

Renormalization Group Evolution and Infra-red Fixed Points in Minimal Supersymmetric Standard Model with Baryon and Lepton Number Violation

B. Ananthanarayan
Centre for Theoretical Studies,
Indian Institute of Science,
Bangalore 560 012, India

P. N. Pandita
Department of Physics,
North-Eastern Hill University,
Shillong 793 022, India

We study the renormalization group evolution and infra-red stable fixed points of the Yukawa couplings and the soft supersymmetry breaking trilinear couplings of the minimal supersymmetric standard model with baryon and lepton number (and R-parity) violation involving the heaviest generations. We show analytically that in the Yukawa sector there is only one infra-red stable fixed point. This corresponds to non-trivial fixed point for the top-, bottom-quark Yukawa couplings and the B violating coupling λ''_{233} , and a trivial one for the τ -Yukawa coupling. All other possible fixed points are either unphysical or unstable in the infra-red region. We then carry out an analysis of the renormalization group equations for the soft supersymmetry breaking trilinear couplings, and determine the corresponding fixed points for these couplings. We also study the quasi-fixed point behaviour, both algebraically and with numerical solutions of the evolution equations, of the third generation Yukawa couplings and the baryon number violating coupling, and those of the soft supersymmetry breaking trilinear couplings. From the analysis of the fixed point behaviour, we obtain upper and lower bounds on the baryon number violating coupling λ''_{233} , as well as on the supersymmetry breaking trilinear couplings.

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I. INTRODUCTION

The Standard Model (SM) is a tremendous success in describing the strong and electroweak interactions based on an underlying gauge principle. However, one of the main weaknesses of the SM is that the masses of the matter particles, the quarks and leptons, are free parameters of the theory. This weakness persists in most extensions of the SM, including the supersymmetric extensions of the Standard Model. The fermions mass problem in the SM and its extensions arises from the presence of many unknown dimensionless Yukawa couplings. On the other hand the minimal supersymmetric version of the SM leads to a successful prediction for the ratio of the gauge couplings with a gauge unification scale $M_G \simeq 10^{16}$ GeV. This has led to the idea that there may be a stage of unification beyond the SM. If so, then it becomes important to perform the radiative corrections in determining all the dimension ≤ 4 terms in the lagrangian. This can be achieved by using the renormalization group equations in finding the values of parameters at the low scale, given their value at a high scale. Thus, considerable attention has recently been focussed on the renormalization group evolution [1] of the various dimensionless Yukawa couplings in the SM and its minimal supersymmetric extension, the Minimal Supersymmetric Standard Model (MSSM). Using the renormalization group evolution, one may attempt to relate the Yukawa couplings to the gauge couplings via the Pendleton-Ross infra-red stable fixed point (IRSFP) for the top-quark Yukawa coupling [2], or via the quasi-fixed point behaviour [3]. The predictive power of the SM and its supersymmetric extensions may, thus, be enhanced if the renormalization group (RG) running of the parameters is dominated by infra-red stable fixed points (IRSFPs). Typically, these fixed points are for ratios like Yukawa couplings to the gauge coupling, or in the context of supersymmetric models, the supersymmetry breaking trilinear A -parameter to the gaugino mass, etc. These ratios do not attain their fixed point values at the weak scale, the range between the GUT (or Planck) scale and the weak scale being too small for the ratios to closely approach the fixed point. Nevertheless, the couplings may be determined by quasi-fixed point behaviour [3] where the value of the Yukawa coupling at the weak scale is independent of its value at the GUT scale, provided the Yukawa coupling at the GUT scale is large. For the fixed point or the quasi-fixed point scenarios to be successful, it is necessary that these fixed points are stable [4–6].

Since supersymmetry [7] requires the introduction of superpartners of all known particles in the SM (in addition to the introduction of at least two Higgs doublets), which transform in an identical manner under the gauge group, there are additional Yukawa couplings in supersymmetric models which violate [8] baryon number (B) or lepton number (L). In MSSM, a discrete symmetry called R-parity (R_p) is invoked to eliminate these B and L violating Yukawa couplings [9]. However, the assumption of R_p conservation at the level of the MSSM appears to be *ad hoc*, since it is not required for the internal consistency of the model. Therefore, the study of the renormalization group evolution of the dimensionless Yukawa couplings in the MSSM, including B and L (and R_p) violation, deserves serious consideration.

Recently considerable attention has been focussed on the study of the RG evolution of the top- and bottom-quark Yukawa couplings, and the R_p violating Yukawa couplings, of the MSSM. This includes the study of quasi-fixed point behaviour [10] as well as the true infra-red fixed points of different Yukawa couplings, and the analysis of their stability [11]. This has led to certain insights and constraints on the fixed point behaviour of some of the R_p violating couplings, involving higher generation indices. As pointed out earlier, for the fixed point or the quasi-fixed point behaviour to be successful, it is necessary that the fixed points are stable [11,4]. It has been shown that in MSSM with only top- and bottom-quark Yukawa couplings, and with the highest generation B or L violation taken into account, only the B -violating coupling λ'_{233} , together with top- and bottom-quark Yukawa couplings, approaches a non-trivial IRSFP, whereas all other non-trivial fixed points are either unphysical or unstable in the infra-red region [11]. This conclusion remains unchanged even if one extends the MSSM by including a Higgs singlet superfield, to the so-called non-minimal supersymmetric standard model [12].

In this paper we carry out a detailed study of the renormalization group evolution of the Yukawa couplings of MSSM, including the B and L violating couplings. We shall include all the third generation Yukawa couplings [13], as well as the highest generation B and L violating couplings in our study, and analyze the situation where all of them could simultaneously approach infra-red fixed points. We shall investigate both the true infra-red fixed points, as well as quasi-fixed points of these couplings. In particular, we shall carry out a detailed stability analysis of the infra-red fixed points of these couplings. Furthermore, corresponding to the B and L (and R_p) violating Yukawa couplings of the MSSM, there are soft supersymmetry breaking trilinear couplings (the A parameters) whose renormalization group evolution and infra-red fixed point structure has not been studied so far. We shall, therefore, also study the renormalization group evolution of these soft supersymmetry breaking A parameters, including those corresponding to the third generation Yukawa couplings, and obtain the simultaneous infra-red fixed points for them.

The plan of the paper is as follows. In Sec. II we describe the model and write down the renormalization group equations of interest to us, which we have rederived since we require the complete set of equations involving baryon and lepton number violation. We then carry out a detailed analytical study of the true infra-red fixed points of

the Yukawa couplings in full generality. Within the context of grand unified theories, one is led to the situation where B and L violating Yukawa couplings may be related at the GUT scale, and one may no longer be able to set one or the other arbitrarily to zero. We, therefore, initially include both baryon and lepton number violation in our RG equations. The fixed point analysis of such a system of RG equations leads to the crucial result that the only stable fixed point is the one with simultaneous non-trivial fixed point values for the top- and bottom-quark Yukawa couplings and the B -violating coupling λ''_{233} , and a trivial one for the τ -Yukawa coupling. Thus, non-trivial simultaneous fixed points for the B and L violating Yukawa couplings are ruled out by our analysis. We then study the fixed points of the corresponding soft supersymmetry breaking trilinear couplings of this model. In Sec. III we algebraically study the simultaneous quasi-fixed points of all the third generation Yukawa couplings of the minimal supersymmetric standard model with B violation, as well as those of the corresponding soft supersymmetry breaking trilinear couplings. Since the quasi-fixed point limit is formally defined as the Landau pole of the Yukawa coupling at the GUT scale, it provides an upper bound on the corresponding Yukawa coupling. In Sec. IV we present the numerical results for the renormalization group evolution and the quasi-fixed points for the minimal supersymmetric standard model with B violation. In Sec. V we summarize our results and present the conclusions.

