Transverse Spin in QCD. I. Canonical Structure

A. Harindranath[*], **Asmita Mukherjee**[†]
Saha Institute of Nuclear Physics, 1/AF, Bidhan Nagar, Calcutta 700064 India
Raghunath Ratabole[‡]
Centre for Theoretical Studies, Indian Institute of Science
Bangalore 560012 India
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Abstract

In this work we initiate a systematic investigation of the spin of a composite system in an arbitrary reference frame in QCD. After a brief review of the difficulties one encounters in equal-time quantization, we turn to light-front quantization. We show that, in spite of the complexities, light-front field theory offers a unique opportunity to address the issue of relativistic spin operators in an arbitrary reference frame since boost is kinematical in this formulation. Utilizing this symmetry, we show how to introduce transverse spin operators for massless particles in an arbitrary reference frame in analogy with those for massive particles. Starting from the manifestly gauge invariant, symmetric energy momentum tensor in QCD, we derive expressions for the interaction dependent transverse spin operators $\mathcal{J}^i$ $(i = 1, 2)$ which are responsible for the helicity flip of the nucleon in light-front quantization. In order to construct $\mathcal{J}^i$, first we derive expressions for the transverse rotation operators F^i. In the gauge $A^+ = 0$, we eliminate the constrained variables. In the completely gauge fixed sector, in terms of the dynamical variables, we show that one can decompose $\mathcal{J}^i = \mathcal{J}^i_I + \mathcal{J}^i_{II} + \mathcal{J}^i_{III}$ where only $\mathcal{J}^i_I$ has explicit coordinate (x^-, x^i) dependence in its integrand. The operators $\mathcal{J}^i_{II}$ and $\mathcal{J}^i_{III}$ arise from the fermionic and bosonic parts respectively of the gauge invariant energy momentum tensor. We discuss the implications of our results.

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[*]e-mail: hari@tnp.saha.ernet.in

[†]e-mail: asmita@tnp.saha.ernet.in

[‡]e-mail: raghu@cts.iisc.ernet.in

I. INTRODUCTION

From the early days of quantum field theory, it has been recognized that the issues associated with the spin of a composite system in an arbitrary reference frame are highly complex and non-trivial [1]. The familiar Pauli-Lubanski operators readily qualify for spin operators *only* in the rest frame of the particle. For a single particle in a moving frame it is known [2] how to construct the appropriate spin operators starting from the Pauli-Lubanski operators. How to construct the spin operators for a composite system in an arbitrary reference frame is a nontrivial problem. In equal-time quantization, the complexities arise from the facts that for a moving composite object, *Pauli-Lubanski operators are necessarily interaction dependent* and, further, it is quite difficult [3] to separate the center of mass and internal variables which is mandatory in the calculation of spin. Due to these difficulties there has been rarely any attempt to study the spin of a moving composite system in the conventional equal time formulation of even simple field theoretic models, let alone Quantum Chromo Dynamics (QCD).

From the early days of light-front field theory, the complications associated with transverse rotation operators F^i have been recognized. They are interaction dependent just like the Hamiltonian. Furthermore, together with the third component of the rotation operator J^3, which is kinematical, F^i do not obey the angular momentum algebra. Instead they obey the algebra of two dimensional Euclidean group which is appropriate only for massless particles. For massive particles, one can define transverse spin operators [4] which together with the third component (helicity) obey the angular momentum algebra. However, they cannot be separated into orbital and spin parts unlike the helicity operator [5]. Most of the studies of the transverse spin operators in light-front field theory, so far, are restricted to free field theory [6]. Even in this case the operators have a complicated structure. However, one can write these operators as a sum of orbital and spin parts, which can be achieved via a unitary transformation, the famous Melosh transformation [7]. In interacting theory, presumably this can be achieved order by order [8] in a suitable expansion parameter which is justifiable only in a weakly coupled theory.

Knowledge about transverse rotation operators and transverse spin operators is mandatory for addressing issues concerning Lorentz invariance in light-front theory. Unfortunately, very little is known [9] regarding the field theoretic aspects of the interaction dependent spin operators, *We emphasize that in a moving frame, the spin operators are interaction dependent irrespective of whether one considers equal-time field theory or light-front field theory.* To the best of our knowledge, in gauge field theory, the canonical structure of spin operators of a composite system in a moving frame has never been studied. In this work we initiate a systematic investigation of the spin of a composite system in a moving frame in QCD. A brief summary of some of our results has been presented in Ref. [10]. We show that, in spite of the complexities, light-front field theory offers a unique opportunity to address the issue of relativistic spin operators in an arbitrary reference frame since boost is kinematical in this formulation.

The plan of this paper is as follows. In Sec. II, first, we briefly review the complexities associated with the description of the spin of a composite system in a moving frame in the conventional equal time quantization. Then we give the canonical structure of light-front Lorentz algebra and light-front spin operators. In this section we also provide a detailed discussion of the transverse spin operators for a massless particle of arbitrary transverse momentum. The explicit form of transverse spin operators in light-front QCD is derived in Sec. III. Summary and conclusions are presented in Sec. IV. For the sake of completeness and clarity, in Appendix A we review the intrinsic spin operators in relativistic quantum mechanics. The explicit form of the kinematical operators and the Hamiltonian in light-front QCD starting from the gauge invariant, symmetric, interaction dependent, energy momentum tensor is derived in Appendix B. A complete discussion of transverse spin operators in free fermion field theory and free massless, spin one boson field theory is presented in detail in Appendices C and D.

