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Eikonalised minijet model analysis of $\sigma_{\gamma\gamma}^{inel}$.

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Abstract

We study the theoretical predictions for the total inelastic $\gamma\gamma$ cross-sections, with an emphasis on the eikonalised minijet model (EMM). In the context of the EMM, we discuss a new ansatz for the overlap function involving the photons. We discuss the dependence of the EMM predictions on various input parameters as well as predictions for $\sigma_{\gamma\gamma}^{\rm inel}$ from a simple extension of the Regge Pomeron Exchange model. We then compare both with the recent LEP data.

The measurement of the total photoproduction cross-sections at HERA [1, 2] and the recent measurements of the hadronic $\gamma\gamma$ cross-sections at LEP [3, 4], have established that all total cross-sections, involving hadrons [5, 6] as well as photons, rise as the c.m. energy of the colliding particles increases. The similarity in the energy dependence of *all* total cross-sections, suitably scaled to take into account the difference between hadrons and a photon, is striking. Fig.(1) demonstrates this. In the figure the cross-sections for photon-induced processes have been multiplied by factors motivated by simple quark-model considerations and the probability for a photon to fluctuate in a $q\bar{q}$ pair $P_\gamma^{\rm had}$, so as to facilitate comparison with pp and $\bar{p}p$ data. We will discuss the choice of $P_\gamma^{\rm had}$ later. In the light of the new $\gamma\gamma$ data the impetus

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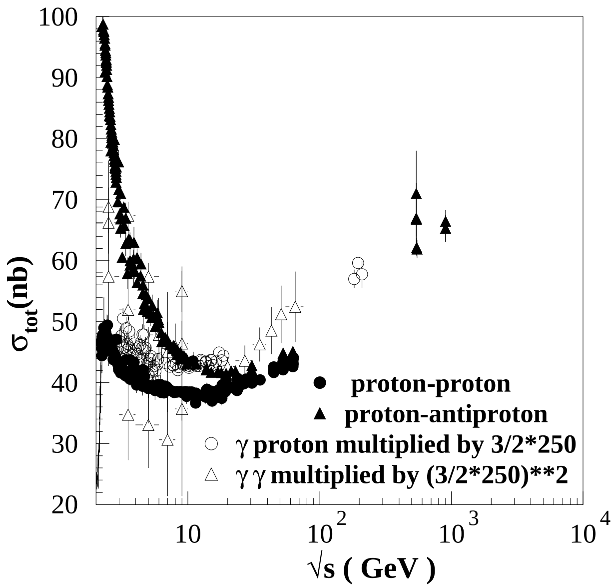


Figure 1: Energy dependence of $\sigma_{ab}^{\rm tot}$

to arrive at a unified description of the energy dependence of the total cross-sections, independent of whether the beam is a hadron or a photon, in terms of a model based on perturbative QCD and the measured parton content of the photon/hadrons, is now ever stronger. The problem of hadronic $\gamma\gamma$ cross-sections at high energy [7, 8] has an additional significance in view of the large potential backgrounds that Beamstrahlung photons could cause at future Linear Colliders [9]. Indeed this was one of the original motivations of this work. The other was to discuss in detail how different input parameters influence the models used to compare with the data. A preliminary version of our results along with the then available high energy data, was presented at PHOTON-97 [10]. It is evident from the most recent literature on $\sigma_{\gamma\gamma}^{\rm had}$ that this issue is still very much debated : the data from the OPAL collaboration and L3 are consistent with each other within errors, but the models which describe the central values of one or the other of the two sets may be rather different. In this paper we shall compare the recent LEP data with the mini-jet model, trying to pinpoint the uncertainties of the model. We shall present elsewhere the detailed analysis resulting from varying all the parameters, here we shall limit ourselves to a summary of such investigation.

