Pre-Equilibrium Evolution Of QCD Plasma
An Appraisal

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1 Introduction

The study of strongly interacting matter at high temperature and density had attracted the interest of people all around the globe over the past couple of years.Now with the availability of the new experimental facilities such as the ep collider of HERA at DESY, Fermi-Lab Tevatron ($p\overline{p}$, p A collision), BNL Relativistic Heavy Ion Collider(RHIC) and LHC of CERN; there is a renewed interest in the field–from experimental as well as theoretical point of view to observe as well as theoreticaly model the production of Quark Gluon Plasma –the elusive state of strongly interacting matter– that is supposed to exist in a chiral symmetry restored and deconfined phase at sufficiently high temperature or density or both together.So it is expected that with the availability of the new experimental data, it will be possible to pin down the uncertainities that existed in the previous estimates and hence provide a better model of the physics involved.The interest in these studies stem from both physics as well as astro-physics point of view, e.g to understand the structure of the interior of a neutron star, millisecond pulsar or for that matter in the early universe cosmology, when the primordial plasma was believed to exist in the form of quark gluon soup all around the universe. A study of this phase of matter in laboratory conditions is expected to shed some light on these issues and hence to the understanding of the universe around us. But to observe or detect the QCD plasma produced in the laboratory one needs to have a proper estimate of the energy density per unit voloume, initial temperature , entropy density and the number density of the produced partons [1]. More over the further space time evolution of the plasma as well as the experimental signatures also depends on these quantities. Therefore it is crucial to have a proper understanding of the Pre-equilibrium production phase of the QCD plasma. The essential goal of these studies is to understand how in a nucleus nucleus collision, the coherent distribution of partons in each nucleus evolve to a highly incoherent state, and finally through self interaction generates a collective QCD excitation –that with the passage of time evolves and re-hadronizes before getting captured in the detectors. Generally, the pre-equilibrium phase is supposed to start from the time of the nuclear contact and last till the system has thermalised. As has already been mentioned,all the planned experiments are expected to cover different energy scales ranging from 200 GeV to 100Tev, so as one moves from the SPS to LHC energy scale different paradigms of QCD opens up, from non-perturbative to perturbative! Since the time taken by the nuclei to pass through each other in LHC scale $\sim$ 0.0005 fm/c and the time of thermalisation(for gluons) ia around 0.25 fm/c, the system spends an appreciable amount of time in the pre-equilibrium phase. Therefore, we in our "qualitative" description of the pre-equilibrium phase of

QGP—will start from the low energy scale where the non-perturbative QCD mechanisms are dominant and move towards the high energy scale where the pQCD based processes dominates the scenario.

1.1 Non-Perturbative Domain: The Flux Tube Model

The idea of creating QGP throgh flux tube decay owes its existence to Low and Nussinov [2]. According to this model in a relativistic heavy ion collision, as the target and the projectile nucleui passes through each other, due to multiple exchange of soft gluons they get color charged–creating a strong chromo-electric field between them. This field pulls out q, $\bar{q}$ pairs from the vacuum by tunneling. A simple kinematic consideration shows, to create a pair of particles from vacuum the amount of energy required is$\sim 2m$ on the other hand as these charged particles move apart over a characteristic length scale, say Compton wave length, the energy gained by the system is $\sim 2E/m$. Now one can deduce from a simple energy conservation argument that critical strength field strength required to produce pairs from vacuum is $E_c \sim \quad 2m^2$[3].In technical language it is said–if the strength of this electric field gE exceeds a critical value $gE_c = 2m^2$, the vacuum becomes unstable against production of $q\ \bar{q}$ pairs of mass m each. Since the system looks like a parallel plate capacitor ,this model is also known as capacitor plate model.In this model the electric field is a c number field and assumed to be constant all through out. The rate of pair production per unit volume and per unit time is,

$$W_p = \frac{\alpha E^2}{\pi^2} e^{\frac{-\pi m^2}{gE}} \tag{1}$$

On the other hand because of self interacting nature of the non-Abelian gauge fields–the color-electric field set up between the receding nuclei can oscillate in time. In fact in this model, the surface charge density on the nuclear plates, in the transverse directions are imagined to be homogeneous, with plates extended up to infinity. As the plates move apart, one can find out the field configuration in the central Baryon free region, by solving the sourceless Yang-Mills (Y.M) equations. In an idealised situation one can study the system in $(1+1)$ dimension with an axial gauge choice to get,

