

Chiral Perturbation Theory for Nuclear Physicists¹

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Abstract

Chiral perturbation theory is the low energy effective theory of the strong interactions for the light pseudoscalar degrees of freedom. This program is based on effective Lagrangian techniques and is an expansion in the powers of the momenta and the powers of the quark masses, which correct the soft-pion theorems. After briefly reviewing these features and some results, we address the implications of this program to $\pi - N$ scattering, the $\pi - N$ sigma term and some recent investigations of the implications of chiral symmetry to nucleon-nucleon forces. We finally look at the implications of chiral perturbation theory to hadron mass relations.

¹Invited talk at the DAE Nuclear Physics Symposium - 1997, December 26-30, 1997, Bangalore, India

1 Introduction

The purpose of this talk is to present in a concise manner some of the results and techniques of modern chiral perturbation theory[1, 2, 3, 4, 5] to highlight the links with nuclear physics. Much of what I will say here is already found in standard textbooks[6, 7], and I will list several references where indepth explanations of the subjects can be found. The pion first posited to explain the forces between two nucleons is of interest to nuclear physics. The pions π^\pm and π^0 are the lightest hadrons and are essentially degenerate in mass and lie in an iso-spin triplet. Their lightness may be understood by regarding them as the Goldstone bosons of spontaneously broken axial vector symmetry of massless QCD; the presence of non-vanishing quark masses shifting the pion pole to $M_\pi^2 = 2\hat{m}B$, where \hat{m} is the average mass of the u- and d- quarks, and B is a measure of the vacuum expectation value $\langle 0|\bar{u}u|0 \rangle = \langle 0|\bar{d}d|0 \rangle = -F_\pi^2 B$ and F_π is the pion decay constant[2]. QCD is the theory of quark and gluon degrees of freedom and exhibits the property of asymptotic freedom in that at large momenta the coupling constant becomes smaller and one may study the theory in a perturbation series in the strong coupling constant $\alpha_S = g^2/(4\pi)$. The lagrangian is:

$$\mathcal{L}_{QCD} = -\frac{1}{4g^2}G_{\mu\nu}^a G^{a\mu\nu} + \bar{q}i\gamma^\mu D_\mu q - \bar{q}\mathcal{M}q \quad (1)$$

If quark mass matrix \mathcal{M} is set to zero then the left- and right- chiral projections may be rotated independently and the quark flavors rotated amongst each other. Thus associated with say N light quark flavors, we have the chiral symmetry given by the group $SU(N)_L \times SU(N)_R$ (we disregard the $U(1)_V$ and the anomalous $U(1)_A$ groups. This chiral symmetry is broken spontaneously to its vector subgroup $SU(N)_V$, where $V = (L+R)$ and corresponding to the $SU(N)_A$ broken symmetry we have $N^2 - 1$ (pseudoscalar) Goldstone bosons.

On the other hand at low energies, the coupling constant is large and the problem

is intractable. In particular, we do not yet know how to obtain the hadronic spectrum from the QCD lagrangian, nor do we know the mechanism by which chiral symmetry is broken spontaneously. Nevertheless, the QCD lagrangian provides the justification for the successful results obtained from PCAC and current algebra. The effective low energy theory of the strong interactions at next to leading order requires the knowledge of the underlying theory and the analysis rests of writing down the generating functional for the currents of the theory which is the vacuum to vacuum transition amplitude in the presence of external sources. The low energy expansion is one that involves derivatives of the external sources a well known example being the Euler-Heisenberg method of analyzing QED. Chiral perturbation theory then is the low-energy effective theory of the strong interactions and involves a simultaneous expansion in the mass of the quarks and the momenta, about the chirally symmetric $SU(2) \times SU(2)$ limit of the massless QCD with the the spontaneous breakdown of this symmetry by the ground state to $SU(2)_V$, the pions corresponding to the Goldstone bosons of the broken $SU(2)_A$ generators. The Goldstone theorem yields