II. RENORMALIZATION GROUP EQUATIONS AND INFRA-RED FIXED POINTS

In this section we study the true infra-red fixed points of the Yukawa couplings and the A parameters of the MSSM with B and L violation. We begin by recalling the basic features of the model. The superpotential of the MSSM is written as

$$W = (h_U)_{ab} Q_L^a \bar{U}_R^b H_2 + (h_D)_{ab} Q_L^a \bar{D}_R^b H_1 + (h_E)_{ab} L_L^a \bar{E}_R^b H_1 + \mu H_1 H_2, \quad (1)$$

where L , Q , \bar{E} , \bar{D} , \bar{U} denote the lepton and quark doublets, and anti-lepton singlet, d-type anti-quark singlet and u-type anti-quark singlet, respectively. In Eq. (1), $(h_U)_{ab}$, $(h_D)_{ab}$ and $(h_E)_{ab}$ are the Yukawa coupling matrices, with a, b, c as the generation indices. Gauge invariance, supersymmetry and renormalizability allow the addition of the following L and B violating terms to the MSSM superpotential (1):

$$W_L = \frac{1}{2} \lambda_{abc} L_L^a L_L^b \bar{E}_R^c + \lambda'_{abc} L_L^a Q_L^b \bar{D}_R^c + \mu_i L_i H_2, \quad (2)$$

$$W_B = \frac{1}{2} \lambda''_{abc} \bar{D}_R^a \bar{D}_R^b \bar{U}_R^c, \quad (3)$$

respectively. The Yukawa couplings λ_{abc} and λ''_{abc} are antisymmetric in their first two indices due to $SU(2)_L$ and $SU(3)_C$ group structures, respectively. Corresponding to the terms in the superpotentials (1), (2) and (3), there are the soft supersymmetry breaking trilinear terms which can be written as

$$\begin{aligned} -V_{\text{soft}} = & \left[(A_U)_{ab} (h_U)_{ab} \tilde{Q}_L^a \tilde{\bar{U}}_R^b H_2 + (A_D)_{ab} (h_D)_{ab} \tilde{Q}_L^a \tilde{\bar{D}}_R^b H_1 + (A_E)_{ab} (h_E)_{ab} \tilde{L}_L^a \tilde{\bar{E}}_R^b H_1 \right] \\ & + \left[\frac{1}{2} (A_\lambda)_{abc} \lambda_{abc} \tilde{L}_L^a \tilde{L}_L^b \tilde{\bar{E}}_R^c + (A_{\lambda'})_{abc} \lambda'_{abc} \tilde{L}_L^a \tilde{Q}_L^b \tilde{\bar{D}}_R^c \right] + \left[\frac{1}{2} (A_{\lambda''})_{abc} \lambda''_{abc} \tilde{\bar{D}}_R^a \tilde{\bar{D}}_R^b \tilde{\bar{U}}_R^c \right], \quad (4) \end{aligned}$$

where a tilde denotes the scalar component of the chiral superfield, and the notation for the scalar component of the Higgs superfield is the same as that of the corresponding superfield. In addition there are soft supersymmetry breaking gaugino mass terms with the masses M_i , with $i = 1, 2, 3$ corresponding to the gauge groups $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively.

The third generation Yukawa couplings are the dominant couplings in the superpotential (1), so it is natural to retain only the elements $(h_U)_{33} \equiv h_t$, $(h_D)_{33} \equiv h_b$, $(h_L)_{33} \equiv h_\tau$ in each of the Yukawa couplings matrices h_U , h_D , h_L , setting all other elements equal to zero. Furthermore, there are 36 independent L violating trilinear couplings λ_{abc} and λ'_{abc} in (2). Similarly, there are 9 independent B violating couplings λ''_{abc} in the baryon number violating superpotential (3). Thus, we would have to consider 39 coupled nonlinear evolution equations for the L violating case and 12 coupled nonlinear equations for the B violating case, respectively. It is clear that there is a need for a radical simplification of these equations before we can think of studying the evolution of the Yukawa couplings in the MSSM with B and L violation.

In order to render the Yukawa coupling evolution equations tractable, we, therefore, need to make certain plausible assumptions. Motivated by the generational hierarchy of the conventional Higgs couplings, we shall assume that an analogous hierarchy amongst the different generations of B and L violating couplings exists. Thus, we shall retain only

the couplings λ_{233} , λ'_{333} , λ''_{233} , and neglect the rest. We note that B and L violating couplings to higher generations evolve more strongly because of larger Higgs couplings in their evolution equations, and hence could take larger values than the corresponding couplings to the lighter generations. We also note that the experimental upper limits are stronger for the B and L violating couplings with lower indices [14].

With these assumptions the one-loop renormalization group equations [15] for the Yukawa couplings, and the B and L violating couplings, following from the various terms in the superpotentials (1), (2) and (3), respectively, can be written as

$$\frac{dh_t}{d \ln \mu} = \frac{h_t}{16\pi^2} \left(6h_t^2 + h_b^2 + \lambda_{333}^{\prime 2} + 2\lambda_{233}^{\prime\prime 2} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right), \quad (5)$$

$$\frac{dh_b}{d \ln \mu} = \frac{h_b}{16\pi^2} \left(h_t^2 + 6h_b^2 + h_\tau^2 + 6\lambda_{333}^{\prime 2} + 2\lambda_{233}^{\prime\prime 2} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right), \quad (6)$$

$$\frac{dh_\tau}{d \ln \mu} = \frac{h_\tau}{16\pi^2} \left(3h_b^2 + 4h_\tau^2 + 4\lambda_{233}^2 + 3\lambda_{333}^{\prime 2} - 3g_2^2 - \frac{9}{15}g_1^2 \right), \quad (7)$$

$$\frac{d\lambda_{233}}{d \ln \mu} = \frac{\lambda_{233}}{16\pi^2} \left(4h_\tau^2 + 4\lambda_{233}^2 + 3\lambda_{333}^{\prime 2} - 3g_2^2 - \frac{9}{15}g_1^2 \right), \quad (8)$$

$$\frac{d\lambda'_{333}}{d \ln \mu} = \frac{\lambda'_{333}}{16\pi^2} \left(h_t^2 + 6h_b^2 + h_\tau^2 + \lambda_{233}^2 + 6\lambda_{333}^{\prime 2} + 2\lambda_{233}^{\prime\prime 2} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right), \quad (9)$$

$$\frac{d\lambda''_{233}}{d \ln \mu} = \frac{\lambda''_{233}}{16\pi^2} \left(2h_t^2 + 2h_b^2 + 2\lambda_{333}^{\prime 2} + 6\lambda_{233}^{\prime\prime 2} - 8g_3^2 - \frac{4}{15}g_1^2 \right), \quad (10)$$

where g_1, g_2, g_3 are the gauge couplings of $U(1)_Y$ (in the GUT normalization), $SU(2)_L$ and $SU(3)_C$ gauge groups, respectively, and μ is the scale parameter. The evolution equations for the gauge couplings are not affected by the presence of B and L violating couplings at the one-loop level, and can be written as

$$16\pi^2 \frac{dg_i}{d \ln \mu} = b_i g_i^3, \quad i = 1, 2, 3, \quad (11)$$

with

$$b_1 = 33/5, \quad b_2 = 1, \quad b_3 = -3. \quad (12)$$

The corresponding one-loop renormalization group equations for the gaugino masses M_i , $i = 1, 2, 3$ can be written as

$$16\pi^2 \frac{dM_i}{d \ln \mu} = 2g_i^2 b_i M_i. \quad (13)$$

We now come to the evolution equations for the soft supersymmetry breaking trilinear parameters in potential (4). The one-loop RGEs for these parameters can be deduced from the general expressions in ref. [15]. In this paper we shall assume the same kind of generational hierarchy for these trilinear parameters as was assumed for the corresponding Yukawa couplings. Thus, we shall consider only the highest generation trilinear couplings $(A_U)_{33} \equiv A_t$, $(A_D)_{33} \equiv A_b$, $(A_L)_{33} \equiv A_\tau$, $(A_\lambda)_{233} \equiv A_\lambda$, $(A_{\lambda'})_{333} \equiv A_{\lambda'}$, $(A_{\lambda''})_{233} \equiv A_{\lambda''}$, setting all other elements equal to zero. With this assumption the RGEs for the soft supersymmetry breaking trilinear parameters can be written as

$$\frac{dA_t}{d \ln \mu} = \frac{1}{8\pi^2} \left(6A_t h_t^2 + A_b h_b^2 + A_{\lambda'} \lambda_{333}^{\prime 2} + 2A_{\lambda''} \lambda_{233}^{\prime\prime 2} - \frac{16}{3}M_3 g_3^2 - 3M_2 g_2^2 - \frac{13}{15}M_1 g_1^2 \right), \quad (14)$$

$$\frac{dA_b}{d \ln \mu} = \frac{1}{8\pi^2} \left(A_t h_t^2 + 6A_b h_b^2 + A_\tau h_\tau^2 + 6A_{\lambda'} \lambda_{333}^{\prime 2} + 2A_{\lambda''} \lambda_{233}^{\prime\prime 2} - \frac{16}{3}M_3 g_3^2 - 3M_2 g_2^2 - \frac{7}{15}M_1 g_1^2 \right), \quad (15)$$

$$\frac{dA_\tau}{d \ln \mu} = \frac{1}{8\pi^2} \left(3A_b h_b^2 + 4A_\tau h_\tau^2 + 4A_\lambda \lambda_{233}^2 + 3A_{\lambda'} \lambda_{333}^{\prime 2} - 3M_2 g_2^2 - \frac{9}{15}M_1 g_1^2 \right), \quad (16)$$

$$\frac{dA_\lambda}{d \ln \mu} = \frac{1}{8\pi^2} \left(4A_\tau h_\tau^2 + 4A_\lambda \lambda_{233}^2 + 3A_{\lambda'} \lambda_{333}^{\prime 2} - 3M_2 g_2^2 - \frac{9}{15}M_1 g_1^2 \right), \quad (17)$$

$$\frac{dA_{\lambda'}}{d \ln \mu} = \frac{1}{8\pi^2} \left(A_t h_t^2 + 6A_b h_b^2 + A_\tau h_\tau^2 + A_\lambda \lambda_{233}^2 + 6A_{\lambda'} \lambda_{333}^{\prime 2} + 2A_{\lambda''} \lambda_{233}^{\prime\prime 2} - \frac{16}{3}M_3 g_3^2 - 3M_2 g_2^2 - \frac{7}{15}M_1 g_1^2 \right), \quad (18)$$

$$\frac{dA_{\lambda''}}{d \ln \mu} = \frac{1}{8\pi^2} \left(2A_t h_t^2 + 2A_b h_b^2 + 2A_{\lambda'} \lambda_{333}^{\prime 2} + 6A_{\lambda''} \lambda_{233}^{\prime\prime 2} - 8M_3 g_3^2 - \frac{4}{15}M_1 g_1^2 \right). \quad (19)$$