$$\begin{aligned}
A_0^1 &= \left[\beta_1 + \beta_1\beta_3 \left[\ e^{i\omega t} + e^{-i\omega t}\right] - i\omega\beta_2 \left[\ e^{i\omega t} - e^{-i\omega t}\right]\right] z + \beta_1 \\
A_0^2 &= \left[\beta_2 + \beta_2\beta_3 \left[\ e^{i\omega t} + e^{-i\omega t}\right] + i\omega\beta_1 \left[\ e^{i\omega t} - e^{-i\omega t}\right]\right] z + \beta_2 \\
A_0^3 &= \left[\beta_3 + {\beta_3}^2 \left[\ e^{i\omega t} + e^{-i\omega t}\right] - \omega^2 \left[\ e^{i\omega t} + e^{-i\omega t}\right]\right] z + \beta_3. \qquad (2)
\end{aligned}$$

Here since $\omega = \sqrt{{\beta_1}^2 + {\beta_2}^2 + {\beta_3}^2}$, the frequency of oscillation is seen to be dependent on amplitude. This is the motivation to re-investigate Schwinger mechanism in an oscillating field. Since finding out W_p using these exact solution, is a formidable task an analysis was carried out in [7], to see effect

of a sinusoidally time varying field on pair production.The result showed that, the production probability–due to the presence of the oscillating field goes like

$$W_g \sim \frac{\alpha_s E^2}{8} \left[\frac{(gE)^2}{4m^2\omega_0^2} \right]^{\frac{2m}{\omega_0}} \tag{3}$$

Since ω_0 is the energy associated with each quanta of the oscillating gluonic field, so m divided by ω_0 is the number of gluon quanta required to produce a pair, hence it is called multi-gluon ionization process. This result is valid even when $E \ll E_c$ and dominates over that of the tunneling rate. A comparison with the perturbative approach also established [7] that for constituent quark mass the multi Gluon ionisation process is dominant over the other two. The effect of time variation of the electric field was also recently considered in [8], for an exponentially decaying Electric filed(Decay due to depletion of the electric field caused by particle production)and the conclusion was, time dependent field increases the production rate. We will discuss this in an appropriate section. Before closing the discussion on nonperturbative production mechanisms its worth mentioning that using similar techniques, one can estimate the decay rate for Gluons too and for an constant external electric field,the rate was found to be [5] similar to the quark case with mass set to equal to zero and a different color factor sitting in front. Few other interesting extensions of this model have been performed in[6] where the effects of finite size correction and the effect of a confinning potential have been studied separately.

2 Hard and semi Hard processes

Having outlined the non-perturbative production mechanisms, we will discuss here the perturbative mechanisms, responsible for the production of quarks and gluons at a higher energy scale. The idea here is, right after collision the coherent distribution of the partons over the nuclei gets disrupted, as the valance partons of each individual nuclei starts colliding with that of the other–producing an incoherent bunch of partonic shower that–decays either forming space like or time like cascades, producing a mini-jetty enviornment . The important input in this theory comes from the nucleonic structure functions measured or predicted for different values of the fractional longitudinal momentum that the colliding parton in question is carrying, as known in the literature as Bjorken x and the energy scale of collision Q^2. The quantity of interest in this model is the jet cross section defined in terms of the double differential cross section for parton parton collisions[9], as,

$$\frac{d\ \sigma_{jet}}{d\ p_T^2 d\ y_1 d\ y_2} = K\Sigma_{a,b} x_1 f_a(x_1, p_T^2) x_2 f_b(x_2, p_T^2) \frac{d\ \sigma_{jet}}{dt} \tag{4}$$

Here the subscripts a and b refer to the different species of partons. The Bjorken variable x_1 and x_2 denotes the longitudinal momentum fraction carried by the partons of respective nucleon with f_a and f_b as their structure

functions. Also $\frac{d\ \sigma_{jet}}{dt}$ is the double-differential cross section associated with partons of type a and b. The energy density is estimated from eqn.(4) by using the relation;

$$\epsilon_h = \frac{d\ E_T^{AA}}{d\ Y \pi R_A^2 \tau_h} = \frac{T_{AA}(b)}{\pi R_A^2 \tau_h} \sigma_{jet} \langle E_T^{pp} \rangle \tag{5}$$

With $T_{AA}(b)$ as the impact parameter and $\sigma_{jet}\langle E_T^{pp}\rangle$ energy moment of the cross section. The parton parton subprocess cross section used here has a pathological divergence, that goes as $\sim {p_t}^{-4}$. In order to get rid of this situation a "saturation criteria" [10] is usually invoked, that gives $p_0 \simeq 2GeV$ for the LHC and $p_0 \simeq 1GeV$ for RHIC for Pb on Pb collision at zero impact parameter. This model predicts that the number density of gluons will out number the quarks.The mean transverse energy of gluons found to be ∼3 GeV with the initial temperature ∼ 1.1 GeV.