$$\langle 0|A_\mu|\pi \rangle = F_\pi p_\mu, \quad (2)$$

and $F_\pi \approx 93$ MeV. To leading order, $O(p^2)$, the effective lagrangian is that of the non-linear sigma model. The effective action is

$$Z_1 = F^2 \int dx \frac{1}{2} \nabla_\mu U^T \nabla^\mu U \quad (3)$$

where U is a four component real $O(4)$ (note that $O(4) \equiv SU(2) \times SU(2)$) unit vector. This model is not renormalizable and the loops of the model lead to divergences which cannot be absorbed into the parameters of the model. In order to absorb the divergences, one is led to introducing higher derivative interactions, which then allows one to extend the predictions at leading order in the momentum or derivative to the next order. The price is the proliferation of coupling constants that must be extracted from experiment

or alternatively from theoretical considerations such as the behaviour of the coupling constants in the chiral limit as well as non-perturbative approaches such as large N_c . The effective lagrangian at $O(p^4)$ [2] and at $O(p^6)$ [8] have been worked out. [When one considers the interactions of pions with nucleons, the chiral power counting is different since the pion is now coupled to an external nucleon.] For completeness we write down the effective lagrangian at $O(p^4)$ [2]:

$$\begin{aligned} \mathcal{L}_4 = & l_1(\nabla^\mu U^T \nabla_\mu U)^2 + l_2(\nabla^\mu U^T \nabla^\nu U)(\nabla_\mu U^T \nabla_\nu U) + l_3(\chi^T U)^2 + l_4(\nabla^\mu \chi^T \nabla_\mu U) + \\ & l_5(U^T F^{\mu\nu} F_{\mu\nu} U) + l_6(\nabla^\mu U^T F_{\mu\nu} \nabla^\nu U) + l_7(\tilde{\chi}^T U)^2 + h_1 \chi^T \chi + h_2 \text{tr} F_{\mu\nu} F^{\mu\nu} + h_3 \tilde{\chi}^T \tilde{\chi} \end{aligned} \quad (4)$$

where $F_{\mu\nu}$ are covariant tensors involving the external fields and their derivatives and the vectors χ and $\tilde{\chi}$ are proportional to the external scalar and pseudoscalar fields. With this effective lagrangian and with the loops generated by the non-linear sigma model and appropriate renormalization, one may obtain the Green's functions of QCD at this order in the momentum expansion. At this order, 10 additional coupling constants enter the effective lagrangian.

Although the number of coupling constants at $O(p^6)$ are very large (> 100), those entering the pion-pion scattering amplitude are still limited in number. Of course, once the coupling constants are fixed from a certain class of experiments, at that order, the theory would have predictions for all other processes at the appropriate level of accuracy. Furthermore, the external field technique permits an off-shell analysis of the Green's functions of the theory and permits one to study the quark mass dependence of the Green's functions.

The important processes of $\pi\pi$ and πN scattering have been analyzed in great detail and methods have been described in standard books[9, 10, 11, 12, 13, 14], and will be of interest to us in this discussion. Note that a good deal of the experimental information on the processes of interest to us has been obtained via dispersion relation analysis of

phase shift information. In fact, there is a rich interplay between the effective lagrangian methods of chiral perturbation theory and dispersion relation theory which we will describe in some of the following sections.

In the following sections, we briefly review the status of $\pi\pi$ scattering, $\pi - N$ scattering and the “sigma” term, and implications of chiral symmetry to hadron mass relations and finally on the status of nucleon-nucleon forces from the view point of chiral symmetry. A few remarks are listed on other subjects of interest with some references to the literature. Considerably more time is spent on the first of the topics as it allows us to set up the framework to discuss the issues raised in this talk. *Above all we catalog a list of references, with some remarks, from which the reader would get a glimpse of this vast and fascinating subject.*

