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Infra-red stable fixed points of R-parity violating Yukawa couplings in supersymmetric models

B. Ananthanarayan,
Centre for Theoretical Studies, Indian Institute of Science,
Bangalore 560 012, India

P. N. Pandita,
Theory Group, Deutsches Elektronen-Synchrotron DESY,
Notkestrasse 85, D 22603 Hamburg, Germany
and
Department of Physics, North-Eastern Hill University,
Shillong 793 022, India[1]

Abstract

We investigate the infra-red stable fixed points of the Yukawa couplings in the minimal version of the supersymmetric standard model with R-parity violation. Retaining only the R-parity violating couplings of higher generations, we analytically study the solutions of the renormalization group equations of these couplings together with the top- and b-quark Yukawa couplings. We show that only the B-violating coupling λ''_{233} approaches a non-trivial infra-red stable fixed point, whereas all other non-trivial fixed point solutions are either unphysical or unstable in the infra-red region. However, this fixed point solution predicts a top-quark Yukawa coupling which is incompatible with the top quark mass for any value of $\tan\beta$.

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[1] Permanent Address

There is considerable interest in the study of infra-red (IR) stable fixed points of the standard model (SM) and its extensions, especially those of the minimal supersymmetric standard model (MSSM). This interest follows from the fact that in the SM (and in the MSSM) there are large number of unknown dimensionless Yukawa couplings, as a consequence of which the fermion masses cannot be predicted. One may attempt to relate the Yukawa couplings to the gauge couplings via the Pendleton-Ross infra-red stable fixed point (IRSFP) for the top-quark Yukawa coupling [1], or via the quasi-fixed point behaviour [2]. The predictive power of the SM and its supersymmetric extensions may, thus, be enhanced if the renormalization group (RG) running of the parameters is dominated by IRSFPs. Typically, these fixed points are for ratios like Yukawa coupling to the gauge coupling, or, in the context of supersymmetric models, the supersymmetry breaking tri-linear A-parameter to the gaugino mass, etc. These ratios do not always attain their fixed points values at the weak scale, the range between the GUT (or Planck) scale and the weak scale being too small for the ratios to closely approach the fixed point. Nevertheless, the couplings may be determined by quasi-fixed point behaviour [2], where the value of the Yukawa coupling at the weak sale is independent of its value at the GUT scale, provided the Yukawa couplings at the unification scale are large. For the fixed point or quasi-fixed point scenarios to be successful, it is necessary that these fixed points be stable [3, 4, 5].

Since supersymmetry [6] necessciates the introduction of superpartners for all known particles in the SM (in addition to the introduction of two Higgs doublets), which transform in an indentical manner under the gauge group, we have additional Yukawa couplings in supersymmetric models which violate [7] baryon number (B) or lepton number (L). In the MSSM a discrete symmetry called R-parity (R_p) is invoked to eliminate these B and L violating Yukawa couplings [8]. However, the assumption of R_p conservation at the level of MSSM appears to be *ad hoc*, since it is not required for the internal consistency of the model. Therefore, the study of MSSM, including R-parity violation, deserves a serious consideration.

Recently attention has been focussed on the study of renormalization group evolution of R_p violating Yukawa couplings of the MSSM [9], and their quasi-fixed points. This has led to certain insights and constraints on the quasi-fixed point behavior of some of the R_p violating Yukawa couplings, involving higher generation indices. We recall that the usefulness of the fixed point and quasi-fixed point scenarios is the existence of *stable* infra-red fixed points. The purpose of this paper is to address the important question of the infra-red fixed points of supersymmetric models with R_p violation, and their stability. Our interest is in the structure of the infra-red stable fixed points, rather than the actual values of the fixed points.

To this end we shall consider the supersymmetric standard model with the minimal particle content and with R_P violation, and refer to it as MSSM with R-parity violation. We begin by recalling some of the basic features of the model. The superpotential of the MSSM is given by

$$W = \mu H_1 H_2 + (h_u)_{ab} Q_L^a \overline{U}_R^b H_2 + (h_d)_{ab} Q_L^a \overline{D}_R^b H_1 + (h_E)_{ab} L_L^a \overline{E}_R^b H_1, \tag{1}$$

to which we add the L and B violating terms

$$W_L = \mu_i L_i H_2 + \frac{1}{2}\lambda_{abc} L_L^a L_L^b \overline{E}_R^c + \lambda'_{abc} L_L^a Q_L^b \overline{D}_R^c, \tag{2}$$