2.1 McLerran Venugopalan Model

Other than these two complementary processes of QGP production at RHIC and LHC energy scales, there also exists a third scenario that recently was pointed out by L.McLarren and Raju VenuGopalan in a series of papers [11], where it was argued that there is also another source of mid rapidity gluons in the central rapidity region of large "A" nucleus nucleus collision. Their argument was, as the surface density of mean charge squared fluctuation i.e μ^2 goes as, $\mu^2 = 1.1 A^{\frac{1}{3}} fm^{-2}$ then for a sufficiently large A nucleus, $\Lambda_{QCD}^2 \ll \mu^2$ such that $\alpha_s(\mu^2) \ll 1$ so that the method of weak coupling expansion would still be valid.In this scenario the soft gluon fields produced by the color charges is classical and can be obtained by solving the classical Y.M equations and from there they compute the distribution function given by,

$$\frac{d\ N}{d\ Y d^2 k_T^2 \tau_h} = \frac{1}{\pi R_A^2} (C_F N_q + C_A N_g)^2 \frac{2 g^6 N_c}{(2\pi)^4} \frac{1}{k^4} L(k_T, \lambda) \tag{6}$$

where

$$L(k_T, \lambda) \sim ln \left(\frac{k_T^2}{\lambda^2} \right) \tag{7}$$

where λ is a cutoff scale associated with dynamical screening effects in dense partonic medium. In the appropriate kinematical domain the predictions of this model has been matched with that of pQCD results [12]. There has also been some effort to calculate the same using the coherent state description instead of the classical radiation formula, and they were shown to agree with each other [13]

3 Space Time Evolution

Once there are sufficient number of partons in the system, they will interact with each other and undergo a space time evolution.This space time evolution of the QCD plasma, should be described by a gauge covariant transport equations.Following ([14]) one can define the gluon and quark distribution functions as.

$$G_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-\frac{1}{2}\mathrm{y.D}(x)} \vec{F}^{\lambda}_{\mu}(x) \right] \left[e^{\frac{1}{2}\mathrm{y.D}(x)} \vec{F}_{\lambda\nu}(x) \right]^{\dagger}, \quad (8)$$

$$W(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-\frac{1}{2}\mathrm{y.D}(x)} \overline{\Psi(x)} \right] \left[e^{\frac{1}{2}\mathrm{y.D}(x)} \Psi(x) \right], \quad (9)$$

And use the equations of motion to arrive at the gauge covariant kinetic equations of Elze, Gyulassy and Vasak.From there one can arrive at the classical non abelian transport equation, after taking an ensemble average of the operator valued equation and then setting terms of $O(\hbar = 0)$:,

$$p^{\mu} D_{\mu} W(x,p) + g/2 p^{\mu} \partial_p{}^{\nu} \left[F_{\mu\nu}, W(x,p) \right]_+ = 0 \quad (10)$$

Here $[,]_+$ means anticommutator. The distribution function W(x,p) , is a hermitian matrix in color space and for the SU(2) case can be written in terms of a color singlet and triplet components as,

$$W(x,p) = \frac{1}{2}\langle G \rangle 1 + \frac{1}{2}\sum_{a=1}^{3} \lambda_a \langle G^a \rangle \quad (11)$$

Here $\langle G \rangle = TrW(x,p)$ and $\langle G_a \rangle = Tr\left[\lambda_a W(x,p)\right]$. Using Eq. (11) one can write Eq. (10) in terms of a set of coupled partial differential equations. The coupling is between the singlet and triplet distribution function for quarks. They are as follows,

$$\begin{aligned} p_{\mu}\partial^{\mu}\langle G \rangle + g p_{\mu} \partial_p{}^{\nu} \left[F_{\mu\nu}{}^{a} \langle G^a \rangle \right] &= 0 \\ p_{\mu}\partial^{\mu}\langle G^a \rangle + g \epsilon_{abc} p^{\mu} A^{b}{}_{\mu} \langle G^c \rangle + g p_{\mu} \partial_p{}^{\nu} \left[2 F_{\mu\nu}{}^{a} \langle G^a \rangle \right] &= 0 \end{aligned} \quad (12)$$