Given the evolution equations (5) - (10) for the Yukawa couplings and the evolution equations (14) - (19) for the A parameters, we are now ready to study the RG evolution and infra-red fixed points of these parameters of MSSM with B and L violation.

A. Infra-red Fixed Points for Yukawa Couplings

We first consider IRFPs for the Yukawa couplings. With the definitions

$$R_t = \frac{h_t^2}{g_3^2}, \quad R_b = \frac{h_b^2}{g_3^2}, \quad R_\tau = \frac{h_\tau^2}{g_3^2}, \quad R = \frac{\lambda_{233}^2}{g_3^2}, \quad R' = \frac{\lambda_{333}^{\prime 2}}{g_3^2}, \quad R'' = \frac{\lambda_{233}^{\prime\prime 2}}{g_3^2}, \quad (20)$$

and retaining only the $SU(3)_C$ gauge coupling constant, we can rewrite the renormalization group equations for the Yukawa couplings as ($\tilde{\alpha}_3 = g_s^2/(16\pi^2)$)

$$\frac{dR_t}{dt} = \tilde{\alpha}_3 R_t \left[\left(\frac{16}{3} + b_3 \right) - 6R_t - R_b - R' - 2R'' \right], \quad (21)$$

$$\frac{dR_b}{dt} = \tilde{\alpha}_3 R_b \left[\left(\frac{16}{3} + b_3 \right) - R_t - 6R_b - R_\tau - 6R' - 2R'' \right], \quad (22)$$

$$\frac{dR_\tau}{dt} = \tilde{\alpha}_3 R_\tau [b_3 - 3R_b - 4R_\tau - 4R - 3R'], \quad (23)$$

$$\frac{dR}{dt} = \tilde{\alpha}_3 R [b_3 - 4R_\tau - 4R - 3R'], \quad (24)$$

$$\frac{dR'}{dt} = \tilde{\alpha}_3 R' \left[\left(\frac{16}{3} + b_3 \right) - R_t - 6R_b - R_\tau - R - 6R' - 2R'' \right], \quad (25)$$

$$\frac{dR''}{dt} = \tilde{\alpha}_3 R'' [(8 + b_3) - 2R_t - 2R_b - 2R' - 6R''], \quad (26)$$

where $b_3 = -3$, the beta function for g_3 in the MSSM, and $t = -\ln \mu^2$.

The renormalization group evolution [10] and the infra-red fixed points [11] of the set of equations (21) - (26) has been studied in the limit of ignoring the τ -Yukawa coupling h_τ , and by considering either the baryon number violating Yukawa coupling λ_{233}' , or the lepton number violating Yukawa couplings λ_{233} and λ_{333}' . In the analysis that follows, we shall consider the evolution equations for h_t and h_b together with the evolution equation for h_τ . Furthermore, we shall also entertain the possibility of simultaneous presence of B and L violating couplings in the renormalization group equations (21) - (26). We do this in order to investigate as to whether such a system of equations does have acceptable infra-red fixed points. Ordering the ratios as $R_i = (R'', R', R, R_\tau, R_b, R_t)$, we rewrite the RGEs (21) - (26) in the form

$$\frac{dR_i}{dt} = \tilde{\alpha}_3 R_i \left[(r_i + b_3) - \sum_j S_{ij} R_j \right], \quad (27)$$

where $r_i = \sum_R 2C_R$, C_R is the QCD Casimir for the various fields ($C_Q = C_{\overline{Q}} = C_{\overline{D}} = 4/3$), the sum is over the representation of the three fields associated with the trilinear coupling that enters R_i , and S is a matrix whose value is fully specified by the wavefunction anomalous dimensions. A fixed point is, then, reached when the right hand side of Eq. (27) is 0 for all i . If we were to write the fixed point solutions as R_i^* , then there are two fixed point values for each coupling: $R_i^* = 0$, or

$$\left[(r_i + b_3) - \sum_j S_{ij} R_j^* \right] = 0. \quad (28)$$

It follows that the non-trivial fixed point solution is

$$R_i^* = \sum_j (S^{-1})_{ij} (r_j + b_3). \quad (29)$$

The anomalous dimension matrix S that enters Eq. (29), which we denote by S_{BL} , is readily seen to be

$$S_{BL} = \begin{bmatrix} 6 & 2 & 0 & 0 & 2 & 2 \\ 2 & 6 & 1 & 1 & 6 & 1 \\ 0 & 3 & 4 & 4 & 0 & 0 \\ 0 & 3 & 4 & 4 & 3 & 0 \\ 2 & 6 & 0 & 1 & 6 & 1 \\ 2 & 1 & 0 & 0 & 1 & 6 \end{bmatrix}. \quad (30)$$

Inverting the matrix (30) and substituting in Eq.(29), we get the following fixed point solution:

$$\begin{aligned} R''^* &= \frac{611}{876}, \quad R'^* = 22/73, \quad R^* = 0, \\ R_\tau^* &= -\frac{285}{292}, \quad R_b^* = 0, \quad R_t^* = \frac{31}{292}. \end{aligned} \quad (31)$$

We note that $R_\tau^* < 0$, and, therefore this fixed point solution is unacceptable. We, therefore, conclude that a simultaneous fixed point for the B and L violating couplings λ''_{233} , and λ'_{333} , λ_{333} , and the Yukawa couplings h_τ , h_b , h_t does not exist.

We next consider the possibility of one of the L violating couplings attaining a zero fixed point value, with all others having non-trivial fixed point values. We first consider the case with $R^* = 0$. The corresponding anomalous dimension matrix we need to consider, denoted by \tilde{S}_{BL} , is

$$\tilde{S}_{BL} = \begin{bmatrix} 6 & 2 & 0 & 2 & 2 \\ 2 & 6 & 1 & 6 & 1 \\ 0 & 3 & 4 & 3 & 0 \\ 2 & 6 & 1 & 6 & 1 \\ 2 & 1 & 0 & 1 & 6 \end{bmatrix}, \quad (32)$$

which is singular. Hence there are no fixed points in this case.

We next consider the possibility of having $R^* = 0$, with all other Yukawa couplings having non-trivial fixed points. The anomalous dimension matrix in this case is denoted by $\tilde{\tilde{S}}_{BL}$, and is given by

$$\tilde{\tilde{S}}_{BL} = \begin{bmatrix} 6 & 0 & 0 & 2 & 2 \\ 0 & 4 & 4 & 0 & 0 \\ 0 & 4 & 4 & 3 & 0 \\ 2 & 0 & 1 & 6 & 1 \\ 2 & 0 & 0 & 1 & 6 \end{bmatrix}, \quad (33)$$

leading to the fixed point values

$$\begin{aligned} R''^* &= \frac{19}{24}, \quad R^* = -\frac{11}{8}, \\ R_\tau^* &= \frac{5}{8}, \quad R_b^* = 0, \quad R_t^* = \frac{1}{8}, \end{aligned} \quad (34)$$

which is physically unacceptable. We conclude that B and L violating couplings of the highest generation cannot simultaneously approach a non-trivial fixed point in the MSSM. This is, perhaps, one of the most important conclusions that we draw from the analysis of the renormalization group equations in this paper.

It is now natural to consider the possibility of having either B or L violation, but not both simultaneously, involving the Yukawa couplings with highest generation indices in the renormalization group evolution in the MSSM.