II. PRELIMINARIES

In this section, first we briefly review the intrinsic spin operator in equal-time quantization. We highlight the difficulties one encounters in constructing the spin operator of a composite system in an arbitrary reference frame in this case. Next, we give the Lorentz generators in light-front formulation and show that with the help of the kinematical boost in the light-front formalism, a relativistic spin operator for a composite system can be defined in an arbitrary reference frame for massive as well as massless particles. We also compare and contrast the spin operators in equal-time and light-front quantization.

A. Intrinsic Spin in Equal Time Quantization

Intrinsic spin operators in an arbitrary reference frame in equal-time quantization are given [2] in terms of the Poincare generators by (see Appendix A for details)

$$\begin{aligned} \mathbf{S} &= \frac{1}{M}\left[\mathbf{W} - \frac{\mathbf{P}W^0}{M+H}\right] \\ &= \mathbf{J}\,\frac{P^0}{M} - \mathbf{K}\times\frac{\mathbf{P}}{M} - \frac{(\mathbf{J}\cdot\mathbf{P})}{M+P^0}\frac{\mathbf{P}}{M} \end{aligned} \tag{2.1}$$

where $\mathbf{W}$ are the space components of the Pauli-Lubanski operator, $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}M_{\nu\rho}P_\lambda$. H,$\vec{P}$ are equal time Hamiltonian and momentum operators respectively obtained by integrating the energy momentum tensor over a spacelike surface and $\vec{J}$ and $\vec{K}$ are the equal time rotation and boost generators respectively, which are obtained by integrating the angular momentum density over a spacelike surface. Since boost $\mathbf{K}$ is dynamical, *all the three components of* $\mathbf{S}$ *are interaction dependent* in the equal time quantization. Nevertheless, the component of $\mathbf{S}$ along $\mathbf{P}$ remains kinematical. This is to be compared with light-front quantization where *the third component of the light-front spin operator* $\mathcal{J}^3$ *is kinematical* (see Sec. IIB). This arises from the facts that boost operators are kinematical on the light front, the interaction dependence of light-front spin operators $\mathcal{J}^i$ arises solely from the rotation operators, and the third component of the rotation operator J^3 is kinematical on the light front.

A further essential complication arises in equal time quantization. In order to describe the intrinsic spin of a composite system, one should be able to separate the center of mass motion from the internal motion. Even in free field theory, this turns out to be quite involved (See Ref. [3] and references therein). On the other hand, in light-front theory, since transverse boosts are simply Galilean boosts, separation of center of mass motion and internal motion is as simple as in non-relativistic theory. (See Appendix A, of Ref. [11] for a detailed example).

B. Light-Front Lorentz Generators and Algebra

In terms of the gauge invariant, symmetric energy momentum tensor $\Theta^{\mu\nu}$, the four-vector P^μ and the tensor $M^{\mu\nu}$ are given by

$$P^\mu = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+\mu}, \tag{2.2}$$

$$M^{\mu\nu} = \frac{1}{2}\int dx^- d^2x^\perp \left[x^\mu \Theta^{+\nu} - x^\nu \Theta^{+\mu}\right]. \tag{2.3}$$

The boost operators are $M^{+-} = 2K^3$ and $M^{+i} = E^i$. The rotation operators are $M^{12} = J^3$ and $M^{-i} = F^i$. The Hamiltonian P^- and the transverse rotation operators F^i are dynamical (depend on the interaction) while other seven operators are kinematical. The rotation operators obey the $E(2)$-like algebra of two dimensional Euclidean group, namely,

$$[F^1, F^2] = 0,\ [J^3, F^i] = i\epsilon^{ij} F^j \tag{2.4}$$

where ϵ^{ij} is the two-dimensional antisymmetric tensor. Thus F^i do not qualify as angular momentum operators. Moreover, F^i are not translationally invariant and hence they do not qualify as intrinsic spin.

C. Transverse Spin Operators: Massive particle

The Pauli-Lubanski spin operator

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} M_{\nu\rho} P_\sigma \tag{2.5}$$

with $\epsilon^{+-12} = -2$. For a massive particle, the transverse spin operators [4] $\mathcal{J}^i$ in light-front theory are given in terms of Poincare generators by

$$M\mathcal{J}^1 = W^1 - P^1\mathcal{J}^3 = \frac{1}{2}F^2P^+ + K^3P^2 - \frac{1}{2}E^2P^- - P^1\mathcal{J}^3, \tag{2.6}$$

$$M\mathcal{J}^2 = W^2 - P^2\mathcal{J}^3 = -\frac{1}{2}F^1P^+ - K^3P^1 + \frac{1}{2}E^1P^- - P^2\mathcal{J}^3. \tag{2.7}$$

The first term in Eqs. (2.6) and (2.7) contains both center of mass motion and internal motion and the next three terms in these equations serve to remove the center of mass motion.

The helicity operator is given by

$$\mathcal{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1 P^2 - E^2 P^1). \tag{2.8}$$

Here, J^3 contain both center of mass motion and internal motion and the other two terms serve to remove the center of mass motion. The operators $\mathcal{J}^i$ obey the angular momentum commutation relations

$$\left[\mathcal{J}^i, \mathcal{J}^j\right] = i\epsilon^{ijk}\mathcal{J}^k. \tag{2.9}$$

They commute with P^μ.

D. Transverse Spin Operators: Massless case

Again, we start from the Pauli-Lubanski spin operator,

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} M_{\nu\rho} p_\sigma. \tag{2.10}$$

For the light-like vector p^μ, usually the collinear choice is made [12,13], namely, $p^+ \neq 0$, $p^\perp = 0$. Then we get, $W^- = 0$, $W^+ = J^3 p^+$, $W^1 = \frac{1}{2}F^2 p^+$, $W^2 = -\frac{1}{2}F^1 p^+$.