The issue of energy dependence of the total cross-section for high energy particle scattering, is not a new one. As a matter of fact more than one model exist which try to explain the observed energy dependence. One successful description of total cross-sections is obtained in the Regge/Pomeron model [11], in which t-channel exchanges in the elastic scattering amplitude lead to the expression

$$\sigma_{ab}^{tot} = Y_{ab}s^{-\eta} + X_{ab}s^{\epsilon}$$

where η and ϵ are related to the intercept at zero of the leading Regge trajectory and of the Pomeron, respectively $\eta \approx 0.5$ and $\epsilon \approx 0.08$. This parametrization applies successfully [11] to photoproduction and is consistent within errors with the lower energy data on $\gamma\gamma$ [7, 12, 13, 14, 15]. Since the lower energy data were characterized by large errors, this left wide margin [16, 17] to predictions about where and by how much the cross-section rises. However, they provided the first check of the hypothesis of factorization of the residues at the poles in the Regge description of elastic and total cross-sections. In fact, assuming the hypothesis of factorization, one can make a prediction for $\gamma\gamma$ total cross-section using

$$Y_{ab}^2 = Y_{aa}Y_{bb} \qquad X_{ab}^2 = X_{aa}X_{bb}$$

and extracting the coefficients X and Y from those for the fit to photoproduction and hadron-hadron data.

An alternative description of total cross-sections is given in terms of semi-hard QCD interactions, called minijet interactions, among the partons of the colliding hadrons with partonic momenta of the order of 1-2 GeV. It was proposed [18] that the rise of $\sigma^{\rm tot}$ with energy is driven by the increase with energy of the number of hard interactions among the proton constituents, i.e., in contemporary language, by the increase with energy of the inclusive productions for these minijets [19]. The lack of unitarity of the original model [18, 19] was cured by the introduction of the eikonalized mini-jet models [20]. These models have an immediate semi-classical derivation, when applied to the semi-hard QCD contribution to σ^{inel}. The eikonalized expression can in fact be obtained semi-classically by considering the number of collisions at fixed impact parameter b. Under the assumption that the collisions are independent of each other at any fixed value of b, one can assume that the distribution of r collisions around their average $n(b,s)$ follow a Poisson distribution $\mathcal{P}(r,n)$ so that the sum over all possible collisions becomes [21]

$$\sigma^{inel}_{QCD} = \int d^2\vec{b} \sum_{r=1}^{\infty} \mathcal{P}(r,n) = \int d^2\vec{b} \left[1 - e^{-n(b,s)}\right] \tag{1}$$

Thus a constant average will give a constant cross-section, increasing number of collisions will produce rising cross-sections.The QCD mini-jet model ascribes to QCD the task of calculating the average number of semi-hard collision, identifying $n(b,s)$ with the total jet cross-section times a suitable function, which is responsible for the b-dependence. Since the jet cross-section depends sensitively on the lowest transverse momentum, p_{tmin}, in the p_t-integration, the calculation is affected by the uncertainty due to the choice of p_{tmin}. For purely hadronic scattering, the other large uncertainty in the mini-jet model is the b-dependence of the partons in the proton, usually described through the Fourier transform of the electromagnetic form factors of the hadrons. In order to describe the rising proton-proton and proton-antiprotron cross-section, and to incorporate the fact that the hadronic structure of all particles, photon included, involves both a perturbative and non-perturbative part with an energy dependent relative weight, the mini-jet model has to be supplemented by the introduction of a non perturbative term in $n(b,s)$, thus introducing further parameters, like a different shape of the b-distribution as the energy increases or even the introduction of a sum of eikonalized functions [7, 26] instead of a single one. The more the terms, the more the parameters that have to be introduced. Since our aim here is to investigate the minimal parameter dependence of the mini-jet model, we restrict ourselves to only one term.

Therefore, apart from the assumption of one or more eikonals, the predictions of the eikonalised mini-jet model in general will depend on 1) the hard jet cross-section $\sigma_{jet} = \int_{p_{tmin}} \frac{d^2\hat{\sigma}}{dp_t^2} dp_t^2$ which in turn depends on the minimum p_t above which one can expect perturbative QCD to hold viz. p_{tmin}, and on the parton densities in the colliding particles a and b, 2) the soft cross–section $\sigma^{\rm soft}_{ab}$ to be introduced to describe the non perturbative region, 3) the overlap function $A_{ab}(b)$ in the $b-$integration. For photons, the model has apparently one more uncertainty, as it has to incorporate [22] the hadronisation probability for the photon to fluctuate itself into a hadronic state, P^{had}_{γ}. For photon induced processes then the eikonalisation leads to