$$W_B = \frac{1}{2}\lambda''_{abc} \overline{D}_R^a \overline{D}_R^b \overline{U}_R^c, \tag{3}$$

respectively, as allowed by gauge invariance and supersymmetry. In Eq. (1), $(h_U)_{ab}$, $(h_D)_{ab}$ and $(h_E)_{ab}$ are the Yukawa coupling matrices, with a, b, c as the generation indices. The Yukawa couplings λ_{abc} and λ''_{abc} are antisymmetric in their first two indices due to $SU(2)_L$ and $SU(3)_C$ group structure. Phenomenological studies of supersymmetric models of this type have placed constraints [10] on the various couplings λ_{abc}, λ'_{abc} and λ''_{abc}, but there is still considerable room left. We note that the simultaneous presence of the terms in Eq. (2) and Eq. (3) is essentially ruled out by the stringent constraints [11] implied by the lack of observation of nucleon decay.

In addition to the dominant third generation Yukawa couplings $h_t \equiv (h_U)_{33}$, $h_b \equiv (h_D)_{33}$ and $h_\tau \equiv (h_E)_{33}$ in the superpotential (1), there are 36 independent R_p violating couplings λ_{abc} and λ'_{abc} in Eq. (2), and 9 independent λ''_{abc} in Eq. (3). Thus, one would have to solve 39 coupled non-linear evolution equations in the L-violating case, and 12 in the B-violating case, in order to study the evolution of the Yukawa couplings in the minimal model with R_p violation. In order to render the Yukawa coupling evolution tractable, we need to make certain plausible simplications. Motivated by the generational hierarchy of the conventional Higgs couplings, we shall assume that an analogous hierarchy amongst the different generations of R_p violating couplings exists. Thus we shall retain only the couplings λ_{233}, λ'_{333} and λ''_{233}, and neglect the rest. We note that the R_p violating couplings to higher generations evolve more strongly because of larger Higgs couplings in their evolution equations, and hence could take larger values than the corresponding couplings to the lighter generations. Furthermore, the experimental upper limits are stronger for the R_p violating Yukawa couplings corresponding to the lighter generations.

We shall first consider the evolution of Yukawa couplings arising from superpotentials (1) and (3), which involve baryon number violation. The one-loop renormalization group equations for h_t, h_b, h_τ and λ''_{233} (all others set to zero) are:

$$\begin{aligned}
16\pi^2 \frac{dh_t}{d(\ln\,\mu)} &= h_t \left(6h_t^2 + h_b^2 + 2\lambda''^2_{233} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right), \\
16\pi^2 \frac{dh_b}{d(\ln\,\mu)} &= h_b \left(h_t^2 + 6h_b^2 + h_\tau^2 + 2\lambda''^2_{233} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right), \\
16\pi^2 \frac{dh_\tau}{d(\ln\,\mu)} &= h_\tau \left(3h_b^2 + 4h_\tau^2 - 3g_2^2 - \frac{9}{5}g_1^2\right), \\
16\pi^2 \frac{d\lambda''_{233}}{d(\ln\,\mu)} &= \lambda''_{233} \left(2h_t^2 + 2h_b^2 + 6\lambda''^2_{233} - 8g_3^2 - \frac{4}{5}g_1^2\right).
\end{aligned} \tag{4}$$

For completeness we list the well-known evolution equations for the gauge couplings, which at one-loop order are identical to those in the MSSM, since the additional Yukawa coupling(s) do not play a role at this order:

$$16\pi^2 \frac{dg_i}{d(\ln \mu)} = b_i g_i^3,\ i = 1, 2, 3, \tag{5}$$

with $b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$. With the definitions

$$R_t = \frac{h_t^2}{g_3^2},\ \ R_b = \frac{h_b^2}{g_3^2},\ \ R_\tau = \frac{h_\tau^2}{g_3^2},\ \ R'' = \frac{\lambda_{233}''^2}{g_3^2}, \tag{6}$$

and retaining only the $SU(3)_C$ gauge coupling constant, we can rewrite the renormalization group equations as ($\tilde{\alpha}_3 = g_3^2/(16\pi^2)$):

$$\frac{dR_t}{dt} = \tilde{\alpha}_3 R_t \left[\left(\frac{16}{3} + b_3\right) - 6R_t - R_b - 2R''\right], \tag{7}$$