To take care of the the effect of collisions along with non perturbative production mechanism one needs to put a source and a collision term on the right hand side of this equation.

$$p^{\mu} D_{\mu} W(x,p) + g/2 p^{\mu} \partial_p{}^{\nu} \left[F_{\mu\nu}, W(x,p) \right]_+ = S_{ource} + C_{ollision} \quad (13)$$

Lots of studies have been performed in the past (see for instance references [15] to [17]) to investigate the space time evolution of the plasma including the effects of source and the collision term or just the source.The popular form of the source used in the colorflux tube model is given as,

$$S_{ource} = \sqrt{\left[E^a(\tau) E_a(\tau) \right]} \times ln \left[1 - e^{\left[-\pi \frac{m^2 + p_T^2}{\sqrt{E_a(\tau) E^a(\tau)}} \right]} \right] \delta(p) \quad (14)$$

And the collision term is usually written [17] with the help of the equilibrium distribution function $W_0(p)$ and the mean collision time τ_0 as:,

$$C_{ollision} = -\frac{W(x,p,t) - W_0(p)}{\tau_0} \tag{15}$$

In all these investigations it was observed that due to production of the particles the external field shows an exponentially damped oscillation in proper time. From the numerical solution of those equations one could see the feature of conversion of fields to particles and vice versa. To take care of this time varying external field, an artificial time dependence is introduced in the (Schwinger inspired) source term. Since in an exponentially decaying external field, the tunneling picture of pair production is not appropriate, the authors of [18] taking this into account derived a modified source term,using perturbation theory (with a decay constant of 0.1 fm). On the other hand if the produced particle current is strong, it would react back on the system. To take care of the effect of quantum back reaction by the particle current a series of investigations were performed by F.cooper,E. Mottola et al. ([19]). And there it was pointed out, that the incorporation of the Pauli blocking in the Schwinger inspired source term almost reproduces the exact field theoretical treatment. The pauli blocked source term they choose was of the form

$$S_{ource} = [1 - 2W(x,p,t)]\sqrt{[E^a(\tau)E_a(\tau)]} \times ln\left[1 - e^{\left[-\pi\frac{m^2+p_T^2}{\sqrt{E_a(\tau)E^a(\tau)}}\right]}\right]\delta(p) \tag{16}$$

The result of their [19] exact calculation is shown in Figure 1. and 2. where the solid line represents the result of the exact field theoretic calculation and the dotted line shows the result of incorporating a Pauli-Blocked Source term (Eq. (16))to the Boltzman Vlasov equation. The half life of the field can

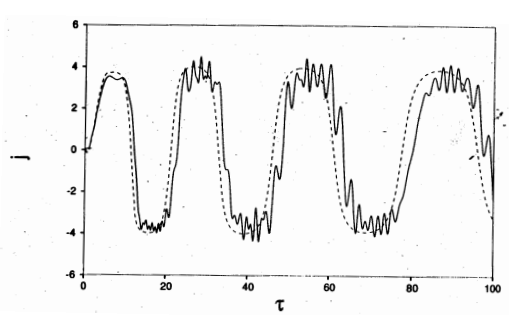

Figure 1: Current vs time

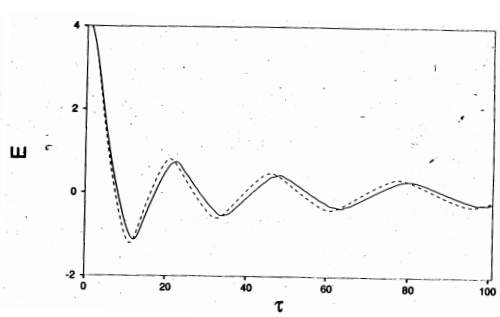

Figure 2: Electric Field vs time

be calculated from Fig. 2, and the value (for a constituent quark mass) comes close to that estimated in ([21]), i.e $\sim$ 4 to 5 fm. On the other hand for current quark mass, the same number comes out to be 240fm! After comparing these numbers it seems that the use of constituent quark mass is more reasonable for this model than the current quark mass. The current in fig 1 is seen to execute an oscillation in proper-time. The origin of this strong oscillation is the perhaps due to the fact that, in presence of an external field the negative and positively charged particles tends to move away in opposite directions hence creating a field with an opposite polarity. As the strength of this field, overcomes that of the external field these charges moves in reverse direction and this prosess keeps on continuing generating an oscillation in time. The peak initial temperature attained, for constituent quark masses,in this model seems to be around 800 MeV, i.e well above the critical energy density required to produce the plasma.