1. Fixed Points with B violation

In this section we consider the possibility of having simultaneous fixed points for the Yukawa couplings h_τ , h_b , h_t , and the B violating coupling λ''_{233} . Ordering the ratios of the Yukawa couplings to the gauge couplings as $R_i = (R_\tau, R'', R_b, R_t)$, the anomalous dimension matrix, denoted by S_B , can be written as

$$S_B = \begin{bmatrix} 4 & 0 & 3 & 0 \\ 0 & 6 & 2 & 2 \\ 1 & 2 & 6 & 1 \\ 0 & 2 & 1 & 6 \end{bmatrix}, \quad (35)$$

leading to the fixed point values

$$\begin{aligned} R_\tau^* &= -\frac{285}{292}, \quad R''^* = \frac{611}{876}, \\ R_b^* &= \frac{22}{73}, \quad R_t^* = \frac{31}{292}, \end{aligned} \quad (36)$$

which must be rejected as being unphysical. We are, therefore, led to the consideration of a fixed point with one of the Yukawa couplings approaching a zero fixed point value, with all others attaining a non-trivial fixed point. We try the fixed point with $R_\tau^* = 0$, with all others obtaining a non-zero fixed point value. In this case we obtain the fixed point values

$$R''^* = \frac{77}{102}, R_b^* = \frac{2}{17}, R_t^* = \frac{2}{17}, \quad (37)$$

which is a physically acceptable fixed point solution. We can also try the possibility $R_b^* = 0$, with all other Yukawa couplings attaining a non-trivial infra-red fixed point value. In this case we find

$$R''^* = \frac{19}{24}, R_t^* = \frac{1}{8}, R_\tau^* = -\frac{3}{4}, \quad (38)$$

which is physically unacceptable.

Next we must try fixed points with two of the Yukawa couplings having a zero fixed point value and others attaining a non-trivial fixed point. We first consider the fixed point with $R_\tau^* = 0, R''^* = 0$. In this case we obtain the fixed point values for the other couplings as

$$R_b^* = R_t^* = \frac{1}{3}, \quad (39)$$

rendering this infra-red fixed point as an acceptable one. The other possibility is $R_\tau^* = 0, R_b^* = 0$, which is relevant for the case when $\tan\beta$ is small. In this case, we get the fixed point values

$$R''^* = \frac{19}{24}, R_t^* = \frac{1}{8}, \quad (40)$$

which is also a theoretically acceptable fixed point. We note that the fixed point values in (37), (39) and (40) are not significantly different from those obtained in [11], where the τ -Yukawa coupling evolution was ignored.

Since there are more than one theoretically acceptable IRFPs in this case, it is necessary to determine, which, if any, of these fixed points is more likely to be realized in nature. To this end, we must examine the stability of each of the fixed point solutions (37), (39) and (40).

The infra-red stability of a fixed point is determined by the sign of the quantities

$$\lambda_i = \frac{1}{b_3} \left[\sum_{j=m+1}^n S_{ij} R_j^* - (r_i + b_3) \right], \quad (41)$$

for those couplings which have a fixed point value zero, $R_i^* = 0, i = 1, 2, \dots, m$, and by the sign of the eigenvalues of the matrix

$$A_{ij} = \frac{1}{b_3} R_i^* S_{ij}, \quad i = m + 1, \dots, n, \quad (42)$$

for those couplings which have a non-trivial IRFP [4], where R_i^* is the set of the non-trivial fixed point values of the Yukawa couplings under consideration, and S_{ij} is the matrix appearing in the corresponding renormalization group equations (27) for the ratios R_i . For stability, we require all the eigenvalues of the matrix (42) to have negative real parts (note that the QCD β -function b_3 is negative). For the infra-red fixed point (37), we find from Eq. (41)

$$\lambda_1 = -\frac{19}{17} \quad (43)$$

corresponding to $R_\tau^* = 0$, and for the eigenvalues of the matrix (42)

$$\lambda_2 = -\frac{10}{51} \simeq -0.2, \quad \lambda_3 = \frac{-273 - \sqrt{43113}}{306} \simeq -1.6, \quad \lambda_4 = \frac{-273 + \sqrt{43113}}{306} \simeq -0.2, \quad (44)$$

corresponding to the non-trivial fixed point values for R''^*, R_b^* and R_t^* , respectively. Since all the $\lambda_i, i = 1, 2, 3, 4$ are negative, the fixed point (37) is infra-red stable.

Next we consider the stability of the fixed point (39). Since in this case $R_\tau^* = R''^* = 0$, we have to obtain the behaviour of these couplings around the origin. This behaviour is determined by the quantities (41) which, in this case, are

$$\lambda_1 = -\frac{4}{3}, \lambda_2 = \frac{11}{9}, \quad (45)$$

thereby indicating that the fixed point is unstable in the infra-red region. For completeness we also obtain the behaviour of R_b and R_t around their respective fixed points governed by the corresponding eigenvalues of the matrix (42). We obtain for the eigenvalues of this matrix

$$\lambda_3 = -\frac{5}{9}, \lambda_4 = -\frac{7}{9}. \quad (46)$$

Although λ_3, λ_4 are negative, the fact that $\lambda_2 > 0$ implies that the fixed point (39) is unstable in the infra-red region. Thus, this infra-red fixed point with trivial fixed point value for the baryon number violating coupling λ''_{233} will never be realized at low energies, and must be rejected. This is to be contrasted with the corresponding result obtained in the absence of B -violation [13], wherein the fixed point (39) is infra-red stable. Thus, the inclusion of the baryon number violating coupling has the effect of making the fixed point (39) unstable.

Similarly, it is straightforward to see that the fixed point (40) is unstable in the infra-red, and must be rejected. We note that this is in contrast to the result found in [16] where the fixed point (40) was found to be stable. The reason for this difference lies in the fact that in [16] the bottom- and the τ -Yukawa couplings, h_b and h_τ , were completely ignored.

At this stage it is interesting to ask whether the baryon number violating stable fixed point solution (37) would still be stable if there were lepton number violation, but with trivial infra-red fixed point value for the lepton number violating coupling. We first consider the case with the lepton number violating coupling λ_{233} being present with $R^* = R_\tau^* = 0$, and the rest of the couplings approaching the fixed point values (37). Stability analysis then yields for the eigenvalues (41)

$$\lambda_1 = -1, \lambda_2 = -\frac{19}{17} \quad (47)$$

corresponding to $R^* = R_\tau^* = 0$, and for the eigenvalues of (42) corresponding to the nontrivial fixed point (37) the results are as in (44). This shows that the baryon number violating fixed point (37) remains stable in presence of trivial fixed point value for the lepton number violating coupling λ_{233} .

We next consider the baryon number violating stable fixed point solution (37) with lepton number violating coupling λ'_{333} , but with this lepton number violating coupling approaching a trivial fixed point, i.e. $R'^* = R_\tau^* = 0$, and the other Yukawa couplings attaining the fixed point values (37). In this case stability analysis leads for the eigenvalues (41)

$$\lambda_1 = 0, \lambda_2 = -\frac{65}{51} \quad (48)$$

corresponding to $R'^* = R_\tau^* = 0$, and for the eigenvalues of (42) corresponding to the nontrivial fixed point (37) the results are as in (44). We conclude that such a fixed point is a saddle point or ultra-violet fixed point, and will never be realized in the infra-red region. Thus, the fixed point (37) ceases to be a stable infra-red stable fixed point in the presence of trivial fixed point value for the lepton-number violating coupling λ'_{333} . The same conclusions are reached when we have both the lepton number violating couplings, λ_{233} and λ'_{333} , but with both approaching a trivial fixed point, and rest of the couplings approaching the fixed point (37).

One may also consider the case where the couplings $\lambda''_{233}, h_\tau, h_b$ attain trivial fixed point values, whereas h_t attains a non-trivial fixed point value. In this case we find $R_t^* = 7/18$, the well-known Pendleton-Ross fixed point [2]. The stability of this fixed point solution is obtained by simply considering

$$\lambda_i = \frac{1}{b_3} [(S_B)_{i4} R_4^* - (r_i + b_3)], \quad i = 1, 2, 3, \quad (49)$$

which yields

$$\lambda_1 = -1, \lambda_2 = \frac{38}{27}, \lambda_3 = \frac{35}{54}, \quad (50)$$

thereby rendering this fixed point unstable.

Finally, one may consider the case where $R''^* = 0$, with R_τ, R_b, R_t attaining non-trivial fixed point values. This is the case of the MSSM with all the third generation Yukawa couplings taken into account. In this case, we find the fixed point solution:

$$R_\tau^* = -\frac{70}{61}, R_b^* = \frac{97}{183}, R_t^* = \frac{55}{183}, \quad (51)$$

which must be rejected as being unphysical.

Thus, we have shown that the only fixed point which is stable in the infra red region is the baryon number, and R_p , violating solution (37). We note that the value of R_t^* corresponding to this solution is lower than the Pendleton-Ross fixed point value of $7/18$ for MSSM with baryon number, and R_p , conservation.