2 $\pi\pi$ Scattering

As an illustration of the techniques used to obtain several results, we describe the $\pi\pi$ process[10, 15] in some detail, although these results are now nearly 40 years old. In axiomatic field theory the validity of dispersion relations have been proved some time ago. In the case of $\pi\pi$ scattering dispersion relations are particularly simple. Phase shift information has been analyzed in the past, well before chiral perturbation theory or QCD were established. Today, an analysis that employs chiral results *ab initio* is required to confront experimental data. Pion-pion scattering may be described in terms of a single function $A(s, t, u)$ of the Mandelstam variables[16], s, t, u . The process is schematically represented by

$$\pi^a(p_1) + \pi^b(p_2) \rightarrow \pi^c(p_3) + \pi^d(p_4) \tag{5}$$

and since iso-spin is conserved by the strong interactions, the transition matrix is given by:

$$A(s, t, u)\delta^{ab}\delta^{cd} + A(t, u, s)\delta^{ac}\delta^{bd} + A(u, s, t)\delta^{ad}\delta^{bc} \quad (6)$$

where the function $A(s, t, u) = A(s, u, t)$ (and is denoted as A_s) due to generalized Bose statistics and $s = (p_1 + p_2)^2$, $t = (p_1 + p_3)^2$ and $u = (p_1 + p_4)^2$, all momenta taken to be incoming. If \sqrt{s} represents the centre of mass energy, then t and u related to the cosine of the centre of mass scattering angle via $\cos\theta = (t - u)/(s - 4)$, $s + t + u = 4$ when we set the particle mass to unity.

Since they lie in an iso-spin triplet, the s-channel amplitudes for definite iso-spin can be written down:

$$\begin{aligned} T_s^0(s, t, u) &= 3A_s + A_t + A_u \\ T_s^1(s, t, u) &= A_t - A_u \\ T_s^2(s, t, u) &= A_t + A_u \end{aligned} \quad (7)$$

which follows from iso-spin coupling. This may be rewritten as $M.A$, where

$$M = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

and $A = [A_s \ A_t \ A_u]^T$. Note that

$$M^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

One convenient representation for dispersion relations for the amplitudes of definite spin in the t-channel, with two subtractions (convergence is guaranteed with this number of subtractions as a result of the Froissart bound) is:

$$T_t^I(s, t, u) = \mu_I(t) + \nu_I(t)(s - u) + \frac{1}{\pi} \int_4^\infty \frac{ds'}{s'^2} \left(\frac{s^2}{s' - s} + (-1)^I \frac{u^2}{s' - u} \right) \sum_{I'} C_{st}^{II'} A_s^{I'}(s', t) \quad (8)$$

where $\mu_I(t)$, $\nu_I(t)$ are unknown t-dependent subtraction constants ($\mu_1 = \nu_0 = \nu_2 = 0$), where now $A_s^I(s', t)$ is the absorptive part of the s-channel amplitude. The matrix C_{st} is the so-called crossing matrix, (embodying the fundamental property of crossing in axiomatic field theory) the entries of which may be written down from the general formula resulting from iso-spin coupling in terms of the Wigner 6-j symbol[17] as:

$$C_{st}(c, d) = (-1)^{(c+d)} (2c + 1) \begin{Bmatrix} 1 & 1 & d \\ 1 & 1 & c \end{Bmatrix} \quad (9)$$

An elementary and alternative derivation of the crossing matrix is presented below (which the present author has been unable to trace in the literature) from considering the simple crossing relations between A_s , A_t and A_u via the matrix relations:

$$C_{st} = M \cdot B_{st} \cdot M^{-1},$$

where B_{st} is given below which crosses the s- and t- channels,

$$B_{st} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

which yields for C_{st} ,

$$C_{st} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6} \\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6} \end{pmatrix}.$$

Analogously we obtain the matrices C_{su} and C_{tu} (not listed here), by replacing $B_{st} \rightarrow B_{su}$, and $B_{st} \rightarrow B_{tu}$ respectively, with

$$B_{su} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } B_{tu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Note that we have not listed the relevant expressions involving 6-j symbols for the s-u and t-u crossing either.