3.1 Tunneling At Finite Temperature

In the previous section we have discussed how the field decays with time. We also had noted from the study of Cooper et. al [19], that by the time the field has decayed to $\frac{1}{e}$th fraction of its original value, the system has attained some temperature. Though this temperature is an oscillating function of proper time, still the time scale of oscillation is much larger that the time scale of production. In ref.([21]),it was argued from the respective time scale analysis of the relevant processes e.g production, depletion and thermalisation that, by the time the system thermalises, some residual mean field will still be present there in the system. The existence of such mean field for a fermionic as well as a bosonic system has also been noted in the studies of Blaizot and Iancu [20]. To see the effect of temperature, on Schwinger's tunneling mechanism, an investigation was carried out in [21], assuming the external filed to be constant and homogeneous every where; the effect of finite size corrections was also taken into account in [22].For cyllindricaly symmetric system of radius R, the tunneling rate at a distance ρ from the axis was found out to be:,

$$
\begin{aligned}
Im \quad F = & \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{gE}{n\pi}\right)^2\sum_{p=1}^{\infty}\left[e^{\frac{-m^2\ n\pi}{gE}-\frac{n\pi[\pi(p)+3\pi/4]^2}{gER^2}}\left(\frac{2R}{(p+3/4)\pi^2\rho}\times\right.\right. \\
& \left.\left. Cos^2\left[\frac{\rho}{R}\left(p+\frac{3}{4}\right)\pi-\frac{\pi}{4}\right]\right)\right]-\frac{1}{2\pi}\sum_{n=1}^{\infty}(-1)^n\sum_{p=1}^{\infty}\frac{2R}{\pi^2\rho\left(p+\frac{3}{4}\right)}\times \\
& Cos^2\left[\frac{\rho}{R}\left(p+\frac{3}{4}\right)\pi-\frac{\pi}{4}\right]p.v\int_0^{\infty}\frac{ds}{s^3}\left((gES)Cot(gES)Sin\frac{n^2\beta^2gE}{4}\right) \\
& e^{-sm^2-n^2\beta^2/4s}e^{-\frac{s\left[\pi p+\frac{3}{4}\pi\right]^2}{R^2}}cosh\left(ng\beta\bar{A}_o\right)+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^n\sum_{p=1}^{\infty}\frac{2R}{\pi^2\rho\left(p+\frac{3}{4}\right)}
\end{aligned}
$$

$$\times Cos^2\left[\frac{\rho}{R}\left(p+\frac{3}{4}\right)\pi-\frac{\pi}{4}\right]\sum_{\ell=1}^{\infty}\left(\frac{gE}{\ell\pi}\right)^2 cosh\left(ng\beta\bar{A}_o\right)Cos\left(\frac{n^2\beta^2gE}{4}\right)$$

$$e^{-\frac{\pi\ell m^2}{gE}-\frac{n^2\beta^2gE}{4\pi\ell}-\frac{\pi\ell[\pi(p+3/4)]^2}{gER^2}} \tag{17}$$

The Fig. 2a shows, how the rate varies with temperature and Fig.2b shows how it varies as one moves radially outward.

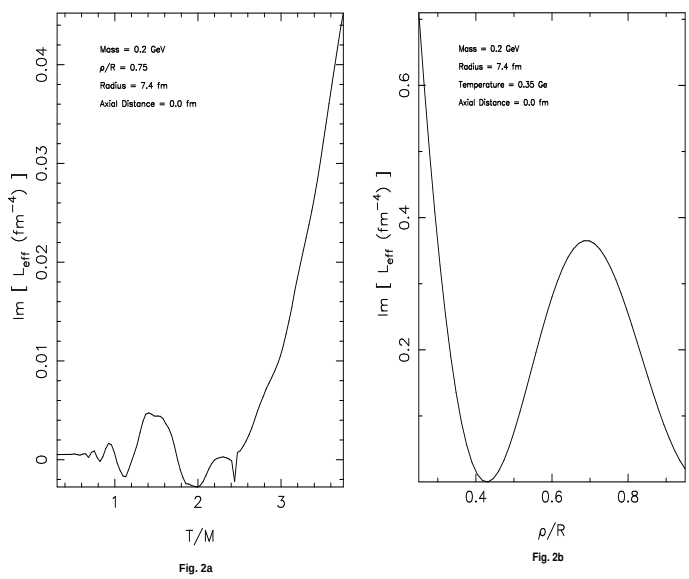

Figure 2: (a) Production rate vs temperature. (b)Production rate vs axial distance