$$\sigma^{inel}_{ab} = P^{had}_{ab} \int d^2\vec{b} [1 - e^{n(b,s)}], \tag{2}$$

with the average number of collisions at a given impact parameter $\vec{b}$ given by

$$n(b,s) = A_{ab}(b)(\sigma_{ab}^{\rm soft} + \frac{1}{P_{ab}^{\rm had}}\sigma_{ab}^{jet}) \tag{3}$$

where P_{ab}^{had} is the probability that the colliding particles a, b are both in a hadronic state, $A_{ab}(b)$ describes the transverse overlap of the partons in the two particles normalised to 1, $\sigma_{ab}^{\rm soft}$ is the non-perturbative part of the cross-section while σ_{ab}^{jet} is the hard part of the cross–section (of order $\alpha_{\rm em}$ or $\alpha_{\rm em}^2$ for γp and $\gamma\gamma$ respectively). Notice that, in the above definitions, $\sigma^{\rm soft}$ is a cross-section of hadronic size since the factor P_{ab}^{had} has already been factored out. Letting

$$P_{\gamma p}^{had} = P_{\gamma}^{had} \quad and \quad P_{\gamma\gamma}^{had} \approx (P_{\gamma}^{had})^2, \tag{4}$$

one can extrapolate the model from photoproduction to photon-photon collisions. Admittedly, this procedure is very simplistic, as the probability P_{γ}^{had} is certainly energy dependent. However, as in the case of the Regge-Pomeron model, this approximation, i.e. eq.(4), allows for a check of factorization ansatz. As for the energy dependence, a scaling property of the eikonal model allows us to include it into $A_{ab}(b)$. Indeed, by a simple change of variables, it is easy to see [23] that one can rewrite the above expression in such a way that mathematically the expression for the cross-section in eq.(2) depends only on the combination $A_{ab}/P_{ab}^{\rm had}$. This helps us reduce the number of parameters which identify one given process and its energy dependence. Thus the above scaling property for $A_{ab}(b)/P_{ab}^{had}$ implies that we can fix a value for P_{ab}^{had} at a given energy and vary only $A_{ab}(b)$.

As we said, in order to clarify the limitations and parameter dependence of this model, we restrict ourselves to a single eikonal. The hard jet cross-sections are calculated in LO perturbative QCD and use photonic parton densities GRV [24] calculated to the leading order as well as SaS densities, mode 1 [25]. We determine $\sigma_{\gamma\gamma}^{\rm soft}$ from $\sigma_{\gamma p}^{\rm soft}$ which in turn is determined by a fit to the photoproduction data. From inspection of the photoproduction data, one can assume that $\sigma^{\rm soft}$ should contain both a constant and a term which falls with energy. Following the suggestion[26]

$$\sigma_{\gamma p}^{\rm soft} = \sigma_{\gamma p}^0 + \frac{\mathcal{A}_{\gamma p}}{\sqrt{s}} + \frac{\mathcal{B}_{\gamma p}}{s}, \tag{5}$$

we then calculate values for $\sigma_{\gamma p}^0, \mathcal{A}_{\gamma p}$ and $\mathcal{B}_{\gamma p}$ from a best fit [27] to the low energy photoproduction data, starting with the Quark Parton Model (QPM) ansatz $\sigma^0_{\gamma p} \approx \frac{2}{3}\sigma^0_{pp}$, where σ^0_{pp} is the constant term in analogous eikonal type fits to proton-proton scattering. For $\gamma\gamma$ collisions, we shall repeat the QPM suggestion and propose

$$\sigma^{\rm soft}_{\gamma\gamma} = \frac{2}{3}\sigma^{\rm soft}_{\gamma p}. \tag{6}$$

Most of the discussion about the mini-jet model predictions for the total inelastic cross-section has so far centered on the uncertainties generated by the choice of p_{tmin} as far as the QCD calculation is concerned, and on $\sigma_{\rm soft}$ for the non-perturbative part, both in the case of photoproduction and $\gamma\gamma$ cross-sections [7, 26, 28]. On the other hand, the effect of $A_{ab}(b)$ and P^{had}_{ab} on the overall validity of this description has not really been assessed. The real emphasis of our work is to discuss this point. In this paper, we shall analyze the simplest and oldest ansatz concerning the overlap function $A_{ab}(b)$

$$A_{ab}(b) = \frac{1}{(2\pi)^2}\int d^2\vec{q}\mathcal{F}_a(q)\mathcal{F}_b(q)e^{i\vec{q}\cdot\vec{b}}, \tag{7}$$