In free field theory, we have explicitly constructed the Poincare generators for a massless spin one particle in $A^+ = 0$ gauge in Appendix D. Consider the single particle state $\mid p\lambda\rangle$ with $p^\perp = 0$. From the explicit form of the operators, we find that

$$\begin{aligned} J^3 \mid p\lambda\rangle &= \lambda \mid p\lambda\rangle, \\ F^i \mid p\lambda\rangle &= 0, \;\; i = 1, 2 \end{aligned} \tag{2.11}$$

since $p^\perp = 0$.

For calculations with composite states (dressed partons, for example) we have to consider light-like particles with arbitrary transverse momenta. Let us try a light like momentum P^μ with $P^\perp \neq 0$, but $P^- = \frac{(P^\perp)^2}{P^+}$ so that $P^2 = 0$. Then we get, as in the case of massive particle,

$$\begin{aligned} W^+ &= J^3 P^+ + E^1 P^2 - E^2 P^1, \\ W^1 &= \frac{1}{2}F^2 P^+ + K^3 P^2 - \frac{1}{2}E^2 P^-, \\ W^2 &= -\frac{1}{2}F^1 P^+ - K^3 P^1 + \frac{1}{2}E^1 P^-, \\ W^- &= F^2 P^1 - F^1 P^2 - J^3 P^-. \end{aligned} \tag{2.12}$$

Thus even though W^1 and W^2 do not annihilate the state, we do get $W^\mu W_\mu (= \frac{1}{2}W^+W^- + \frac{1}{2}W^-W^+ - (W^1)^2 - (W^2)^2) \mid k\lambda\rangle = 0$ as it should be for a massless particle.

Just as in the case of massive particle, we have the helicity operator for the massless particle,

$$\mathcal{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1 P^2 - E^2 P^1). \tag{2.13}$$

In analogy with the transverse spin for massive particles, we define the transverse spin operators for massless particles as

$$\mathcal{J}^i = W^i - P^i \mathcal{J}^3. \tag{2.14}$$

They do satisfy

$$\mathcal{J}^i \mid k\lambda\rangle = 0,$$
$$\mathcal{J}^3 \mid k\lambda\rangle = \lambda \mid k\lambda\rangle, \tag{2.15}$$

where k is an arbitrary momentum. The operators $\mathcal{J}^i$ and $\mathcal{J}^3$ obey the $E(2)$-like algebra

$$\left[\mathcal{J}^1, \mathcal{J}^2\right] = 0, \ \ \left[\mathcal{J}^3, \mathcal{J}^1\right] = i\mathcal{J}^2, \ \ \left[\mathcal{J}^3, \mathcal{J}^2\right] = -i\mathcal{J}^1. \tag{2.16}$$

E. Comments

In order to calculate the transverse spin operators, first we need to construct the Poincare generators P^+, P^i, P^-, K^3, E^i, J^3 and F^i in light-front QCD. The explicit form of the operator J^3 is given Ref. [5]. The construction of F^i which is algebraically quite involved is carried out in the next section. The construction of the rest of the kinematical operators is given in Appendix B. In this appendix we also present the Hamiltonian in a manifestly Hermitian form.

In order to have a physical picture of the complicated situation at hand it is instructive to calculate the spin operator in free field theory. The case of free massive fermion is carried out in Appendix C. In free field theory one can indeed show that (see Appendix C) $\mathcal{J}^i \mid k\lambda\rangle = \frac{1}{2}\sum_{\lambda'} \sigma^i_{\lambda'\lambda} \mid k\lambda'\rangle$. The case of free massless spin one particle is carried out in Appendix D.

III. THE TRANSVERSE ROTATION OPERATOR IN QCD

In this section we derive the expressions for interaction dependent transverse rotation operators in light-front QCD starting from the manifestly gauge invariant energy momentum tensor. It is extremely interesting to compare and contrast the situation in the equal time and light-front case. The angular momentum density

$$\mathcal{M}^{\alpha\mu\nu} = x^\mu \Theta^{\alpha\nu} - x^\nu \Theta^{\alpha\mu}. \tag{3.1}$$

In equal time theory, generalized angular momentum

$$M^{\mu\nu} = \int d^3x \mathcal{M}^{0\mu\nu}. \tag{3.2}$$

The rotation operators are $J^i = \epsilon^{ijk} M^{jk}$. Thus in a non-gauge theory, all the three components of the rotation operators are manifestly interaction independent. However, the spin operators S^i for a composite system in a moving frame involves, in addition to J^i, the boost operators $K^i = M^{0i}$ which are interaction dependent. *Thus all the three components of S^i become interaction dependent.*

A gauge invariant separation of the nucleon angular momentum is performed in Ref. [14]. However, as far the spin operator in an arbitrary reference frame is concerned, the analysis of this reference is valid only in the rest frame where spin coincides with total angular momentum operator and in an arbitrary reference frame the need to project out the center of mass motion, which is quite complicated in equal time theory is not emphasized. Moreover, the distinction between the longitudinal and transverse components of the spin is never made. It is crucial to make this distinction since physically the longitudinal and transverse components of the spin carry quite distinct information (as is clear, for example, from the spin of a massless particle). Moreover, even for the third component of the spin of a composite system in a moving frame, there is crucial difference between equal time and light front cases. $\mathcal{J}^3$ (helicity) is interaction independent whereas S^3 is interaction dependent in general except when measured along the direction of $\mathbf{P}$.