3.2 Color Oscillation

In the limit of very low number density and very weak external field, one can study the collective oscillation of the plasma, by taking the momentum moments of equations (12) and from there generating a set of color hydrodynamic equations. Before we go for generating the hydrodynamic equations, we normalize the 4-velocities for quarks in the following way

$$U^\mu U_\mu = 1 \qquad \text{with} \qquad mU^\mu = \frac{\int p^\mu G_a\left(x,p\right)d^4p}{\int G_a\left(x,p\right)d^4p} = \frac{\int p^\mu G\left(x,p\right)d^4p}{\int G\left(x,p\right)d^4p} \tag{18}$$

After taking the zeroth moment of eqn.(12) we arrive at

$$\partial_\mu\left[G^\mu\right]=0 \quad \text{and} \quad \partial_\mu\left[{G_a}^\mu\right]+g\epsilon_{abc}{A_\mu}^b{G_c}^\mu=0 \tag{19}$$

Following Stewart [24] if one decomposes the 4-vectors $G^\mu = mnU^\mu$ and $G_a{}^\mu = mn_aU^\mu$ one arrives at the mass and color continuity equations,

$$\partial_\mu \left[nU^\mu\right] = 0 \quad \text{and} \quad \partial_\mu \left[U^\mu n_a\right] - g\epsilon_{abc}A_\mu{}^b n_c U^\mu = 0 \tag{20}$$

Further by defining the ratio of the two charge densities as $Q^a = \frac{n_a}{n}$ and using eq.(20) one can obtain the color evolution equation , namely

$$U^\mu \partial_\mu \left[Q_a\right] - g\epsilon_{abc}A_\mu{}^b Q_c U^\mu = 0 \tag{21}$$

Similarly from the first momentum moment of equation (12) one arrives at,

$$U^\mu \partial_\mu U^\nu = \frac{g}{m} F_a{}^{\mu\nu} U_\mu n_a = \frac{g}{m} F_a{}^{\mu\nu} J_\mu{}^a \tag{22}$$

Here repeated indices are summed up and $J^\mu{}_a = nU_\mu Q^a$. [2] These single flavour equations are very similar to the hydrodynamic equations of Kajantie and Montonen [25] and they can easily be generalised to the multiflavor case for quarks.

3.3 Gluon Hydro-dynamical Equations

To derive the gauge covariant transport equations for Gluon one follows, the same procedure as before–starting from eqn(8). Upon using the equation of motion,and setting terms O($\hbar$) to zero one arrives at the classical kinetic equation of the gluons,i.e,

$$p^\mu . D_\mu G_{\mu\nu} + \frac{g}{2} P^\sigma \partial_p^\tau \left[\ \mathcal{F}_{\sigma\tau}, G_{\mu\nu}\ \right]_+ \ = g\left(\mathcal{F}_{\mu\sigma} G^\sigma_\nu - \quad G_{\mu\sigma}\mathcal{F}^\sigma_\nu\right) \tag{23}$$

Here $[,]_+$ means anti-commutator and $D_\mu = \partial_\mu - ig\left[\mathcal{A}_\mu,\right]$ where, $\mathcal{A}^{ab}_\mu \equiv -if_{\text{abc}}A^c_\mu$, $\mathcal{F}^{ab}_{\mu\nu} \equiv -if_{\text{abc}}F^c_{\mu\nu}$and f_{abc} is the antisymmetric structure constant for $SU(2)$ and g is the coupling constant. In order to simplify things a bit further, one can assume the quantity $G_{\mu\nu}$ to be symmetric w.r.t the Lorenz indices, which is technically known as spin equilibration ansatz, to write

$$G_{\mu\nu}(x,p) = p_\mu p_\nu G(x,p), \tag{24}$$

where $G(x,p)$ is a Lorentz scalar, but is a 3×3 matrix in color space. One can check that with the ansatz of spin equilibration, the right hand side of equation[23] vanishes and the Gluon kinetic equation in component (color) notation takes the form :

$$p^\mu \partial_\mu G^{mn} + gp^\mu . A^c_\mu \left[\, f_{\text{cma}} G^{an} - G^{ma} f_{\text{can}}\,\right] + i\frac{g}{2} p^\sigma \partial_p^\tau \left[\, f_{\text{ema}} G^{an} + f_{\text{ean}} G^{ma}\,\right] \times F^e_{\sigma\tau} = 0 \tag{25}$$

[2] By multiplying equation (22) by U^ν one can show $U^\mu U_\mu = constant$;Normalising the U's appropriately Eqn.(18) is satisfied.

