3 = \frac{d^4 p_3}{(2\pi)^4} \delta(p_3^2 - m_0^2)[\Theta(p_{30}) - N(|p_{30}|)] \tag{5}$$

Here $m_0 = 0$ as the external quarks are massless and the factor $[\Theta(p_{40}) - N(|p_{40}|)]$ takes care of Pauli blocking [5].

Then the total scattering cross-section is given by

$$\begin{aligned} \sigma(i \to f) &= \int \frac{1}{p_1 \cdot p_2} < |M|^2 > \delta(p_1 + p_2 - p_3 - p_4)(2\pi)^4 \\ &\quad \frac{d^4 p_4}{(2\pi)^4} \delta(p_4^2)[\Theta(p_{40}) - N(|p_{40}|)] \frac{d^4 p_3}{(2\pi)^4} \delta(p_3^2)[\Theta(p_{30}) - N(|p_{30}|)] \\ &= \frac{1}{s} \int \frac{d\Omega_{p_4}}{(2\pi)^4} < |M|^2 > [1 - N(\frac{\sqrt{s}}{2})]^2 \\ &\sim \frac{1}{s^2} < |M|^2 > [1 - N(\frac{\sqrt{s}}{2})]^2 \end{aligned} \tag{6}$$

as, $d\Omega = \frac{d\phi d(-t)}{s/2}$.

Therefore,

$$\frac{d\Sigma}{dt} \sim \frac{1}{4\pi^3} \frac{1}{s^2} < |M|^2 > \left[1 - N(\sqrt{s})\right]^2 \tag{7}$$

Since, to leading order, $< |M|^2 > \sim O(\frac{1}{t}) + ..$, which has a *log divergence* in small momentum transfer we conclude that *infra-red divergence is NOT cured in two-gluon exchange level by introducing an effective mass to the thermal gluons only.* As σ diverges for small t, we once again have the

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situation of vanishing transport coefficients! One is saved from such absurd results by considering non-zero mass for the external quarks. In a chiral symmetry broken phase one could introduce the mass following the standard procedure and that cures the problem. But in a chirally symmetric phase one needs to consider the chirally invariant mass of the fermions [6]. A detailed report of this work will be presented elsewhere [7].

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