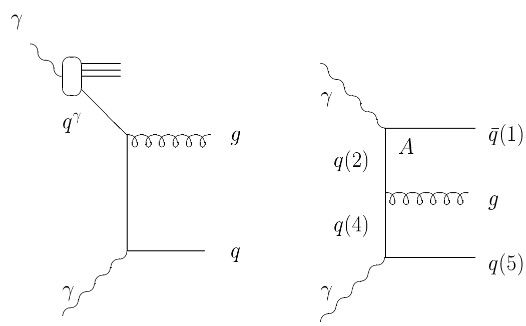
where $\mathcal{F}$ is the Fourier transform of the b-distribution of partons in the colliding particles and can be obtained using for $\mathcal{F}$ the electromagnetic form factors of the colliding hadrons. For protons this is given by the dipole expression

$$\mathcal{F}_{prot}(q) = [\frac{\nu^2}{q^2+\nu^2}]^2, \tag{8}$$

with $\nu^2 = 0.71$ GeV^2. For photons a number of authors [26, 29], on the basis of Vector Meson Dominance, have assumed the same functional form as for pion, i.e. the pole expression

$$\mathcal{F}_{pion}(q) = \frac{k_0^2}{q^2+k_0^2}, \tag{9}$$

with $k_0 = 0.735$ GeV from the measured pion form factors, changing the value of the scale parameter k_0, if necessary in order to fit the data. We would like to adopt here a different philosophy, i.e. that the b-space distribution of partons is the Fourier transform of the transverse momentum distribution of the colliding system [30]. To leading order, this transverse momentum distribution can be entirely due to an intrinsic transverse momentum of partons in the parent hadron, but while the intrinsic transverse momentum (k_T) distribution of partons in a proton is normally taken to be Gaussian, a choice



(a) (b)

Subprocess $q^\gamma + \gamma \to g + q$

Figure 2: A single resolved process and corresponding perturbative diagram.

which can be justified in QCD based models [31], in the case of photon the origin of all partons can, in principle, be traced back to the hard vertex $\gamma\ q\bar{q}$. Therefore, also in the intrinsic transverse momentum philosophy, one can expect the k_T distribution of photonic partons to be different from that of the partons in the proton. The expected functional dependence can be deduced using the origin of photonic partons from the $\gamma \to q\bar{q}$ splitting. We present below a discussion of the same and then proceed to assess the effect of this ansatz for $A_{ab}(b)$.

To do this consider a single resolved diagram (say) for $\gamma\gamma \to 2$ hard jets given below in Fig.(2.a). Perturbatively the same diagram can be drawn as shown in Fig.(2.b). The quark (2) in this figure is 'almost' on shell and that is why it can be looked upon as q^γ in Fig.(2.a). Due to the hard $\gamma\bar{q}(1)q(2)$ vertex at 'A' in Fig.(2.b) $\bar{q}(1)$ and hence $q(2)$ can appear with sizable transverse momentum. Hence in the structure function language one can talk of an intrinsic transverse momentum (k_T) of the photonic partons. Assuming $\bar{q}(1)$ to be on shell the 4-mmta. of the three particles $\gamma, \bar{q}(1)$ and $q(2)$ are

$$\gamma = E_\gamma(1,0,0,1); \tag{10}$$

$$1 = (E_\gamma(1-x_\gamma), 0,\ k_T, \sqrt{{E_\gamma}^2(1-x_\gamma)^2 - k_T^2}); \tag{11}$$

$$2 = (E_\gamma x_\gamma, 0, -\ k_T, E_\gamma - \sqrt{{E_\gamma}^2(1-x_\gamma)^2 - k_T^2}). \tag{12}$$

The virtuality of parton '2' is

$$(2)^2 \equiv t(2) = 2{E_\gamma}^2(1-x_\gamma)\left[-1+\sqrt{1-\frac{k_T^2}{{E_\gamma}^2(1-x_\gamma)^2}}\right]. \tag{13}$$