A very convenient form of dispersion relations has been found[18] which eliminates the unknown functions μ_I, ν_I in favour of the S- wave scattering lengths a_0^0 and a_0^2 , where the scattering lengths a_l^I arise in the threshold expansion for the partial wave amplitudes $\text{Re}f_l^I(\nu) = \nu^l(a_l^I + b_l^I\nu + \dots)$, where the partial wave expansion is given by $T_s^I(s, t, u) = \sum(2l + 1)f_l^I(s)P_l((t - u)/(s - 4))$, $\nu = (s - 4)/4$. This form is:

$$\begin{aligned} T_s^I(s, t) &= \sum_{I'} \frac{1}{4}(s \mathbf{1}^{II'} + t C_{st}^{II'} + u C_{su}^{II'}) T_s^{I'}(4, 0) \\ &+ \int_4^\infty ds' g_2^{II'}(s, t, s') A_s^{I'}(s', 0) + \int_4^\infty ds' g_3^{II'}(s, t, s') A_s^{I'}(s', t). \end{aligned} \quad (10)$$

For our purposes, it is convenient to write the kernels in the form

$$\begin{aligned} g_2(s, t, s') &= -\frac{t}{\pi s' (s' - 4)} (u C_{st} + s C_{st} C_{tu}) \left(\frac{\mathbf{1}}{s' - t} + \frac{C_{su}}{s' - 4 + t} \right) \\ g_3(s, t, s') &= -\frac{s u}{\pi s' (s' - 4 + t)} \left(\frac{\mathbf{1}}{s' - s} + \frac{C_{su}}{s' - u} \right). \end{aligned} \quad (11)$$

Furthermore, $T_s(4, 0) = (a_0^0, 0, a_0^2)$.

The property of crossing places constraints on the absorptive parts of the amplitudes. However, the presence of 2 subtractions in these dispersion relations implies that S- and P- waves do not face any constraints. It has been shown that the dispersive representation for the amplitude in the approximation that only S- and P- waves saturate the absorptive parts of the amplitudes lends itself to a straightforward comparison with chiral amplitudes[19].

Nevertheless the higher partial waves have to be treated with care. The problem must be accounted for when we saturate fixed-t dispersion relations with absorptive parts which are modelled theoretically, perhaps in terms of resonance propagators, Pomeron exchange, say the Veneziano model, etc. Alternatively, one may write down dispersion relations in terms of homogeneous variables[20] which manifestly enforce crossing symmetry; however

there might be a dependence on parameters which parametrize the curves in the plane of the homogeneous variables on which the dispersion relations are written down. While the actual implications of these constraints may appear academic, present day chiral computations are seeking to make very accurate predictions for, say threshold parameters of scattering. The effective lagrangian technique produces manifestly crossing symmetric results for the amplitudes and thus a comparison which is made must ensure that crossing constraints are respected. At $O(p^4)$ 4 additional coupling constants, scale free coupling constants $\bar{l}_{1,2,3,4}$ enter the $\pi\pi$ scattering amplitude[2], while we note that at leading order we have the simple form for the scattering amplitude

$$A(s, t, u) = \frac{s - 1}{32\pi F_\pi^2} \quad (12)$$

which leads to the prediction for $a_0^0 = 7/(32\pi F_\pi^2) \simeq 0.16$, (note that $m_\pi = 139$ MeV, has been set to unity). At $O(p^4)$ the presence of the infrared singularities of the theory modifies this prediction substantially and may be expressed in terms of the four \bar{l} 's, in addition to F_π . Estimates for these quantities from disparate sources such as D- wave scattering lengths [alternatively from $\pi\pi$ phase information directly], $SU(3)$ mass relations and the ratio of the decay constants F_K/F_π gives a correction of about 25% to the leading order prediction, $a_0^0 = 0.20 \pm 0.01$ [2] (the experimental value for this number is quoted as 0.26 ± 0.05 [21]). Note that the scattering amplitudes in chiral perturbation theory are perturbatively unitarity; at one-loop order the loops contribute to the scattering amplitude terms that have the correct analytic structure corresponding to producing them from the tree level amplitude by elastic unitarity.