In light-front theory, generalized angular momentum

$$M^{\mu\nu} = \frac{1}{2}\int dx^- d^2x^\perp \mathcal{M}^{+\mu\nu}. \tag{3.3}$$

J^3 which is related to the helicity is given by

$$J^3 = M^{12} = \frac{1}{2}\int dx^1 d^2x^\perp [x^1\Theta^{+2} - x^2\Theta^{+1}] \tag{3.4}$$

and is interaction independent. On the other hand, the transverse rotation operators which are related to the transverse spin are given by

$$F^i = M^{-i} = \frac{1}{2}\int dx^- d^2x^\perp [x^-\Theta^{+i} - x^i\Theta^{+-}].$$

They are interaction dependent even in a non-gauge theory since Θ^{+-} is the Hamiltonian density.

In light-front theory we set the gauge $A^+ = 0$ and eliminate the dependent variables ψ^- and A^- using the equations of constraint. In this paper we restrict to the topologically trivial sector of the theory and set the boundary condition $A^i(x^-, x^i) \to 0$ as $x^{-,i} \to \infty$. This completely fixes the gauge and put all surface terms to zero.

The transverse rotation operator

$$F^i = \frac{1}{2}\int dx^- d^2x^\perp \Big[x^-\Theta^{+i} - x^i\Theta^{+-}\Big]. \tag{3.5}$$

The symmetric, gauge invariant energy momentum tensor

$$\Theta^{\mu\nu} = \frac{1}{2}\overline{\psi}\Big[\gamma^\mu iD^\nu + \gamma^\nu iD^\mu\Big]\psi - F^{\mu\lambda a}F^{\nu a}_{\lambda} - g^{\mu\nu}\Big[-\frac{1}{4}(F_{\lambda\sigma a})^2 + \overline{\psi}(\gamma^\lambda iD_\lambda - m)\psi\Big], \tag{3.6}$$

where

$$\begin{aligned} iD^\mu &= \frac{1}{2}\, i\overleftrightarrow{\partial^\mu} + gA^\mu, \\ F^{\mu\lambda a} &= \partial^\mu A^{\lambda a} - \partial^\lambda A^{\mu a} + gf^{abc}A^{\mu b}A^{\lambda c}, \\ F^{\nu a}_{\lambda} &= \partial^\nu A^a_\lambda - \partial_\lambda A^{\nu a} + gf^{abc}A^{\nu b}A^c_\lambda. \end{aligned} \tag{3.7}$$

First consider the fermionic part of $\Theta^{\mu\nu}$:

$$\Theta^{\mu\nu}_F = \frac{1}{2}\overline{\psi}\Big[\gamma^\mu iD^\nu + \gamma^\nu iD^\mu\Big]\psi - g^{\mu\nu}\overline{\psi}(\gamma^\lambda iD_\lambda - m)\psi. \tag{3.8}$$

The coefficient of $g^{\mu\nu}$ vanishes because of the equation of motion.

Explicitly, the contribution to F^2 from the fermionic part of $\Theta^{\mu\nu}$ is given by

$$\begin{aligned} F^2_F =& \frac{1}{2}\int dx^- d^2x^\perp \left[x^- \frac{1}{2}\overline{\psi}(\gamma^+ iD^2 + \gamma^2 iD^+)\psi - x^2\frac{1}{2}\overline{\psi}(\gamma^+ iD^- + \gamma^- iD^+)\psi\right], \\ =& F^2_{F(I)} + F^2_{F(II)}, \end{aligned} \tag{3.9}$$

where

$$F^2_{F(I)} = \frac{1}{2}\int dx^- d^2x^\perp x^- \Big[\psi^{+\dagger}\frac{1}{2} i\overleftrightarrow{\partial^2} \psi^+ + \psi^{+\dagger} gA^2 \psi^+ + \frac{1}{4}\overline{\psi}\gamma^i\, i\overleftrightarrow{\partial^+}\, \psi\Big], \tag{3.10}$$

$$F^2_{F(II)} = -\frac{1}{2}\int dx^- d^2x^\perp x^2 \Big[\psi^{+\dagger}\Big(\frac{1}{2} i\overleftrightarrow{\partial^-} + gA^-\Big)\psi^+ + \frac{1}{4}\psi^{-\dagger}\gamma^i\, i\overleftrightarrow{\partial^+}\, \psi^-\Big]. \tag{3.11}$$

We have the equation of constraint

$$i\partial^+\psi^- = [\alpha^\perp\cdot(i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+, \tag{3.12}$$

and the equation of motion

$$i\partial^-\psi^+ = -gA^-\psi^+ + [\alpha^\perp\cdot(i\partial^\perp + gA^\perp) + \gamma^0 m]\frac{1}{i\partial^+}[\alpha^\perp\cdot(i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+. \tag{3.13}$$

Using the Eqs. (3.12) and (3.13) we arrive at free (g independent) and interaction (g dependent) parts of F^2_F. The free part of F^2_F is given by

$$\begin{aligned} F^2_{F(free)} =& \frac{1}{2}\int dx^- d^2x^\perp \Bigg\{ x^-\Bigg[\xi^\dagger\Big[i\partial^2\xi\Big] - \Big[i\partial^2\xi^\dagger\Big]\xi\Bigg] \\ & - x^2\Bigg[\xi^\dagger\Big[\frac{-(\partial^\perp)^2+m^2}{i\partial^+}\xi\Big] - \Big[\frac{-(\partial^\perp)^2+m^2}{i\partial^+}\xi^\dagger\Big]\xi\Bigg] \\ & + \Bigg[\xi^\dagger\Big[\sigma^3\partial^1 + i\partial^2\Big]\frac{1}{\partial^+}\xi + \Big[\frac{1}{\partial^+}(\partial^1\xi^\dagger\sigma^3 - i\partial^2\xi^\dagger)\Big]\xi\Bigg] \\ & + m\Bigg[\xi^\dagger\Big[\frac{\sigma^1}{i\partial^+}\xi\Big] - \Big[\frac{1}{i\partial^+}\xi^\dagger\sigma^1\Big]\xi\Bigg]\Bigg\}. \end{aligned} \tag{3.14}$$