2. Fixed Points with L violation

Having established a stable infra-red fixed point with baryon number violation, we now investigate the possibility of having stable fixed points with lepton number violation. We first consider the possibility of having simultaneous fixed points for h_τ, h_b, h_t , and the two lepton number violating couplings λ_{233} and λ'_{333} . Ordering the ratios of the squares of the Yukawa couplings to the square of the gauge coupling g_3 as $R_i = (R, R', R_\tau, R_b, R_t)$, the anomalous dimension matrix, S_L , for this case is given by

$$S_L = \begin{bmatrix} 4 & 3 & 4 & 0 & 0 \\ 1 & 6 & 1 & 6 & 1 \\ 4 & 3 & 4 & 3 & 0 \\ 0 & 6 & 1 & 6 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}, \quad (52)$$

leading to the fixed point values

$$R^* = 0, R'^* = \frac{97}{183}, R_\tau^* = -\frac{70}{61}, R_b^* = 0, R_t^* = \frac{55}{183}. \quad (53)$$

Since $R_\tau^* < 0$, this fixed point is unacceptable. Thus, a simultaneous IRFP for both the L violating couplings $\lambda_{233}, \lambda'_{333}$, and the third generation Yukawa couplings is not possible.

We now consider the possibility of the two L violating couplings separately, i.e. we shall take either $\lambda_{233} \ll \lambda'_{333}$, or $\lambda'_{333} \ll \lambda_{233}$, respectively. In the first case we reorder the couplings as $R_i = (R', R_\tau, R_b, R_t)$, so that the anomalous dimension matrix, denoted as S_{L1} , now reads

$$S_{L1} = \begin{bmatrix} 6 & 1 & 1 & 1 \\ 3 & 4 & 3 & 0 \\ 6 & 1 & 6 & 1 \\ 1 & 0 & 1 & 6 \end{bmatrix}. \quad (54)$$

Since the determinant of this matrix vanishes, there are no fixed points in this case. We, thus, conclude that a simultaneous fixed point for $\lambda'_{333}, h_\tau, h_b$, and h_t does not exist. We note that the vanishing of the determinant corresponds to a solution with a fixed line or surface.

It must also be noted that even if we were to consider a trivial fixed point for $R_\tau, R_\tau^* = 0$, we still have a singular anomalous dimension matrix. Thus, there are no fixed points for the L violating couplings λ'_{333} , and h_b and h_t , with a trivial fixed point for the τ -Yukawa coupling. We must, however, consider a trivial fixed point for h_b , and non-trivial fixed point for other couplings. We then obtain the fixed point values

$$R_b^* = 0, R'^* = \frac{97}{183}, R_\tau^* = -\frac{70}{61}, R_t^* = \frac{55}{183}, \quad (55)$$

which must, however, be rejected as unphysical.

We must now consider a situation where two of the couplings approach a trivial fixed point value, with the other two approaching a non-trivial fixed point value. We first consider trivial fixed points for h_τ and h_b , with non-trivial fixed points for λ'_{233} and h_t , which is relevant for low values of $\tan \beta$. In this case we obtain the fixed point values

$$R_b^* = R_\tau^* = 0, R'^* = R_t^* = \frac{1}{3}. \quad (56)$$

We must now study the stability of the fixed point solution (56). The stability of $R_b^* = R_\tau^* = 0$ is determined by the sign of the quantities λ_i in (41), which are calculated to be

$$\lambda_1 = 0, \lambda_2 = -\frac{4}{3}, \quad (57)$$

from which we conclude that the fixed point (56) will never be reached in the infra-red region. The fixed point is either a saddle point or an ultra-violet fixed point. Next we consider the case in which λ'_{333} and h_τ approach trivial fixed point values, whereas h_b and h_t attain nontrivial fixed points. We obtain

$$R'^* = R_\tau^* = 0, R_b'^* = R_t^* = \frac{1}{3}. \quad (58)$$

The stability of the fixed point (58) is determined in a manner analogous to that of the fixed point (56). We find that the fixed point (58) is either a saddle point, or an ultra-violet stable fixed point. That the stability properties of the fixed points (56) and (58) are identical is a consequence of the symmetry of the renormalization group equations (21) - (26). We conclude that there are no non-trivial stable fixed points in the infra-red region for the L violating coupling λ'_{333} .

Finally we consider the case when $\lambda'_{333} \ll \lambda_{233}$. We reorder the couplings as $R_i = (R, R_\tau, R_b, R_t)$, with the anomalous dimension matrix S_{L2} given by

$$S_{L2} = \begin{bmatrix} 4 & 4 & 0 & 0 \\ 4 & 4 & 3 & 0 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}, \quad (59)$$

which leads to the fixed point solution

$$R^* = -\frac{97}{36}, R_\tau^* = \frac{35}{18}, R_b^* = 0, R_t^* = \frac{7}{18}, \quad (60)$$

which must be rejected as a fixed point. We, therefore, try a fixed point with $R_\tau^* = 0$, with the result

$$R^* = -\frac{3}{4}, R_b^* = \frac{1}{3}, R_t^* = \frac{1}{3}, \quad (61)$$

which too must be rejected. We then try a trivial fixed point for the b-quark Yukawa coupling $R_b^* = 0$, and non-trivial fixed points for all other coupling. From the matrix (59), we see that the corresponding matrix is singular, so that there are no fixed points. We finally try the fixed point with $R_b^* = R_\tau^* = 0$. For this case we obtain the solution

$$R_b^* = R_\tau^* = 0, R^* = -\frac{3}{4}, R_t^* = \frac{7}{18}, \quad (62)$$

which too is unacceptable. We have checked that the trivial fixed point for λ_{233} , h_b , h_τ , and the Pendleton-Ross type fixed point for h_t , is unstable in the infra-red region. We, therefore, conclude that there are no fixed point solutions for the L violating coupling λ_{233} .

To sum up, we have found that there are no IRSFPs in the MSSM with the highest generation lepton number violation. This result, together with the result on the fixed point with baryon number violation, shows that only the simultaneous non-trivial fixed point for the baryon number violating coupling λ''_{233} , and the top- and bottom-quark Yukawa couplings, h_t and h_b , is stable in the infra-red region.

It is appropriate to examine the implications of the value of the top-quark mass predicted by our fixed point analysis. From (37), it is readily seen that the fixed point value for the top-quark Yukawa coupling translates into a top-quark (pole) mass of about $m_t \simeq 70 \sin \beta$ GeV, which is incompatible with the measured value of [17] of top mass, $m_t \simeq 174$ GeV, for any value of $\tan \beta$. It follows that the true fixed point obtained here provides only a lower bound on the baryon number violating coupling $\lambda''_{233} \gtrsim 0.97$.

B. Infra-red Fixed Points for the Trilinear Soft Supersymmetry Breaking Parameters

We now consider the renormalization group evolution and the fixed point structure for the soft supersymmetry breaking trilinear parameters A_i . As we have seen previously, there is only one IRSFP in the MSSM with B and L violation. We shall, therefore, consider the IRFPs for the A parameters corresponding to this case only, i.e. for A_t , A_b , A_τ and $A_{\lambda''}$.

Retaining only these parameters, and defining $\tilde{A}_i = A_i/M_3$ ($A_i = A_t, A_b, A_\tau, A_{\lambda''}$), we obtain from Eq. (14) - Eq. (19) the relevant renormalization group equations for \tilde{A}_i (neglecting the $SU(2)_L$ and $U(1)_Y$ gauge couplings):

$$\frac{d\tilde{A}_t}{d(-\ln\mu^2)} = \tilde{\alpha}_3 \left[\frac{16}{3} - (6R_t - b_3)\tilde{A}_t - R_b\tilde{A}_b - R''\tilde{A}_{\lambda''} \right], \quad (63)$$

$$\frac{d\tilde{A}_b}{d(-\ln\mu^2)} = \tilde{\alpha}_3 \left[\frac{16}{3} - R_t\tilde{A}_t - (6R_b - b_3)\tilde{A}_b - R_\tau\tilde{A}_\tau - 2R''\tilde{A}_{\lambda''} \right], \quad (64)$$

$$\frac{d\tilde{A}_\tau}{d(-\ln\mu^2)} = \tilde{\alpha}_3 \left[-3R_b\tilde{A}_b - (4R_\tau - b_3)\tilde{A}_\tau \right], \quad (65)$$

$$\frac{d\tilde{A}_{\lambda''}}{d(-\ln\mu^2)} = \tilde{\alpha}_3 \left[8 - 2R_t\tilde{A}_t - 2R_b\tilde{A}_b - (6R'' - b_3)\tilde{A}_{\lambda''} \right], \quad (66)$$

which can be written in the following compact form:

$$\frac{d\tilde{A}_i}{d(-\ln\mu^2)} = \tilde{\alpha}_3 \left[r_i - \sum_j K_{ij}\tilde{A}_j \right], \quad (67)$$

where r_i have been defined in the discussion following Eq. (27), and where K is a matrix whose entries are fully specified by the wave function anomalous dimensions and R_i . A fixed point for A_i is reached when the right hand side of Eq. (67) vanishes for all i . Denoting this fixed point solution by \tilde{A}_i^* , we have

$$r_i - \sum_j K_{ij}^*\tilde{A}_j^* = 0, \quad (68)$$

where K^* is the matrix K evaluated when R_i take their fixed point values R_i^* . With the ordering $\tilde{A}_i = (\tilde{A}_\tau, \tilde{A}_{\lambda''}, \tilde{A}_b, \tilde{A}_t)$, we see from Eq. (63) - Eq. (66) that the matrix K can be written as

$$K^* = \begin{bmatrix} 4R_\tau^* - b_3 & 0 & 3R_b^* & 0 \\ 0 & 6R''^* - b_3 & 2R_b^* & 2R_t^* \\ R_\tau^* & 2R''^* & 6R_b^* - b_3 & R_t^* \\ 0 & 2R''^* & R_b^* & 6R_t^* - b_3 \end{bmatrix}, \quad (69)$$

with $R_\tau^* = 0$, and R''^*, R_b^*, R_t^* given by their fixed point values in (37). The fixed points \tilde{A}_i^* are given by the solution of

$$\tilde{A}_i^* = \sum_j (K^{*-1})_{ij} r_j, \quad (70)$$

with the result

$$\tilde{A}_\tau^* = -\frac{2}{17}, \quad \tilde{A}_{\lambda''}^* = \tilde{A}_b^* = \tilde{A}_t^* = 1. \quad (71)$$

We note that the fixed point values for \tilde{A}_b and \tilde{A}_t are the same as in MSSM with baryon number conservation [6]. However, the fixed point value of the A parameter corresponding to the τ -Yukawa coupling is affected by the presence of the B -violating parameter $\tilde{A}_{\lambda''}$. We further note that the fixed point (71) is infra-red stable because of the general results connecting the stability of a set of A parameters to the stability of the corresponding set of Yukawa couplings [6].

III. QUASI-FIXED POINTS

The infra-red fixed points that we have discussed in the previous section are the true IRFPs of the renormalization group equations. However, these fixed points may not be reached in practice, the range between the GUT scale and the weak scale being too small for the ratios to closely approach the fixed point values. In that case, the various couplings may be determined by the quasi-fixed point behaviour [3], where the value of various couplings at the weak scale is independent of its value at the GUT scale, provided the Yukawa couplings at the unification scale are large.

In this section, we shall discuss the quasi-fixed point behaviour of the Yukawa couplings and the A parameters of the MSSM with B (and R_p) violation.

Before proceeding with the discussion of quasi-fixed point behaviour for the MSSM with baryon and lepton number (and R_p) violation, it is instructive to consider the case of MSSM with R_p conservation, and with only one dominant Yukawa and gauge coupling, h_t and g_3 , respectively. In this case the renormalization group equation (5) for the top-quark Yukawa coupling can be written as

$$\frac{dh_t^2}{d\ln\mu} = \frac{h_t^2}{8\pi^2} \left(6h_t^2 - \frac{16}{3}g_3^2 \right). \quad (72)$$

If at some large scale $\mu = \Lambda$, the top-quark Yukawa coupling is large compared to the strong gauge coupling, $h_t^2(\Lambda) \gg g_3^2(\Lambda)$, then the running of h_t^2 is driven by the term $6h_t^2$ in the right hand side of (72). Thus, we can replace $16g_3^2/3$ in (72) by some constant average value $16\bar{g}_3^2/3$. In the evolution of h_t^2 , with μ running towards the infra-red region, a transient slowing down in the running of h_t^2 is expected in the region of μ where the right hand side of (72), and consequently $dh_t^2/d\ln\mu$, vanishes, leading to the quasi-fixed point [3] for the top Yukawa coupling [18]

$$h_t^{2*} \approx \frac{8}{9}\bar{g}_3^2. \quad (73)$$

It is clear that the quasi-fixed point (73) is not a genuine fixed point, since its position depends on the large scale Λ as well as the initial $g_3^2(\Lambda)$, and it is not attractive for smaller initial values of $h_t(\Lambda)$. In practice, of course, one must evolve $h_t^2(\mu)$, together with $g_3^2(\mu)$, for large initial values of h_t to find the quasi infra-red fixed point. We now apply this procedure to MSSM with all the third generation Yukawa couplings, and with baryon and lepton number violation.

A. Yukawa couplings

Since the simultaneous fixed point for the third generation Yukawa couplings and the baryon number violating coupling $\tilde{A}_{\lambda''}$ is stable, we shall consider the quasi-fixed points for these couplings only, *viz.* for the couplings h_t, h_b, h_τ and λ''_{233} . Similarly, we shall consider the quasi-fixed points for the corresponding A parameters only. For this purpose, we define

$$Y_i = \frac{h_i^2}{4\pi}, \quad i = t, b, \tau, \quad (74)$$

$$Y'' = \frac{\lambda''_{233}}{4\pi}, \quad (75)$$

and write the RG equations for these quantities as

$$\frac{dY_t}{d(\ln\mu)} = \frac{1}{2\pi}Y_t \left(6Y_t + Y_b + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1 \right), \quad (76)$$

$$\frac{dY_b}{d(\ln\mu)} = \frac{1}{2\pi}Y_b \left(Y_t + 6Y_b + Y_\tau + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_1 \right), \quad (77)$$

$$\frac{dY_\tau}{d(\ln\mu)} = \frac{1}{2\pi}Y_\tau \left(3Y_b + 4Y_\tau - 3\alpha_2 - \frac{9}{5}\alpha_1 \right), \quad (78)$$

$$\frac{dY''}{d(\ln\mu)} = \frac{1}{2\pi}Y'' \left(2Y_t + 2Y_b + 6Y'' - 8\alpha_3 - \frac{4}{5}\alpha_1 \right). \quad (79)$$

The existence of the quasi-fixed point requires [10]

$$\frac{dY_t}{d(\ln\mu)} \simeq \frac{dY_b}{d(\ln\mu)} \simeq \frac{dY_\tau}{d(\ln\mu)} \simeq \frac{dY''}{d(\ln\mu)} \simeq 0. \quad (80)$$

The solution of these conditions leads to

$$Y_\tau^* = -\frac{3}{730} [121\alpha_1 + 100(\alpha_2 - \alpha_3)] \simeq -0.02, \quad (81)$$

where we have used $\alpha_1 \simeq 0.017$, $\alpha_2 \simeq 0.033$, $\alpha_3 \simeq 0.1$ at the effective supersymmetry scale, which we take to be 1 TeV. This solution is clearly unphysical. We are, therefore, led to set $Y_\tau^* = 0$, yielding the solution

$$Y_t^* = \frac{47\alpha_1 + 225\alpha_2 + 200\alpha_3}{425} \simeq 0.066, \quad (82)$$

$$Y_b^* = \frac{13\alpha_1 + 225\alpha_2 + 200\alpha_3}{425} \simeq 0.065, \quad (83)$$

$$Y''^* = \frac{2(11\alpha_1 - 45\alpha_2 + 130\alpha_3)}{255} \simeq 0.092, \quad (84)$$

which implies

$$h_t^* \simeq 0.91, \quad (85)$$

$$h_b^* \simeq 0.90, \quad (86)$$

$$\lambda_{233}''^* \simeq 1.08. \quad (87)$$

We note that these quasi-fixed point values are not significantly different from those obtained in a situation when τ -Yukawa coupling is ignored [10].