In particular, today the chiral amplitudes to two-loops have been computed[22, 23, 24]. The work of [23] in standard chiral perturbation theory affords an accurate prediction for the parameter a_0^0 which is a soft quantity from the point of view of dispersion relations and work is in progress to this end. For some of the latest information on the experimental

as well as theoretical aspects of the subject, see Ref.[25].

The methods of chiral perturbation theory are also of great use for decays, strong, semi-leptonic, electromagnetic, etc. One important example where all the methods of effective lagrangians as well as those of dispersion relations are of utility are in the decay $\eta \rightarrow 3\pi$ [27].

3 Pion-Nucleon Scattering

An excellent introduction to the subjects of particle and nuclear physics at their interface is Ref.[26]. The earliest evidence for the existence of (partially) conserved currents in the strong interactions came from the analysis of pion-nucleon scattering. The Goldberger-Treiman relation has been since the earliest days, the cornerstone of current algebra. This relation relates the pion-nucleon coupling constant, $g_{\pi N}(= \sqrt{4\pi}13.5 \simeq 13)$ to the nucleon axial-vector coupling constant $g_A(= 1.26)$ and F_π .

$$g_{\pi N} \simeq \frac{m_N g_A}{F_\pi} \quad (13)$$

We have dealt with the example of $\pi\pi$ scattering at some length, in order to establish the framework for πN scattering, one must account for the differences in the iso-spin coupling scheme as well as taking into account the fermion spin. While the principles remain analogous, the details vary with regard to crossing principles, subtractions for the dispersion relations, the partial wave expansion to name a few aspects. The process that is represented by

$$\pi^a(q_1) + N(p_1) \rightarrow \pi^b(q_2) + N(p_2) \quad (14)$$

may be represented in terms of four invariant amplitudes D^\pm, B^\pm :

$$T_{ab} = T^+ \delta_{ab} - T^- i\epsilon_{abc} \tau_c$$

$$T^\pm = \bar{u}(p_2) \left[D^\pm(\nu, t) + \frac{i}{2m_N} \sigma^{\mu\nu} q_{2\mu} q_{1\nu} B^\pm(\nu, t) \right] u(p_1) \quad (15)$$

with $s = (p_1 + q_1)^2$, $t = (q_1 - q_2)^2$, $u = (p_1 - q_2)^2$, $\nu = (s - u)/(4m_N)$. At a given order, one wishes to compute the 4 invariant amplitudes to the appropriate orders (which differ by 2). Note however that since the pions are the lightest hadrons, the dispersion relations in that context prove to be the simplest. The inclusion of all the aspects of dispersion relations can be easily found in the literature[14]. The methods of current algebra, PCAC and in the early days of phenomenological lagrangians, the pion-nucleon system has been studied in detail. An excellent introduction and review of these topics may be found in Chapter 19 of Ref.[6]. For instance, we have the lowest order predictions for the isospin 3/2 and 1/2 scattering lengths of -0.075 and 0.15 which compare favorably with the data[21].

In the framework of modern chiral perturbation theory, in a relativistic framework this process has been first considered in[28]. In this work, the authors have extended the analysis of the Green's functions of QCD with an external nucleon. Furthermore this environment is an important one for the tests of chiral predictions[29, 30].

More recently the program of heavy baryon chiral perturbation theory[31] has been advocated which takes into account the fact that the nucleon is much more massive than the pion. Within this framework there have been results for the scattering amplitude[30].