We have introduced the two-component field ξ,

$$\psi^+ = \begin{bmatrix}\xi \\ 0\end{bmatrix}. \tag{3.15}$$

The interaction dependent part of $F^2_{F(I)}$ is

$$F^2_{F(I)int} = g\int dx^- d^2x^\perp x^- \xi^\dagger A^2 \xi$$
$$+ \frac{1}{4} g \int dx^- d^2x^\perp \Big[\xi^\dagger \frac{1}{\partial^+}[(-i\sigma^3 A^1 + A^2)\xi] + \frac{1}{\partial^+}[\xi^\dagger(i\sigma^3 A^1 + A^2)]\xi\Big]. \tag{3.16}$$

The interaction dependent part of $F^2_{F(II)}$ is

$$\begin{aligned}
F^2_{F(II)int} = & \frac{1}{4} g \int dx^- d^2x^\perp \Big[\xi^\dagger \frac{1}{\partial^+}[(-i\sigma^3 A^1 + A^2)\xi] + \frac{1}{\partial^+}[\xi^\dagger(i\sigma^3 A^1 + A^2)]\xi\Big] \\
& -\frac{1}{2} g \int dx^- d^2x^\perp x^2 \Bigg[\frac{\partial^\perp}{\partial^+}[\xi^\dagger(\tilde\sigma^\perp \cdot A^\perp)]\tilde\sigma^\perp \xi + \xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)\frac{1}{\partial^+}(\tilde\sigma^\perp\cdot\partial^\perp)\xi \\
& + (\frac{\partial^\perp}{\partial^+}\xi^\dagger)\tilde\sigma^\perp(\tilde\sigma^\perp\cdot A^\perp)\xi + \xi^\dagger\frac{1}{\partial^+}(\tilde\sigma^\perp\cdot\partial^\perp)(\tilde\sigma^\perp\cdot A^\perp)\xi \\
& - m\frac{1}{\partial^+}[\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)]\xi + m\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)\frac{1}{\partial^+}\xi \\
& + m(\frac{1}{\partial^+}\xi^\dagger)(\tilde\sigma^\perp\cdot A^\perp)\xi - m\xi^\dagger\frac{1}{\partial^+}[(\tilde\sigma^\perp\cdot A^\perp)\xi]\Bigg] \\
& -\frac{1}{2} g^2 \int dx^- d^2x^\perp x^2 \Bigg[\xi^\dagger\tilde\sigma^\perp\cdot A^\perp \frac{1}{i\partial^+}\tilde\sigma^\perp\cdot(A^\perp\xi) - \frac{1}{i\partial^+}(\xi^\dagger\tilde\sigma^\perp\cdot A^\perp)\tilde\sigma^\perp\cdot A^\perp\xi\Bigg].
\end{aligned} \tag{3.17}$$

We have introduced $\tilde\sigma^1 = \sigma^2$ and $\tilde\sigma^2 = -\sigma^1$.

Next consider the gluonic part of the operator F^2:

$$F^2_g = \frac{1}{2}\int dx^- d^2x^\perp \Big[x^- \Theta^{+2}_g - x^2 \Theta^{+-}_g\Big], \tag{3.18}$$

where

$$\begin{aligned}
\Theta^{+2}_g &= -F^{+\lambda a}F^{2a}_{\lambda}, \\
\Theta^{+-}_g &= -F^{+\lambda a}F^{-a}_{\lambda} + \frac{1}{4}g^{+-}(F_{\lambda\sigma a})^2.
\end{aligned} \tag{3.19}$$

Using the constraint equation

$$\frac{1}{2}\partial^+ A^{-a} = \partial^i A^{ia} + g f^{abc}\frac{1}{\partial^+}(A^{ib}\partial^+ A^{ic}) + 2g\frac{1}{\partial^+}\Big(\xi^\dagger T^a \xi\Big), \tag{3.20}$$

we arrive at

$$F^2_g = F^2_{g(free)} + F^2_{g(int)} \tag{3.21}$$

where

$$\begin{aligned}
F^2_{g(free)} = & \frac{1}{2}\int dx^- d^2x^\perp \Bigg\{x^-\Big(A^{ja}\partial^+\partial^j A^{2a} - A^{2a}\partial^+\partial^j A^{ja} + A^{ja}\partial^+\partial^2 A^{ja}\Big) \\
& - x^2\Big(A^{ka}(\partial^j)^2 A^{ka}\Big)\Bigg\} \\
& - 2\int dx^- d^2x^\perp A^{2a}\partial^1 A^{1a}.
\end{aligned} \tag{3.22}$$

The interaction part

$$F^2_{g(int)} = \frac{1}{2}\int dx^- d^2x^\perp x^- \Big\{ g f^{abc}\partial^+ A^{ia} A^{2b} A^{ic}$$
$$+ g\Big(f^{abc}\frac{1}{\partial^+}(A^{ib}\partial^+ A^{ic}) + 2\frac{1}{\partial^+}(\xi^\dagger T^a \xi)\Big)\partial^+ A^{2a}\Big\}$$
$$- \frac{1}{2}\int dx^- d^2x^\perp x^2 \Big\{ 2g f^{abc}\partial^i A^{ja} A^{ib} A^{jc} + \frac{g^2}{2} f^{abc} f^{ade} A^{ib} A^{jc} A^{id} A^{je}$$
$$+ 2g\partial^i A^{ia}\frac{1}{\partial^+}\Big(f^{abc} A^{jb}\partial^+ A^{jc} + 2\xi^\dagger T^a \xi\Big)$$
$$+ g^2\Big(f^{abc}\frac{1}{\partial^+}(A^{ib}\partial^+ A^{ic}) + 2\frac{1}{\partial^+}\xi^\dagger T^a \xi\Big)\Big(f^{ade}\frac{1}{\partial^+}(A^{jd}\partial^+ A^{je}) + 2\frac{1}{\partial^+}\xi^\dagger T^a \xi\Big)\Big\}. \quad (3.23)$$