B. Trilinear Couplings

We now turn our attention to the renormalization group equations (14) - (19) for the A parameters, and their quasi-fixed points. Since the quasi-fixed point of interest is for $h_\tau = 0$, we cannot determine the quasi-fixed point value for A_τ and therefore ignore Eq. (16). In the remaining equations, we substitute $h_\tau = 0$, and obtain the equations that the remaining A parameters must satisfy in order for us to determine the quasi-fixed point solution:

$$6Y_t A_t + Y_b A_b + 2Y'' A_{\lambda''} - \frac{16}{3}\alpha_3 M_3 - 3\alpha_2 M_2 - \frac{13}{15}\alpha_1 = 0, \quad (88)$$

$$Y_t A_t + 6Y_b A_b + 2Y'' A_{\lambda''} - \frac{16}{3}\alpha_3 M_3 - 3\alpha_2 M_2 - \frac{7}{15}\alpha_1 = 0, \quad (89)$$

$$2Y_t A_t + 2Y_b A_b + 6Y'' A_{\lambda''} - 8\alpha_3 M_3 - \frac{4}{5}\alpha_1 = 0. \quad (90)$$

These yield the following quasi-fixed point solution:

$$A_t^* = \frac{47\alpha_1 M_1 + 225\alpha_2 M_2 + 200\alpha_3 M_3}{425Y_t^*} \simeq 0.77m_{\bar{g}}, \quad (91)$$

$$A_b^* = \frac{13\alpha_1 M_1 + 225\alpha_2 M_2 + 200\alpha_3 M_3}{425Y_b^*} \simeq 0.78m_{\bar{g}}, \quad (92)$$

$$A_{\lambda''}^* = \frac{2(11\alpha_1 M_1 - 45\alpha_2 M_2 + 130\alpha_3 M_3)}{255Y''^*} \simeq 1.02m_{\bar{g}}, \quad (93)$$

where $m_{\bar{g}}$ is the gluino mass ($= M_3$) at the weak scale. We have used the fact that the gaugino masses scale as the square of the gauge couplings, and that $\alpha_G \simeq 0.041$ at the scale $M_G \simeq 10^{16}\text{GeV}$. One must compare these quasi-fixed point values with the true fixed-point values (71). We note that the quasi-fixed point values (91) - (93) provide a lower bound on the corresponding A parameters.

IV. NUMERICAL RESULTS AND DISCUSSION

In the previous section we have obtained the approximate quasi-fixed point values for the Yukawa couplings and the A parameters by an algebraic solution of the corresponding RG equations. The RG equations are a set of coupled first-order differential equations that must be solved numerically to obtain accurate values for the fixed points. We have numerically solved the RG equations for the Yukawa couplings, and the A parameters. We now present the results of such a numerical analysis.

In Fig. 1 we show the fixed point behaviour of the top-quark Yukawa coupling as a function of the logarithm of the scale parameter μ . We have included the evolution equations for the b-quark as well as the τ -lepton Yukawa

couplings, as well as the B -violating coupling λ''_{233} , in the numerical solution. It is seen that for all $h_t \gtrsim 1$ at the GUT scale, the top-quark Yukawa coupling approaches its quasi-fixed point at the weak scale. We note that the numerical evolution of fixed point approaches but does not exactly reproduce the approximate analytical value in (85). In Figs. 2 and 3 we present the corresponding approach to the infra-red fixed point for the couplings h_b and λ''_{233} , respectively. These infra-red fixed points provide a model independent theoretical upper bound on the B -violating coupling λ''_{233} .

In Figs. 4, 5 and 6, we present the fixed point behaviour of the corresponding A parameters. We notice the remarkable focussing property seen in the fixed point behaviour of all the A parameters. Again, we notice that the numerical evolution of the fixed point approaches, but does not actually reproduce, the approximate analytical values of Eqs. (91), (92), and (93). Since the quasi-fixed point value for the A parameter is inversely proportional to the quasi-fixed point value of the Yukawa coupling, it provides a lower bound on the corresponding A parameter.

V. SUMMARY AND CONCLUSIONS

We have carried out a detailed renormalization group analysis of the MSSM with all the third generation Yukawa couplings, and with highest generation baryon and lepton number violation. We have shown that the simultaneous fixed point for the top- and bottom-Yukawa couplings, and the B -violating coupling λ''_{233} , is the only fixed point that is stable in the infra-red region. However, the top-quark mass predicted by this fixed point is incompatible with measured value of the top mass. This fixed point, therefore, provides a process-independent lower bound on the baryon number violating coupling at the electroweak scale.

We have shown that all other possible fixed point solutions are either unphysical, or unstable, in the infra-red region. In particular there is no infra-red fixed point with simultaneous B and L violation.

We have also carried out the renormalization group analysis of the corresponding trilinear soft supersymmetry breaking parameters. We have obtained the true fixed points for these parameters, which serve as upper bounds on these parameters.

Since the true fixed points may not be reached in practice at the electroweak scale, we have also obtained the quasi-fixed points of the Yukawa couplings and the trilinear parameters. The quasi-fixed point values for the Yukawa couplings are numerically very close to the values obtained previously by ignoring the τ Yukawa coupling. Since the quasi-fixed points are reached for large initial values of the couplings at the GUT scale, these reflect on the assumption of perturbative unitarity, or the absence of Landau poles, of the corresponding couplings. These quasi-fixed points, therefore, provide an upper bounds on the relevant Yukawa couplings, especially the baryon number violating coupling λ''_{233} . From the true fixed point and the quasi-fixed point analysis we are able to constrain the baryon number violating coupling $0.97 \lesssim \lambda''_{233} \lesssim 1.08$ in a model independent manner.

We have complemented the quasi-fixed point analysis of the Yukawa couplings with an analysis of the corresponding soft supersymmetry breaking trilinear couplings. We have shown that the A parameters for the top- and bottom-quark Yukawa couplings, and the baryon number violating couplings all show striking convergence properties. This strong focussing property is quite independent of the input parameters at the unification scale (or equivalently the pattern of supersymmetry breaking), and the A parameters are, therefore, fully determined in the quasi-fixed regime. However, the actual values of the A parameters in the quasi-fixed regime are significantly different from the case when B -violating Yukawa couplings are ignored. In particular, we have constrained the A parameters to be $0.77 \lesssim A_t/m_g \lesssim 1$, $0.78 \lesssim A_b/m_g \lesssim 1$, and $A_{\lambda''}/m_g \simeq 1$.

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Figure Captions

Fig. 1. Renormalization group evolution of the top-quark Yukawa coupling h_t as a function of the logarithm of the energy scale. We have taken the initial values of h_t at the scale $M_G \sim 10^{16}$ to be 5.0, 4.0, 3.0, 2.0, and 1.0. The initial values of other Yukawa couplings are $h_b = 0.91$, $h_\tau = 0$, and $\lambda''_{233} = 1.08$.

Fig. 2. Renormalization group evolution of the bottom-quark Yukawa coupling h_b as a function of the logarithm of the energy scale. The initial values of h_b at M_G are 5.0, 4.0, 3.0, 2.0, and 1.0. The initial values of other Yukawa couplings are $h_t = 0.92$, $h_\tau = 0$, $\lambda''_{233} = 1.08$.

Fig. 3. Renormalization group evolution of the baryon number violating Yukawa coupling λ''_{233} as a function of the logarithm of the energy scale. The initial values are $\lambda''_{233} = 5.0, 4.0, 3.0, 2.0, 1.0$. The initial values of other Yukawa couplings are $h_t = 0.92$, $h_b = 0.91$, $h_\tau = 0$.

Fig. 4. Renormalization group evolution of ratio A_t/m_g as a function of the logarithm of the energy scale for several different initial values at M_G . The initial values for other parameters at M_G are $h_t = 5.0$, $h_b = 0.91$, $h_\tau = 0$, $\lambda''_{233} = 1.08$, and $A_b/m_g = 1.94$, $A_{\lambda''}/m_g = 2.57$.

Fig. 5. Renormalization group evolution of the ratio A_b/m_g as a function of the logarithm of the energy scale. Other parameters at the scale M_G are $h_b = 5.0$, $h_t = 0.92$, $h_\tau = 0$, $\lambda''_{233} = 1.08$, and $A_t/m_g = 1.92$, $A_{\lambda''}/m_g = 2.57$.

Fig. 6. Renormalization group evolution of the trilinear coupling $A_{\lambda''}/m_g$. Other parameters at the scale M_G are $\lambda''_{233} = 5.0$, $h_t = 0.92$, $h_b = 0.92$, $h_\tau = 0$, and $A_t/m_g = 1.92$, $A_b/m_g = 1.94$.