$$\frac{dR_b}{dt} = \tilde{\alpha}_3 R_b \left[\left(\frac{16}{3} + b_3\right) - R_t - 6R_b - R_\tau - 2R''\right], \tag{8}$$

$$\frac{dR_\tau}{dt} = \tilde{\alpha}_3 R_\tau \left[b_3 - 3R_b - 4R_\tau\right], \tag{9}$$

$$\frac{dR''}{dt} = \tilde{\alpha}_3 R'' \left[(8 + b_3) - 2R_t - 2R_b - 6R''\right], \tag{10}$$

where $b_3 = -3$ is the beta function for g_3 in the MSSM, and $t = -\ln \mu^2$. Ordering the ratios as $R_i = (R'', R_\tau, R_b, R_t)$, we rewrite the RG equations (7) - (10) in the form [3]

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i \left[(r_i + b_3) - \sum_j S_{ij} R_j\right], \tag{11}$$

where $r_i = \sum_R 2\, C_R$, C_R is the QCD Casimir for the various fields ($C_Q = C_{\overline{U}} = C_{\overline{D}} = 4/3$), the sum is over the representation of the three fields associated with the trilinear coupling that enters R_i, and S is a matrix whose value is fully specified by the wavefunction anomalous dimensions. A fixed point is then reached when the right hand side of Eq. (11) is 0 for all i. If we were to write the fixed-point solutions as R_i^*, then there are two fixed point values for each coupling: $R_i^* = 0$, or

$$\left[(r_i + b_3) - \sum_j S_{ij} R_j^*\right] = 0. \tag{12}$$

It follows that the non-trivial fixed point solution is

$$R_i^* = \sum_j (S^{-1})_{ij} (r_j + b_3). \tag{13}$$

Since we shall consider the fixed points of the couplings h_t, h_b and $\lambda_{233}^{''}$ only, we shall ignore the evolution equation (9). However, the coupling h_τ does enter the evolution

Eq. (8) of h_b, but it can be related to h_b at the weak scale (which we take to be the top-quark mass), since

$$h_\tau(m_t) = \frac{\sqrt{2} m_\tau(m_t)}{\eta_\tau v \cos\beta}, \tag{14}$$

and

$$h_\tau(m_t) = \frac{m_\tau(m_\tau)}{m_b(m_b)} \frac{\eta_b}{\eta_\tau} h_b(m_t) = 0.6 h_b(m_t), \tag{15}$$

where η_b gives the QCD or QED running [12] of the b-quark mass $m_b(\mu)$ between $\mu = m_b$ and $\mu = m_t$ (similarly for η_τ), and $\tan\beta = v_2/v_1$ is the usual ratio of the Higgs vacuum expectation values in the MSSM, with $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV. The anomalous dimension matrix S can, then, be written as

$$S = \begin{pmatrix} 6 & 2 & 2 \\ 2 & 6+\eta & 1 \\ 2 & 1 & 6 \end{pmatrix}, \tag{16}$$

where $\eta = h_\tau^2(m_t)/h_b^2(m_t) \simeq 0.36$ is the factor coming from Eq. (15). We, therefore, get the following fixed point solution for the ratios:

$$\begin{aligned} R_1^* \equiv R''^* &= \frac{385 + 76\eta}{3(170 + 32\eta)} \simeq 0.76, \\ R_2^* \equiv R_b^* &= \frac{20}{170 + 32\eta} \simeq 0.11, \\ R_3^* \equiv R_t^* &= \frac{20 + 4\eta}{170 + 32\eta} \simeq 0.12. \end{aligned} \tag{17}$$

Since each of the R_i's is positive, this is a theoretically acceptable fixed point solution.

We next try to find a fixed point solution with $R''^* = 0$, with R_b and R_t being given by their non-zero solutions. We need to consider only the lower right hand 2×2 sub-matrix of the matrix S in Eq. (16) to obtain the fixed point solutions for R_b and R_t in this case. We then have

$$\begin{aligned} R_1^* \equiv R''^* &= 0, \\ R_2^* \equiv R_b^* &= \frac{35}{3\,(35 + 6\eta)} \simeq 0.36, \\ R_3^* \equiv R_t^* &= \frac{7(5 + \eta)}{3(35 + 6\eta)} \simeq 0.34. \end{aligned} \tag{18}$$