So the full transverse rotation operator in QCD can be written as,

$$F^2 = F^2_I + F^2_{II} + F^2_{III}, \quad (3.24)$$

where

$$F^2_I = \frac{1}{2}\int dx^- d^2x^\perp [x^- \mathcal{P}^2_0 - x^2(\mathcal{H}_0 + \mathcal{V})], \quad (3.25)$$

$$F^2_{II} = \frac{1}{2}\int dx^- d^2x^\perp \Big[\xi^\dagger\big[\sigma^3\partial^1 + i\partial^2\big]\frac{1}{\partial^+}\xi + \big[\frac{1}{\partial^+}(\partial^1\xi^\dagger\sigma^3 - i\partial^2\xi^\dagger)\big]\xi\Big]$$
$$+ \frac{1}{2}\int dx^- d^2x^\perp m\Big[\xi^\dagger\big[\frac{\sigma^1}{i\partial^+}\xi\big] - \big[\frac{1}{i\partial^+}\xi^\dagger\sigma^1\big]\xi\Big]$$
$$+ \frac{1}{2}\int dx^- d^2x^\perp g\Big[\xi^\dagger\frac{1}{\partial^+}[(-i\sigma^3 A^1 + A^2)\xi] + \frac{1}{\partial^+}[\xi^\dagger(i\sigma^3 A^1 + A^2)]\xi\Big], \quad (3.26)$$

$$F^2_{III} = -\int dx^- d^2x^\perp 2(\partial^1 A^1) A^2$$
$$- \frac{1}{2}\int dx^- d^2x^\perp g\frac{4}{\partial^+}(\xi^\dagger T^a\xi)A^{2a} - \frac{1}{2}\int dx^- d^2x^\perp g f^{abc}\frac{2}{\partial^+}(A^{ib}\partial^+ A^{ic})A^{2a} \quad (3.27)$$

where $\mathcal{P}^i_0$ is the free momentum density, $\mathcal{H}_o$ is the free Hamiltonian density and $\mathcal{V}$ are the interaction terms in the Hamiltonian in manifestly Hermitian form (see Appendix B). The operators F^2_{II} and F^2_{III} whose integrands do not explicitly depend upon coordinates arise from the fermionic and bosonic parts respectively of the gauge invariant, symmetric, energy momentum tensor in QCD. The above separation is slightly different from that in [10]. From Eq. (2.6) in Sec. II it follows that the transverse spin operators $\mathcal{J}^i$, $(i = 1, 2)$ can also be written as the sum of three parts, $\mathcal{J}^i_I$ whose integrand has explicit coordinate dependence, $\mathcal{J}^i_{II}$ which arises from the fermionic part, and $\mathcal{J}^i_{III}$ which arises from the bosonic part of the energy momentum tensor.

IV. SUMMARY AND CONCLUSIONS

We have initiated the study of spin operators in QCD. In equal time quantization, one encounters two major difficulties in the description of the spin of a composite system in an arbitrary reference frame. They are 1) the complicated interaction dependence arising from dynamical boost operators and 2) the difficulty in the separation of center of mass motion from the internal motion. Due to these severe difficulties, there have been hardly any attempt to study spin operators of a moving composite system in the conventional equal time formulation of quantum field theory.

In light-front theory, on the other hand, the longitudinal spin operator (light-front helicity) is interaction independent and the interaction dependence of transverse spin operators arises solely from that of transverse rotation operators. Moreover, in this case the separation of center of mass motion from internal motion is trivial since light-front transverse boosts are simple Galilean boosts.

We have investigated the case of transverse spin operators for both massive and massless particles. A novel feature here is the introduction of transverse spin operators for massless particles with arbitrary transverse momentum. *To the best of our knowledge, this is done for the first time in light-front field theory.* To provide physical intuition for transverse spin operators which have a complicated structure in interaction theory, we have provided the explicit form of these operators in Fock space basis for both free fermion field theory and free massless spin one field theory.

In QCD, our starting point is the formula for transverse rotation operators expressed as the integral of generalized angular momentum density given in terms of gauge invariant, symmetric, energy momentum tensor. We have emphasized the differences between spin operators in field theory in equal time and light-front quantization schemes.

Appropriate to light-front quantization, we choose the light-front gauge. We use the constraint equations for ψ^- and A^- to eliminate them in favor of dynamical degrees of freedom. In this initial study, we restrict to topologically trivial sector of QCD and set the requirement that the transverse gauge fields vanish as $x^{-,i} \rightarrow \infty$. This eliminates the surface terms and completely fixes the gauge. In the gauge fixed theory we found that the transverse rotation operators can be decomposed as the sum of three distinct terms: F^i_I which has explicit coordinate dependence in its integrand, and F^i_{II} and F^i_{III} which have no explicit coordinate dependence in their integrand. Further, F^i_{II} and F^i_{III} arise from the fermionic and bosonic parts of the energy momentum tensor. Since transverse spin is responsible for the helicity flip of the nucleon in light-front theory, we now have identified the complete set of operators responsible for the helicity flip of the nucleon.