The structure function language makes sense when k_T is small. Then expanding the root we get

$$t(2) = -\frac{k_T^2}{(1-x_\gamma)}. \tag{14}$$

The dominant part of the perturbative diagram in Fig.(2.b) which is approximated by the single resolved diagram in the structure function language, is given by small values of $t(2)$, as the cross-section (in the leading log approximation) is

$$\frac{d\sigma}{dt(2)} \propto \frac{1}{t(2)} \tag{15}$$

This immediately tells us that

$$\frac{dq^\gamma}{dk_T^2} \propto \frac{1}{k_T^2} \tag{16}$$

Of course the distribution has to be regularised. One expects the nonperturbative effects to keep quark (2) always (slightly) off mass shell. Phenomenologically this would imply an intrinsic k_T distribution given by

$$f(\ k_T) = \frac{C}{(k_T^2 + k_0^2)}$$

Finally, we have to choose the normalisation C such that the eventual $A_{ab}(b)$ satisfies

$$\int d^2\vec{b} A_{ab}(\vec{b}) = 1.$$

and this gives

$$f(k_T) = \frac{1}{2\pi}\frac{k_0^2}{(k_T^2 + k_0^2)}.$$

To summarize, while for the proton the transverse momentum distribution model for $A_{ab}(b)$ would correspond to use of a Gaussian distribution instead of the dipole expression of eq.(8), for the photon one can argue that the intrinsic transverse momentum ansätze [32] would imply the use of a different value of the parameter k_0, which is extracted from data involving 'resolved' photon interactions [33], in the pole expression for the form factor. By varying k_0 one can then explore various possibilities, i.e. the VMD/pion hypothesis if $k_0 = 0.735$ GeV, or the intrinsic transverse distribution case for other values of k_0. Still another possibility, not in contradiction with the above, is that $A_{ab}(b)$ is the Fourier transform of the transverse momentum distribution of the initial collinear parton pair, to be evaluated using soft gluon summation techniques [30]. However, in this letter we limit ourselves to models with the intrinsic transverse momentum ansatz for the photon. Assuming the functional expression described above, the overlap function for $A_{ab}(b)$ becomes

$$A_{\gamma\gamma}(b) = \frac{1}{4\pi} k_0^3 b \mathcal{K}_1(bk_0) \tag{17}$$

To show the dependence of $A_{\gamma\gamma}(b)$ from the scale parameter k_0 (which in principle could be energy dependent, we stress again), we plot in Fig.(3) the funtion $A_{ab}(b)$ for $\gamma\gamma$ scattering for various values of k_0. Notice that the region most important to this calculation is for large values of the parameter b, after the overlap function changes trend, and where fall with b is faster for larger k_0 values. This is in agreement with the intuitive idea that higher k_0 values correspond to hardening of the scale of the subprocesses.

As for P_γ^{had}, which is clearly expected to be $\mathcal{O}(\alpha_{\rm em})$, VMD prescriptions would suggest

$$P_\gamma^{had} = P_{VMD} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha_{\rm em}}{f_V^2} \approx \frac{1}{250} \tag{18}$$

We shall fix the value in such a way as to obtain a good fit to the photoproduction data and then use factorization for the comparison with the $\gamma\gamma$ cross-section. This corresponds to a value [29] of 1/204, which includes a non-VMD contribution of $\approx 20\%$. We shall not discuss this point at length, since for any given value, P_{had} can be absorbed into a redefinition of the scale parameters k_0 and ν through a simple change of variables [23]. If P_γ^{had} were to be energy dependent, this would then result into energy dependence of the scale parameters.