This is also a theoretically acceptable solution, as all the fixed point values are non-negative. We must also consider the fixed point with $R_b^* = 0$, which is relevant for the low values of the parameter $\tan\beta$. In this case, we have to reorder the couplings as $R_i = (R_b,\, R'', R_t)$, so that we have the anomalous dimension matrix (in this case denoted as $\tilde{S}$)

$$\tilde{S} = \begin{pmatrix} 6+\eta & 2 & 1 \\ 2 & 6 & 2 \\ 1 & 2 & 6 \end{pmatrix}. \tag{19}$$

Since $R_b^* = 0$, we have to determine the non-zero fixed point values for R'' and R_t only. For this we consider the lower right hand 2×2 submatrix of the matrix in (19) to obtain

$$\begin{aligned} R_1^* &\equiv R_b^* = 0, \\ R_2^* &\equiv R''^* = \tfrac{19}{24} \simeq 0.79, \\ R_3^* &\equiv R_t^* = \frac{1}{8} \simeq 0.12. \end{aligned} \tag{20}$$

which is an acceptable fixed point solution as well. Since there are more than one theoretically acceptable IRSFPs in this case, it is important to determine which, if any, is more likely to be realized in nature. To this end, we must examine the stability of each of the fixed point solutions.

The infra-red stability of a fixed point solution is determined by the sign of the eigenvalues of the matrix A whose entries are (i not summed over) [3]

$$A_{ij} = \frac{1}{b_3} R_i^* S_{ij}, \tag{21}$$

where R_i^* is the set of the fixed point solutions of the Yukawa couplings under consideration, and S_{ij} is the matrix appearing in the corresponding RG equations (11) for the ratios R_i. For stability, we require all the eigenvalues of the matrix Eq. (21) to have negative real parts (note that the QCD β-function b_3 is negative). Considering the fixed point solution (17), the matrix A can be written as

$$A = -\frac{1}{3} \begin{pmatrix} 6R_1^* & 2R_1^* & 2R_1^* \\ 2R_2^* & (6+\eta)R_2^* & R_2^* \\ 2R_3^* & R_3^* & 6R_3^* \end{pmatrix}, \tag{22}$$

where R_i^* are given in Eq. (17). The eigenvalues of the matrix Eq. (22) are calculated to be

$$\lambda_1 = -1.6,\ \lambda_2 = -0.2,\ \lambda_3 = -0.2, \tag{23}$$

which shows that the fixed point (17) is an infra-red stable fixed point. We note that the eigenvalue λ_1 is larger in magnitude as compared to the other eigenvalues in (23), indicating that the fixed point for λ''_{233} is more attractive, and hence more relevant.

Next, we consider the stability of the fixed point solution (18). Since in this case the fixed point of the coupling $R''^* = 0$, we have to obtain the behaviour of this coupling around the origin. This behaviour is determined by the eigenvalue [3]

$$\lambda_1 = \frac{1}{b_3} \left[\sum_{j=2}^{3} S_{1j} R_j^* - (r_1 + b_3) \right], \tag{24}$$

where $r_1 = 2(C_{\overline{U}} + C_{\overline{D}}) = 8$, the Cs are the quadratic Casimirs of the fields occuring in the B-violating terms in the superpotential (3), and the S_{ij} is the matrix (16), with the fixed points R_i^*, $i = 1, 2, 3$ given by Eq. (18). Inserting these values in Eq. (24), we find

$$\lambda_1 = \frac{385 + 76\eta}{9\,(35 + 6\eta)} > 0, \tag{25}$$

thereby indicating that the fixed point is unstable in the infra-red. The behaviour of the couplings R_b and R_t around their respective fixed points is governed by the eigenvalues of the the 2×2 lower submatrix of the matrix A in Eq. (22)

$$-\frac{1}{3}\begin{pmatrix} (6+\eta)R_2^* & R_2^* \\ R_3^* & 6R_3^* \end{pmatrix}, \tag{26}$$

which we find to be

$$\lambda_2 = -0.78,\ \lambda_3 = -0.56. \tag{27}$$

Although λ_2 and λ_3 are negative, because of the result (25), the fixed-point solution (18) is unstable in the infra-red. In other words, the R_p conserving fixed point solution (18) will never be achieved at low energies and must be rejected.