It is extremely interesting to contrast the cases of longitudinal and transverse spin operators in light-front field theory. In the case of longitudinal spin operator (light-front helicity), in the gauge fixed theory, the operator is interaction independent and can be separated into orbital and spin parts for quarks and gluons. It is known for a long time that the transverse spin operators in light-front field theory cannot be separated into orbital and spin parts except in the trivial case of free field theory. *In this work, we have shown that, in spite of the complexities, a physically interesting separation is indeed possible for the transverse spin operators* which is quite different from the separation into orbital and spin parts in the rest frame familiar in the equal time picture.

In light-front theory, in addition to the Hamiltonian, transverse spin operators also contain interactions and have a complicated structure. Since transverse rotational symmetry is not manifest in light-front theory a study of these operators is essential for questions regarding Lorentz invariance in the theory [15]. An important issue in the case of transverse spin operators concerns renormalization. Since they are interaction dependent, they will acquire divergences in perturbation theory just like the Hamiltonian. It is of interest to find the physical meaning of these divergences and their renormalization. We address these issues in Ref. [11] by computing the expectation value of the transverse spin operators in a dressed quark state.

In this work we have explored in detail the theoretical aspects of spin operators in quantum field theory in the context of QCD and their consequences. Our construction and decomposition of the transverse spin operators in QCD also have important phenomenological consequences. Elsewhere, we have shown [10] that nucleon expectation values of F^i_{II} and F^i_{III} are directly related to the integrals of quark and gluon distribution functions that appear in transversely polarized deep inelastic scattering. Our results show that one can relate nucleon expectation values of operators appearing in the transverse spin to transversely polarized deep inelastic scattering. It is interesting to establish a transverse spin sum rule in analogy to the helicity sum rule and explore its phenomenological consequences [11].

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APPENDIX A: INTRINSIC SPIN IN RELATIVISTIC QUANTUM MECHANICS

In this appendix, for the sake of clarity and completeness we review the intrinsic spin operators in relativistic quantum mechanics [16]. The unitary representations of the Poincare group can be usefully classified on the basis of sign of M^2, where $M^2 = P^\mu P_\mu$ (and further by the sign of H in case $M^2 \geq 0$). We consider two classes of representations which are of physical importance:

- Positive time-like representations: $M^2 > 0 \;\; H > 0$
- Positive light-like representations: $M^2 = 0 \;\; H > 0$

In either cases we do not demand that the representations be irreducible (this allows us to deal with elementary and composite systems simultaneously).

a. Positive time-like representations

Beginning from the basic generators $H, \mathbf{P}, \mathbf{J}$, and $\mathbf{K}$ one can construct an operator $\mathbf{S}$ such that it is translationally invariant, transforms as a three vector under pure rotations and within itself obeys $SU(2)$ commutation relations.

$$[S^j, P^\mu] = 0, \quad [J^j, S^k] = i\epsilon^{jkl} S^l, \quad [S^j, S^k] = i\epsilon^{jkl} S^l. \tag{A1}$$

A suitable solution to the above requirements is provided by

$$\mathbf{S} = \frac{1}{M}\left[\mathbf{W} - \frac{\mathbf{P}W^0}{M+H}\right]$$
$$= \mathbf{J}\,\frac{P^0}{M} - \mathbf{K}\times\frac{\mathbf{P}}{M} - \frac{(\mathbf{J}\cdot\mathbf{P})}{M+P^0}\frac{\mathbf{P}}{M} \tag{A2}$$

where $\mathbf{W}$ are the space components of the Pauli-Lubanski operator, $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}M_{\nu\rho}P_\lambda$.

The operators $\mathbf{S}$ cease to be defined when M tends to zero. The commutation relations among $\mathbf{P}, \mathbf{S}$ and M are given by

$$[P^j, S^k] = 0, \quad [S^j, S^k] = i\epsilon^{jkl}S^l, \quad [S^j, M] = 0. \tag{A3}$$

Since $\mathbf{P}$ and M stand for the momentum and invariant mass of the system, the above relations make clear that $\mathbf{S}$ should represent 'intrinsic spin' of the system.

The invariant W^2 can be completely expressed in terms of M and $\mathbf{S}$ as

$$W^2 = -M^2\mathbf{S}^2. \tag{A4}$$

b. Positive light-like representations

Begining from the basic generators $\mathbf{P}$, $\mathbf{J}$ and $\mathbf{K}$ (here $H = |\mathbf{P}|$) one has to construct operators S, $\mathcal{T}^1$ and $\mathcal{T}^2$ such that they commute with four momentum P^μ and amongst themselves satisfy $E(2)$ commutation relations:

$$[S, \mathcal{T}^1] = i\mathcal{T}^2, \; [S, \mathcal{T}^2] = -i\mathcal{T}^1, \; [\mathcal{T}^1, \mathcal{T}^2] = 0. \tag{A5}$$

A suitable solution consistent with the above requirements is:

$$S = \frac{W^0}{|\,\mathbf{P}\,|},$$
$$\mathcal{T}^1 = W^1 - P^1\frac{(W^3 + W^0)}{(|\,\mathbf{P}\,| + P^3)},$$
$$\mathcal{T}^2 = W^2 - P^2\frac{(W^3 + W^0)}{(|\,\mathbf{P}\,| + P^3)}. \tag{A6}$$

Note that although $\mathcal{T}_1$ and $\mathcal{T}_2$ coincide with the front definitions, the difference lies in the remaining component. Note that here S is the component of angular momentum in the direction of motion. To further bring out the difference, we note in passing that S is a scalar under pure spatial rotation, while shows complicated behaviour under pure boosts. Contrast this with the fact that $\mathcal{J}^3$ is front boost invariant.

c. Comments

The generators for a multi-particle relativistic system have been analyzed by several authors [3]. The expressions obtained are too complicated to be used in any practical calculations and the generators cannot be easily separated into the center of mass and internal variables. Moreover, the derivations have been done neglecting the field theoretical effects such as pair creation and crossing and so are expected to be valid in the relatively low energy region where an expansion in $\frac{v}{c}$ is permissible. Interactions are to be incorporatated by introducing an effective potential which vanish sufficiently rapidly for large distance.