After this rather general introduction, whose aim was to establish the range of variability of the quantities involved in the mini-jet calculation of

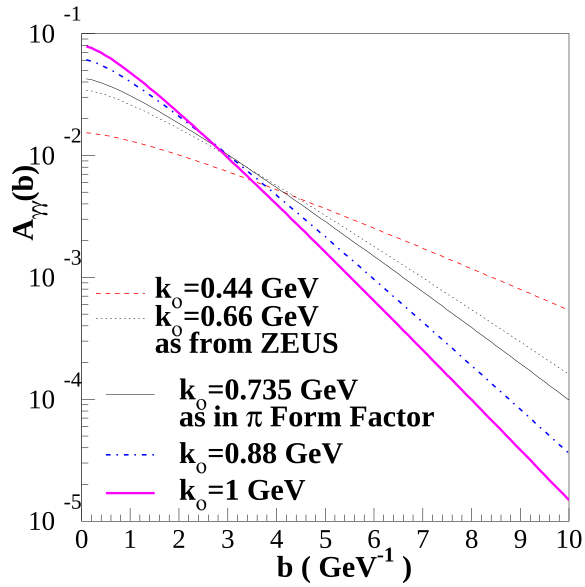


Figure 3: The overlap function for $\gamma\gamma$ collisions according to the two different models discussed in the text. The value $k_0 = 0.735$ GeV corresponds to using the pion form factor for the photon overlap function. The other values correspond to the experimental value given by the ZEUS Collaboration: 0.66 ± 0.22 GeV. The value of 1 GeV corresponds approximately to the maximum value at 90% confidence level.

total inelastic photonic cross sections, we shall now present the predictions of the eikonalized minijet model with respect to the presently available data for photon-photon scattering.

We start with the photoproduction cross-section from HERA, using GRV (LO) densities and different values of p_{tmin} for the jet cross-section. For $\sigma_{\rm soft}$ we proceed as described, obtaining two good fits. We choose the set

$$\sigma^0_{\gamma p} = 31.2 \text{ mb}, \mathcal{A}_{\gamma p} = 0.0 \text{ mb} \ , \mathcal{B}_{\gamma p} = 63.1 \text{ mb GeV}^2 \tag{19}$$

We show the mini-jet result in Fig.(4), using the form factor model for $A_{\gamma p}(b)$, i.e. eq.(7) with k_0 modified so as to take into account the intrinsic transverse momentum hypothesis for the photon and the findings by the ZEUS Collaboration [33], i.e. $k_0 = 0.66 \pm 0.22$ GeV. The curves are obtained summing both the resolved and the direct contribution which is order $\alpha_{\rm em}$

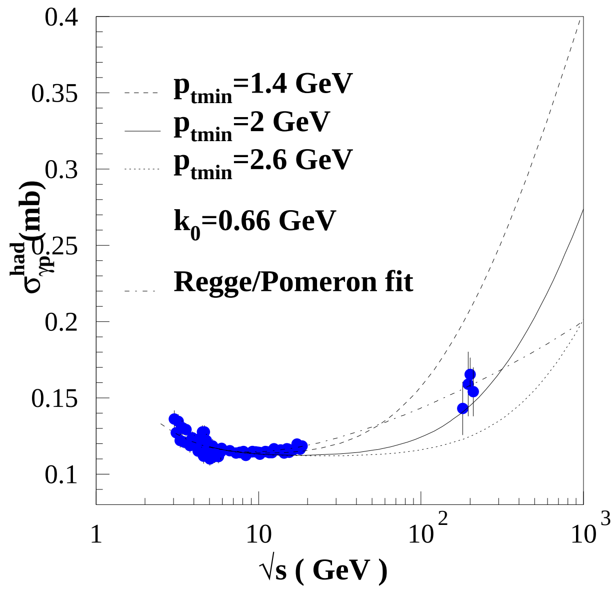


Figure 4: Total inelastic photon-proton cross-section for GRV densities and various p_{tmin} values, compared with data and Regge/Pomeron parametrization [11]

relative to the hadronic term. We observe that it is very difficult with a single eikonal, the same $A_{\gamma p}(b)$ and the same set of parton densities, GRV for the proton as well as for the photon, to fit both the beginning of the rise of the cross-section and the high energy points. If one chooses to fit the high energy rise, then a good choice could be $p_{tmin} = 2$ GeV, but the low energy region would be better described by a smaller p_{tmin}, like $p_{tmin} \leq 1.4$ GeV. This is the most difficult point of the mini-jet model, i.e. whether it is possible to understand both the beginning of the rise as well the rise in the high energy region. Whether this dificulty has to be ascribed to our still rough understanding of the impact parameter description or to non perturbative effects in the transition between σ^{soft} and σ^{jet} or to a combination of both these effects, will be investigated in future papers. Here, for a comparison, we show in Fig.(4) the fit from the Regge/Pomeron exchange model [11].