Fig. 1

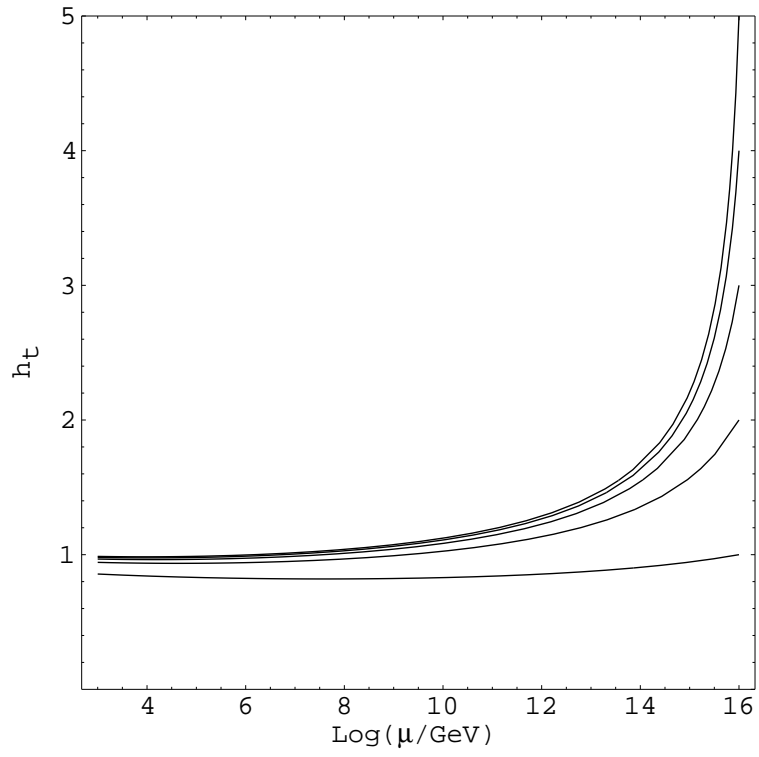


Fig. 2

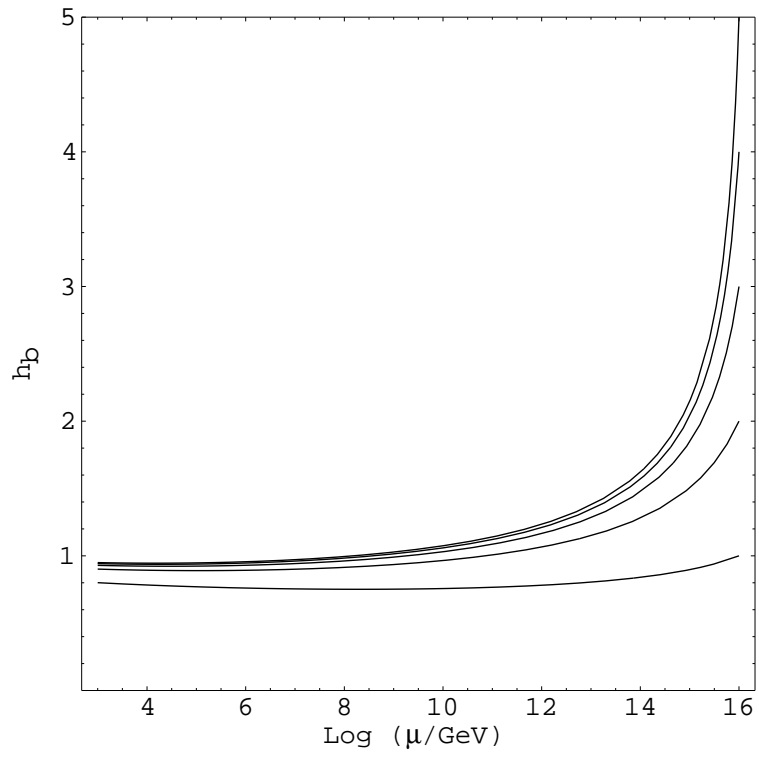


Fig. 3

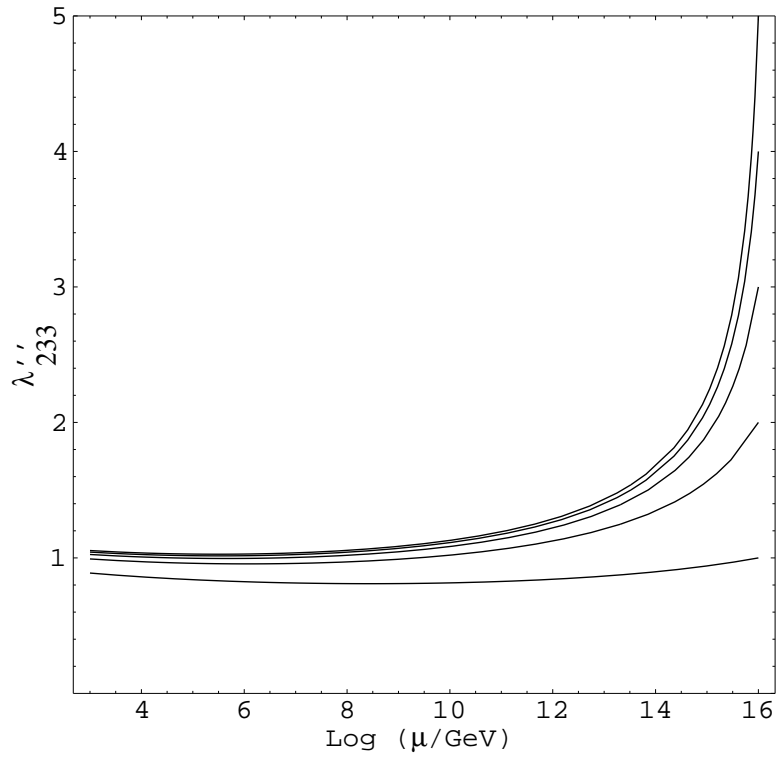


Fig. 4

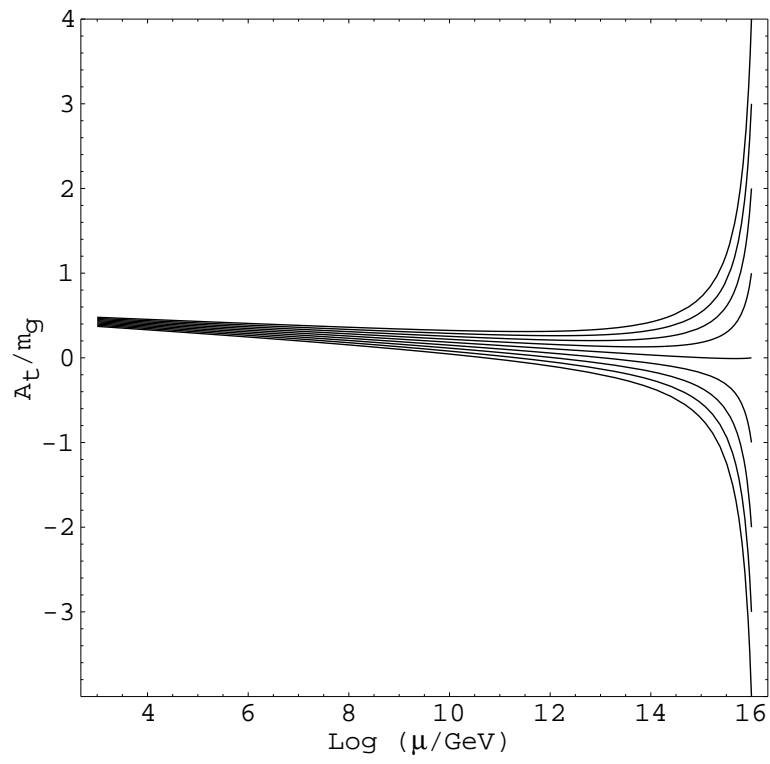


Fig. 5

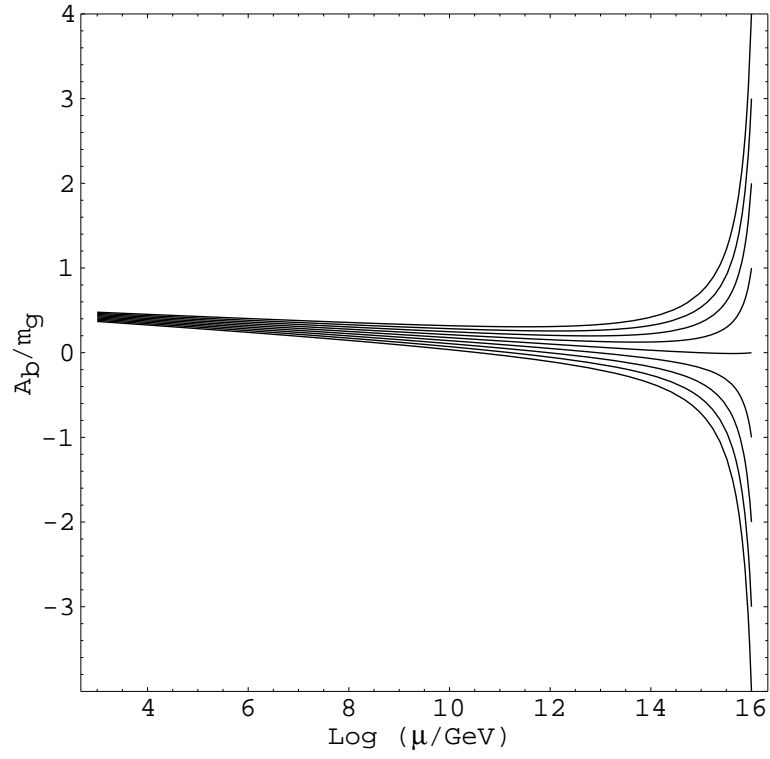


Fig. 6