In the above equation all repeated indices are summed over. From equation (24) we define the diagonal, antisymmetric and symmetric components as follows :

$$\begin{aligned} G^{11} &= 2G^1, \quad 2S^1 = G^{23} + G^{32}, \quad 2iQ^1 = G^{23} - G^{32} \\ G^{22} &= 2G^2, \quad 2S^2 = G^{31} + G^{13}, \quad 2iQ^2 = G^{31} - G^{13} \\ G^{33} &= 2G^3, \quad 2S^1 = G^{12} + G^{21}, \quad 2iQ^3 = G^{12} - G^{21} \end{aligned} \tag{26}$$

We want to emphasize here that the most general the hydrodynamic equations for Gluons would not be the same as the quarks,it is an artifact of the spin equilibration ansatz that they seem to come out to be the same here. In order to make some more progress now we will further assume that all the symmetric combinations of the distribution function are zero. With this assumption one can show that the equations (25) reduce to :

$$\begin{aligned} p_\mu \partial^\mu Q^1 - gp^\mu \left[A^2_\mu Q_3 - A^3_\mu Q_2\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 \left(G^2 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^2 - gp^\mu \left[A^3_\mu Q_1 - A^1_\mu Q_3\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^2 \left(G^1 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^3 - gp^\mu \left[A^1_\mu Q_2 - A^2_\mu Q_1\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 \left(G^1 + G^2\right)\right] &= 0 \end{aligned} \tag{27}$$

and

$$\begin{aligned} p_\mu \partial^\mu G^1 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \\ p_\mu \partial^\mu G^2 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^1 Q^1\right] &= 0 \\ p_\mu \partial^\mu G^3 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 Q^1 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \end{aligned} \tag{28}$$

The set of equations for studying the hydrodynamic evolution of the plasma can be obtained as in the quark case after taking moments of the one particle distribution function as before with the only exception that for the massless gluons one will have:,

$$U^\mu U_\mu = 0. \tag{29}$$

Instead of going into the details of the derivation, we shall write down here final set of Gluon hydrodynamic equations:, The continuity equations are,

$$\partial_\mu \left[nU^\mu\right] = 0 \qquad U^\mu \partial_\mu \left[Q_a\right] - g\epsilon_{abc} A_\mu{}^b Q_c U^\mu = 0 \tag{30}$$

And the force balance equation is

$$U^\mu \partial_\mu U^\nu = \frac{g}{E} F_a{}^{\mu\nu} U_\mu Q^a \tag{31}$$

For studying the collective behavior of the system one needs to solve Eq.(30) and Eq.(31) along with the Yang Mills equation

$$D_\mu F_a{}^{\mu\nu} = J_a{}^\mu = gnQ_a U^\mu \tag{32}$$

self consistently. The basic idea here is that due to the self interaction the high momentum modes generate a low momentum long wavelength mean field which in turn acts back as a source term for a mean Yang-Mills field equation.In order to study the collective properties one needs to solve these equations self consistently, to describes the proper evolution of the system. We have tried to solve these equations numerically, in 2+1 dimension. Initially susch a study was performed in [26] in the stationary frame of the plasma in (1+1) dimension, and it showed–depending upon the initial conditions— a rich variety of regular as well as chaotic solutions. The motivation of going to one extra dimension was to understand how the system behaves under a small perturbation in one direction with an objective to understand the role of chaotic dynamics in equilibrating the system.