We now apply the criteria and parameter set used in γp collisions to the case of photon-photon collisions, i.e. $P_\gamma^{had} = 1/204$, $p_{tmin} = 2$ GeV, $A_{\gamma\gamma}(b)$ from eq.(7), and the value $k_0 = 0.66$ GeV for the photon scale parameter.

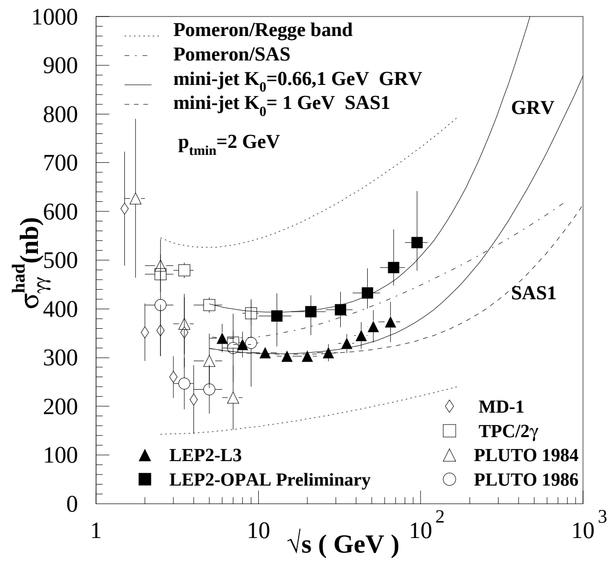


Figure 5: Total inelastic photon-photon cross-section in the eikonalized mini-jet model with $p_{tmin} = 2$ GeV, compared with data [3, 4, 12, 13, 14, 15] and Regge/Pomeron parametrization (see text). L3 data do not include the quoted [3] uncertainties on scale and overall normalization. The two lower mini-jet curves correspond to $k_0 = 1$ GeV with GRV and SAS1 densities. The highest one is for GRV densities and $k_0 = 0.66$ GeV.

We show the results in Fig.(5). Clearly, all the quantities involved in photoproduction appear here squared, so that the possibilty of measuring the photon related parameters in photon-photon scattering is enhanced by the sensitivity of the cross-section to these parameters. To show the dependence of the model upon the uncertainties we have just discussed, we have obtained also fits varying the parton densities and/or k_0. In particular, we can check the intrinsic transverse momentum ansätze through variations in k_0 values. Fits obtained with a value of k_0, of order 1 GeV, in keeping with the hypothesis of a harder scale are also shown. For this value, we plot the result with both GRV and SAS1 parton densities. The highest of the two full lines corresponds exactly to the same parameter set used in the photoproduction case, Fig.(4), and appears, in this model, to be in good agreement with the preliminary results from the OPAL [4] Collaboration, whereas the

L3 results, everything else being the same, would favour a higher k_0 value. Notice however that if one include the errors due to normalization and scale to the ones explicitly indicated in Table 2 of ref. ([3]), then the two sets of data are consistent with each other within one standard deviation.

All the predictions of the minijet model are also compared with predictions (Pomeron/SaS) based on a Pomeron/Regge type parametrization[7], using factorization of the residues as described before. and using for η and ϵ the average values from the Particle Data Group[34], and for X and Y the average between pp and $p\bar{p}$. Finally, the dashed band has been obtained using the errors on X and Y, whereupon the large errors on Y coming from photoproduction are responsible for the large band shown. One notices the same occurrence as in the photoproduction data, i.e. that the curvature with which the cross-sections rise is quite different in mini-jet and Pomeron/Regge models, as observed already in all total cross-sections.

In conclusion, we have compared recent data on photon-photon total cross-section with predictions from both a Regge/Pomeron type parametrization as well as from the mini-jet models. We stress the fact that our mini-jet analysis is based on currently used QCD parton densities and on a clear specification of the variable parameters used. We see here that the data are well described by the minijet model where the same set of parameters affords a acceptable description of both , the photoproduction as well as $\gamma\gamma$ inelastic cross-sections.

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