Finally we come to the question of the stability of the fixed point solution (20). The behaviour of the coupling R_b^* around the origin is determined by the eigenvalue

$$\lambda_1 = \frac{1}{b_3}\left[\sum_{j=2}^{3} \tilde{S}_{1j} R_j^* - (r_1 + b_3)\right], \tag{28}$$

where $r_1 = 2(C_{\overline{Q}} + C_{\overline{D}}) = 16/3$, and $\tilde{S}$ is the matrix (19). Inserting these numbers, we find

$$\lambda_1 = \frac{5}{24} \simeq 0.21 > 0, \tag{29}$$

with the other two eigenvalues for determining the stability given by the eigenvalues of the matrix which is obtained from the lower 2×2 submatrix of the matrix $\tilde{S}$ in (19). This submatrix can be written as

$$\tilde{A} = -\frac{1}{3}\begin{pmatrix} 6R_2^* & 2R_2^* \\ 2R_3^* & 6R_3^* \end{pmatrix}, \tag{30}$$

where R_2^* and R_3^* are given by Eq. (20). The eigenvalues are

$$\lambda_2 = -1.61,\ \lambda_3 = -0.22. \tag{31}$$

It follows, once again, that the fixed point solution given in (20) is not stable in the infra-red and is, therefore, never reached at low-energies.

One may also consider the case where the couplings λ''_{233} and h_b attain trivial fixed point values, whereas h_t attains a non-trivial fixed point value. In this case we have $R_3^* \equiv R_t^* = 7/18$, the well-known Pendleton-Ross [1] top-quark fixed point of the MSSM. To study the stability of this solution in the present context, we must consider the eigenvalues

$$\lambda_i = \frac{1}{b_3}(S_{i3}R_3^* - (r_i + b_3)),\ i = 1, 2,$$

where S_{i3} are read off from the matrix (16), which yields

$$\lambda_1 = \frac{38}{27},\ \lambda_2 = \frac{35}{54}.$$

Since the sign of each of λ_1 and λ_2 is positive, this solution is also unstable in the infra-red region. However, from our discussion of infra-red fixed point solution (17), it is clear that the Pendelton-Ross fixed point would be stable in case h_b and λ''_{233} are small, though negligible at the GUT scale. In this case, these would, of course, evolve away from zero at the weak scale, though realistically they would still be small (but not zero) at the weak scale. Thus, the only true infra-red stable fixed point solution is the baryon number, and R_p, violating solution (17). This is one of the main conclusions of this paper. We note that the value of R_t^* in (17) is lower than the corresponding value of 7/18 in MSSM with R_p conservation.

It is appropriate to examine the implications of the value of $h_t(m_t)$ predicted by our fixed point analysis for the top-quark mass. From (17), and $\alpha_3(m_t) \simeq 0.1$, the fixed point value for the top-Yukawa coupling is predicted to be $h_t(m_t) \simeq 0.4$. This translates into a top-quark (pole) mass of about $m_t \simeq 70 \sin\beta$ GeV, which is incompatible with the measured value [13] of top mass, $m_t \simeq 174$ GeV, for any value of $\tan\beta$. It follows that the true fixed point obtained here provides only a qualitative understanding of the top quark mass in MSSM with R_p violation.

We now turn to the study of the renormalization group evolution for the lepton number violating, and R_p, violating couplings in the superpotential (2). Here we shall consider the dimensionless couplings λ_{233} and λ'_{333} only. The relevant one-loop renormalization group equations are:

$$
\begin{aligned}
16\pi^2 \frac{dh_t}{d(\ln\,\mu)} &= h_t \left(6h_t^2 + h_b^2 + \lambda'^2_{333} - \frac{16}{3} g_3^2\right), \\
16\pi^2 \frac{dh_b}{d(\ln\,\mu)} &= h_b \left(h_t^2 + 6h_b^2 + h_\tau^2 + 6\lambda'^2_{333} - \frac{16}{3} g_3^2\right), \\
16\pi^2 \frac{dh_\tau}{d(\ln\,\mu)} &= h_\tau \left(3h_b^2 + 4h_\tau^2 + 4\lambda^2_{233} + 3\lambda'^2_{333}\right), \\
16\pi^2 \frac{d\lambda_{233}}{d(\ln\,\mu)} &= \lambda_{233} \left(4h_\tau^2 + 4\lambda^2_{233} + 3\lambda'^2_{333}\right), \\
16\pi^2 \frac{d\lambda'_{333}}{d(\ln\,\mu)} &= \lambda'_{333} \left(h_t^2 + 6h_b^2 + h_\tau^2 + \lambda^2_{233} + 6\lambda'^2_{333} - \frac{16}{3} g_3^2\right).
\end{aligned} \tag{32}
$$