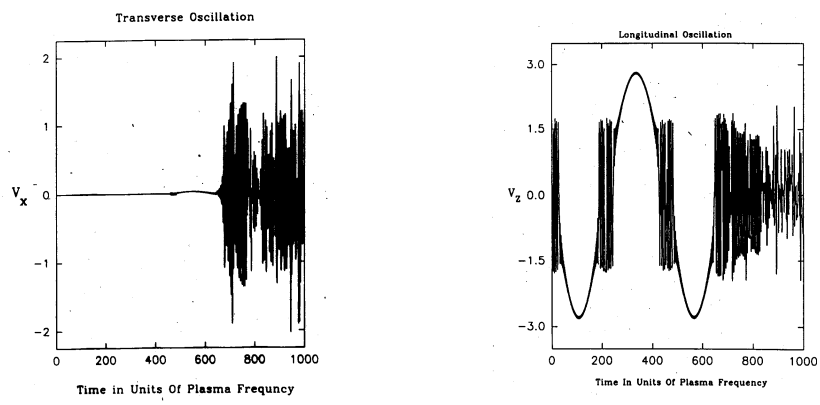

Figure 3: Oscillation Profile:Left panel V_x vs time. Right panel V_z vs time

4 Study of Collective Oscillation of the Plasma

The figures five and six shows the generic effect of a very small transverse perturbation on the otherwise regular longitudinal oscillation. As can be seen from the figures(3a) and (3b) that upto time $= 600\omega_p$ the velocity profiles remain same, but after that there is a catastrophic change in the velocity profile of V_x and simultaneously the coherent oscillation in V_z breaks up into a chaotic one. A fast Fourier transform of these oscillations had shown that the most dominant frequency for both V_x and V_z components are the same, suggesting energy equilibration with the onset of chaotic motion. The most intersting investigation here would be to see if this energy equilibration is also accompanied by color equilibration.There have been some effort in that direction by Gyulassy [27], but we will talk about it in the appropriate section. Having described the role of soft dynamics in RHIC, we'll now briefly introduce the corresponding picture in the still higher energy scale. That is the parton cascade model. As one moves to higher and higher energy, it's believed that the hard processes will dominate and produced partons will screen the long-range

field. Therefore to get the picture of partonic evolution in parton Cascade Model, one needs to modify equation (13), by putting the external field to be equal to zero.Its worth noting that as a result of this, the two coupled partial differential equations decouples from each other and one is left with:,

$$p^{\mu}\partial_{\mu}W\left(x,p,t\right)=C_{ollision} \tag{33}$$

This collision term describes, all the collision-al effects due to $2 \rightarrow 2$ and $2 \rightarrow 1$ and the reverse processes, with appropriate modification of distribution functions due to pauli blocking and bose enhancement, along with virtuality factors to take into account the space like and time like cascading. The pathological low p_t divergence is cured by self consistently introducing a screening mass for the partons.This selfconsistently modified parton cascade model is known as the self screened parton cascade model [28] ,for the sake of brevity we won't discuss it here, the details can be found in [23].

4.1 Discussion and outlook

Having introduced the basic mechanisms and models of parton production in relativistic heavy ion collision, we will go over to the discussion of the prediction's of these models –valid at different energy scales. As before we start from the low energy non-perturbative soft process dominated regime. In the Boltzman Vlasov description with a Schwinger like source term, it appears that the initial temperature at the onset of equilibrium is dependent on the initial field strength that goes into the description. According to refrence [8] for $E_0^2 = 20GeV/fm^3$ the initial temperature at the onset of thermalisation was found to be varying between 225 to 250 MeV. On the other hand according to reference ([19] fig. a constituent quark of mass .5 GeV the initial peak temperature comes out to be around 4.5 Gev, though for current quark mass, the temperature comes out much less. According to reference ([30]) the initial temperature in RHIC (LHC) for gluons are estimated to be 500 MeV (660 MeV) the same for quarks is 200 MeV (260 MeV) and the thermalisation time for Gluons varies from 0.3 to 0.25 fm/c in RHIC and LHC energy scales. For gluons and light quarks the predictions from [30] or [31] see to be close to each other and they suggest that the system undergoes a multistage thermalisation. The quarks are believed to be less equilibrated than gluons[32]. Although for charm or bottom quarks the thermalisation time $\langle$ 2 fm/c. The Initial Conditions as obtained from Self Screened Parton Model [33] for Gold on Gold at BNL RHIC and CERN LHC energies–prior to hydrodynamic expansion is given in Table I.

Table I.

Self Screened Parton Cascade	τ_i(fm/c)	T_i	ϵ_i (GeV/fm^3)
RHIC	0.25	0.668	61.4
LHC	0.25	1.02	425

Coming back to the question of color equilibration, though according to [27], color equilibrates takes place before the system reaches thermal equilibration, but it would be nice to demonstrate this result using the color kinetic equations, retaining their full nonlinearity. It is generally felt that such an investigation would shed light on the question, whether the onset of chaotic oscillation has really got some thing to do with the equilibration procedure or not. And lastly it's worth mentioning that all the previous studies reveal that in RHIC the system would not come chemical equilibrium though at LHC energy scale it might get very close to it.

5 Acknowledgement

The author would like to thank Prof. B.C Sinha for providing him with the local hospitality of SINP Calcutta–during which a part of this work was done. He also would like to thank Dinesh K. Srivastava and Rajiv Bhalerao for numerous illuminating communications on various issues related to this work. Lastly I would like to thank S.Lal for his invaluable help while preparing the manuscript.

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