4 $\pi - N$ Sigma term

The $\pi - N$ sigma term[32] is defined as a certain matrix element that plays a significant role in the testing of chiral predictions for the pion-nucleon system. The sigma term is the matrix element:

$$\sigma = \frac{\hat{m}}{2m_p} \langle p | \bar{u}u + \bar{d}d | p \rangle \quad (16)$$

where m_p is the proton mass and $|p\rangle$ is the physical one-proton state. Its value may be inferred from mass relations for the Σ -hyperons and the cascades (Ξ) and the strangeness

content of the proton. Furthermore, chiral symmetry implies that this is also related to one of the pion-nucleon invariant amplitudes evaluated at an unphysical (but onshell) point, the Cheng-Dashen point, which requires the extrapolation of the scattering data. The comparison of these two completely independent measures of the sigma term implies a very delicate test of the implications of chiral symmetry as well as methods of treatment of the experimental information, which is why it is a subject of great interest[29]. Furthermore, it behaves as a probe of the contribution of the strange sea to the proton mass and as a result is a quantity worthy of considerable theoretical study.

5 Nucleon-Nucleon Forces

Recent work on the application of chiral symmetry to nucleon-nucleon interactions has been spurred by proposals by Weinberg[33, 34]. The approach taken here extends the ordering of diagrams in field theory by numbers of powers of soft pion momenta to processes involving nucleons as well. A guide to the description of forces from chiral viewpoint is to enforce the Adler consistency condition on scattering amplitudes which may be used to rule out or modify models. In Ref.[35], an effective field theory approach to compute specific scattering amplitudes has been pursued. This involves a modified expansion which does not correspond to an expansion in m_π . A different approach to the problem has been advocated in Ref.[36]; it is observed that loop effects dominate over tree level effects and that small explicit breaking of chiral symmetry controls long range attraction in the scalar-isoscalar channel for NN forces. Analogous observations have been made for multinucleon forces which differ from other conclusions[33, 34].

6 CHPT and Hadron Mass Relations

The presence of non-analytic corrections to the mass spectrum of the hadrons is an important subject which leads to the estimates for the quark mass[5]. In the absence of iso-spin breaking, one has at tree level algebraic relations between the masses of the pseudoscalar octet mesons which yields ratios for the quark masses in terms of the mass ratios of the pseudoscalar masses. Details of the spectrum are then fed in to extract the corrections. Analogously iso-spin breaking is then treated to obtain the mass splitting between the u- and d- quarks in terms of the mass splitting of the pion triplet. This is then extended to the baryon sector as well. Well known examples in the quark model are the Gell-Mann Okubo mass formula for the octet and the baryonic sector. More recently in heavy baryon chiral perturbation theory, these have been revisited as well[37, 38].

7 Other Avenues

The chiral principles being as important as they are in the study of strong interaction physics can be used as a guide for physics outside the realm of perturbation theory as well. An interesting application of chiral principles has been recently proposed[39] in order to study the behaviour of pion amplitudes in the nuclear medium. Combining the chiral principle of the presence of an Adler zero with the Veneziano model, implies a modification of the Regge slope for the the dual resonance formula of the scattering amplitudes. This approach is found to imply overall consistency between experimentally observed features such as the drop in the ρ mass, $\Delta - N$ mass difference, etc., in terms of a universal parameter. In the heavy fermion formalism of chiral perturbation theory for meson exchange currents in nuclei has been developed and applied to nuclear axial charge transitions[40]. Interesting issues are discussed in Ref.[41] regarding the possible

discrepancy between the results of chiral perturbation theory and those of PCAC-current algebra approach in dense matter. For a discussion on the nucleon and strongly interacting material, see Ref.[42]. In the context of nucleon-nucleon potentials and chiral symmetry, see. Ref.[43]

8 Epilogue

Chiral perturbation theory is the effective low energy theory of the strong interactions which takes into account the known symmetry structure of the QCD lagrangian and the properties of the spectrum. In particular, the interactions of the mesonic sector are well understood while the baryonic sector offers theoretical challenges. We are in an era when nuclear physics takes all these inputs into account to finally produce a framework for nucleon-nucleon forces and interactions.

Acknowledgements: It is a pleasure to thank Drs. P. Büttiker, N. D. Hari Dass, J. Pasupathy and M. Sainio for discussions. I also thank Drs. Büttiker and Sainio for reading the manuscript and making invaluable suggestions.

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