Defining the new ratios

$$
R = \frac{\lambda^2_{233}}{g_3^2}, \quad R' = \frac{\lambda'_{333}}{g_3^2}, \tag{33}
$$

we may now rewrite the equations (32) as

$$
\frac{dR}{dt} = \tilde{\alpha_3} R \left[b_3 - 4R - 3R' - 4R_\tau\right], \tag{34}
$$

$$
\frac{dR'}{dt} = \tilde{\alpha_3} R' \left[\left(\frac{16}{3} + b_3\right) - R - 6R' - R_\tau - 6R_b - R_t\right], \tag{35}
$$

$$
\frac{dR_\tau}{dt} = \tilde{\alpha_3} R_\tau \left[b_3 - 4R - 3R' - 3R_b - 4R_\tau\right], \tag{36}
$$

$$\frac{dR_b}{dt} = \tilde{\alpha}_3 R_b \left[\left(\frac{16}{3} + b_3\right) - 6R' - R_\tau - 6R_b - R_t\right], \tag{37}$$

$$\frac{dR_t}{dt} = \tilde{\alpha}_3 R_t \left[\left(\frac{16}{3} + b_3\right) - R' - R_b - 6R_t\right]. \tag{38}$$

Ordering the ratios as $R_i = (R, R', R_\tau, R_b, R_t)$, we can write these RG equations as:

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i \left[(r_i + b_3) - \sum_j S_{ij} R_j\right], \tag{39}$$

where $r_i = \sum_R 2C_R$, with C_R denoting the quadartic Casimir of the each of the fields, the sum being over the representation of fields that enter R_i, and S fully specified by the respective wavefunction anomalous dimensions. It follows that there are two fixed point values for each coupling: $R_i^* = 0$, or the non-trivial fixed point solution

$$R_i^* = \sum_j (S^{-1})_{ij}(r_j + b_3). \tag{40}$$

We shall be interested in the fixed-point solutions of the couplings λ_{233}, λ'_{333}, h_b, h_t only, and shall not consider the h_τ coupling. Therefore, we replace it, as we did earlier, by $h_\tau(m_t) = 0.6h_b(m_t)$ at the weak scale in the determination of the fixed point solutions (40). The anomalous dimensions matrix can then be written as:

$$S = \begin{pmatrix} 4 & 3 & 4\eta & 0 \\ 1 & 6 & 6+\eta & 1 \\ 0 & 6 & 6+\eta & 1 \\ 0 & 1 & 1 & 6 \end{pmatrix} \tag{41}$$

This leads to the fixed point values for the ratios:

$$\begin{aligned} R_1^* &\equiv R^* = 0, \\ R_2^* &\equiv R'^* = \frac{315 + 194\eta}{366\eta - 315}, \\ R_3^* &\equiv R_b^* = -\frac{140}{122\eta - 105}, \\ R_4^* &\equiv R_t^* = \frac{110\eta - 105}{366\eta - 315}. \end{aligned} \tag{42}$$

We note that $R_2^* \equiv R'^* < 0$, and therefore, this fixed point solution is not an acceptable fixed point. We, thus, see that a simultaneous fixed point for the lepton number violating couplings λ_{233}, λ'_{333}, and h_b, h_t does not exist.

We now consider the two L-violating couplings separately, i.e., we shall take either $\lambda_{233} \ll \lambda'_{333}$, or $\lambda'_{333} \ll \lambda_{233}$, respectively. In the case when λ'_{333} is the dominant of the couplings, we order the couplings as $R_i = (R', R_b, R_t)$, so that the matrix S that enters Eq. (40) for this case can be written as

$$S = \begin{pmatrix} 6 & 6+\eta & 1 \\ 6 & 6+\eta & 1 \\ 1 & 1 & 6 \end{pmatrix}. \tag{43}$$

Since the determinant of this matrix vanishes, there are no fixed points in this case. We thus conclude that a simultaneous non-zero fixed point for the coupling λ'_{233}, h_b, h_t does not exist. We note that the vanishing of the determinant corresponds to a solution with a fixed line or surface.

If h_b is small (e.g., for the case of small $\tan\beta$) we may reorder the couplings $R_i = (R_b, R', R_t)$, and the matrix S, to find the fixed point solution

$$\begin{aligned} R_1^* &\equiv R_b^* = 0, \\ R_2^* &\equiv R'^* = \frac{1}{3}, \\ R_3^* &\equiv R_t^* = \frac{1}{3}. \end{aligned} \tag{44}$$

In order to study the stability of this solution, we must obtain the behaviour of the coupling R_b^* around the origin from the eigenvalue

$$\lambda_1 = \frac{1}{b_3}\left[\sum_{j=2}^{3} S_{1j}R_j^* - (r_1 + b_3)\right], \tag{45}$$

where $r_1 = 16/3$. Inserting the relevant R_i^*'s into (45), we get

$$\lambda_1 = 0, \tag{46}$$

from which we conclude that the fixed point Eq. (44) will never be reached in the infra-red region. This fixed point is either a saddle point or an ultra-violet fixed point. We conclude that there are no non-trivial stable fixed points in the infra-red region for the lepton number violating coupling λ'_{333}.

Finally, we consider the case when $\lambda'_{333} \ll \lambda_{233}$. We find the fixed point solution

$$\begin{aligned} R_1^* &\equiv R^* = \frac{-315 - 194\eta}{12(35 + 6\eta)}, \\ R_2^* &\equiv R_b^* = \frac{35}{3(35 + 6\eta)}, \\ R_3^* &\equiv R_t^* = \frac{7(5 + \eta)}{3(35 + 6\eta)}, \end{aligned} \tag{47}$$

which is unphysical. We, therefore, try a fixed point with $R_b^* = 0$. We find

$$\begin{aligned} R_1^* &\equiv R_b^* = 0, \\ R_2^* &\equiv R^* = -3/4, \\ R_3^* &\equiv R_t^* = 7/18, \end{aligned} \tag{48}$$

which, again, is unphysical. We have also checked that: (1) trivial fixed points for λ_{233} and h_b and the Pendleton-Ross type fixed point for the top-quark Yukawa coupling, or (2) trivial fixed points for λ'_{333} and h_b and the Pendleton-Ross fixed point for the top-quark

Yukawa coupling, are, both, unstable in the infra-red region. We, therefore, conclude that there are no fixed point solutions for the lepton number violating coupling λ_{233}.

To summarize, we have analyzed the one-loop renormalization group equations for the evolution of Yukawa couplings in MSSM with R_p violating couplings to the heaviest generation, taking into account B and L violating couplings one at a time. The analysis of the system with R_p, and the baryon number, violating coupling λ''_{233} yields the surprising and important result that only the simultaneous non-trivial fixed point for this coupling and the top-quark and b-quark Yukawa couplings h_t and h_b is stable in the infra-red region. However, the fixed point value for the top-quark coupling here is lower than its corresponding value in the MSSM, and is incompatible with the measured value of the top-quark mass. The R_p conserving solution with λ''_{233} attaining its trivial fixed point, with h_t and h_b attaining non-trivial fixed points, is infra-red unstable, as is the case for trivial fixed points for λ''_{233} and h_b, with a non-trivial fixed point for h_t. Our analysis shows that the usual Pendelton-Ross type infra-red fixed point of MSSM is unstable in the presence of R_p violation, though for small, but negligible, values of h_b and $\lambda^{''}_{233}$ it could be stable. The system with L, and R_p, violating couplings does not possess a set of non-trivial fixed points that are infra-red stable. Our results are the first in placing strong theoretical constraints on the nature of R_p violating couplings from fixed-point and stability considerations: the fixed points that are unstable, or the fixed point that is a saddle point, cannot be realized in the infra-red region. The fixed points obtained in this work are the true fixed points, in contrast to the quasi-fixed points of [9], and serve as a lower bound on the relevant R_p violating Yukawa couplings. In particular, from our analysis of the simultaneous (stable) fixed point for the baryon number violating coupling $\lambda^{''}_{233}$ and the top and bottom Yukawa couplings, we infer a lower bound on $\lambda^{''}_{233} \gtrsim 0.98$.

Note addded: After this paper was submitted for publication, another paper [14] which considers the question of infra-red fixed points in the supersymmetric standard model with R_p violation has appeared. The fixed points for $\lambda^{'}_{333}$, $\lambda^{''}_{233}$ and h_t, neglecting all other Yukawa couplings, is considered. Their results, where there is an overlapp, agree with ours. However, unlike in the present work, the stability of the fixed points has not been considered in [14].

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