APPENDIX B: POINCARE GENERATORS IN LIGHT-FRONT QCD

In this appendix we derive the manifestly hermitian kinematical Poincare generators (except J^3) and the Hamiltonian in light-front QCD starting from the gauge invariant symmetric energy momentum tensor $\Theta^{\mu\nu}$. To begin with, $\Theta^{\mu\nu}$ is interaction dependent. In the *gauge fixed* theory we find that the seven kinematical generators are manifestly independent of the interaction.

We shall work in the gauge $A^+ = 0$ and ignore all surface terms. Thus we are working in the completely gauge fixed sector of the theory [5]. The explicit form of the operator J^3 in this case is given in Ref. [5] which is manifestly free of interaction at the operator level. The rotation operators are given in Sec. III.

At $x^+ = 0$, the operators K^3 and E^i depend only on the density Θ^{++}. A straightforward calculation leads to

$$\Theta^{++} = \psi^{+\dagger}\, i\overleftrightarrow{\partial^+}\, \psi^+ + \partial^+ A^i \partial^+ A^i. \tag{B1}$$

Then, longitudinal momentum operator,

$$\begin{aligned} P^+ &= \frac{1}{2}\int dx^- d^2x^\perp \Theta^{++} \\ &= \frac{1}{2}\int dx^- d^2x^\perp \left[\psi^{+\dagger}\, i\overleftrightarrow{\partial^+}\, \psi^+ + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{B2}$$

Generator of longitudinal scaling,

$$\begin{aligned} K^3 &= -\frac{1}{4}\int dx^- d^2x^\perp x^- \Theta^{++}, \\ &= -\frac{1}{4}\int dx^- d^2x^\perp x^- \left[\psi^{+\dagger}\, i\overleftrightarrow{\partial^+}\, \psi^+ + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{B3}$$

Transverse boost generators,

$$\begin{aligned} E^i &= -\frac{1}{2}\int dx^- d^2x^\perp x^i \Theta^{++}, \\ &= -\frac{1}{2}\int dx^- d^2x^\perp x^i \left[\psi^{+\dagger}\, i\overleftrightarrow{\partial^+}\, \psi^+ + \partial^+ A^j \partial^+ A^j\right]. \end{aligned} \tag{B4}$$

The transverse momentum operator

$$P^i = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+i} \tag{B5}$$

which appears to have explicit interaction dependence. Using the constraint equations for ψ^- and A^-, we have

$$\begin{aligned} \Theta^{+i} &= \Theta^{+i}_F + \Theta^{+i}_G, \\ \Theta^{+i}_F &= 2\psi^{+\dagger} i\partial^i \psi^+ + 2g\psi^{+\dagger} A^i \psi^+, \end{aligned} \tag{B6}$$

$$\Theta^{+i}_G = \partial^+ A^j \partial^i A^j - \partial^+ A^j \partial^j A^i + \partial^+ A^i \partial^j A^j - 2g\psi^{+\dagger} A^i \psi^+. \tag{B7}$$

Thus

$$P^i = \frac{1}{2}\int dx^- d^2x^\perp \Big[\psi^{+\dagger}\, i\overleftrightarrow{\partial^i}\, \psi^+ + A^j\partial^+\partial^j A^i - A^i\partial^+\partial^j A^j - A^j\partial^+\partial^i A^j\Big]. \tag{B8}$$

Thus we indeed verify that all the kinematical operators are explicitly independent of interactions.

Lastly, the Hamiltonian operator can be written in the manifestly Hermitian form as,

$$P^- = \frac{1}{2}\int dx^- d^2x^\perp \Theta^{+-} = \frac{1}{2}\int dx^- d^2x^\perp (\mathcal{H}_0 + \mathcal{H}_{int}) \tag{B9}$$

where $\mathcal{H}_0$ is the free part given by,

$$\mathcal{H}_0 = -A^j_a(\partial^i)^2 A^j_a + \xi^\dagger\Big[\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\Big]\xi - \Big[\frac{-(\partial^\perp)^2 + m^2}{i\partial^+}\xi^\dagger\Big]\xi. \tag{B10}$$

The interaction terms are given by,

$$\mathcal{H}_{int} = \mathcal{H}_{qqg} + \mathcal{H}_{ggg} + \mathcal{H}_{qqgg} + \mathcal{H}_{qqqq} + \mathcal{H}_{gggg}, \tag{B11}$$

where,

$$\begin{aligned}\mathcal{H}_{qqg} = -4g\xi^\dagger\frac{1}{\partial^+}(\partial^\perp.A^\perp)\xi + g\frac{\partial^\perp}{\partial^+}[\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)]\tilde\sigma^\perp\xi + g\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)\frac{1}{\partial^+}(\tilde\sigma^\perp\cdot\partial^\perp)\xi \\ + g(\frac{\partial^\perp}{\partial^+}\xi^\dagger)\tilde\sigma^\perp(\tilde\sigma^\perp\cdot A^\perp)\xi + g\xi^\dagger\frac{1}{\partial^+}(\tilde\sigma^\perp\cdot\partial^\perp)(\tilde\sigma^\perp\cdot A^\perp)\xi \\ - mg\frac{1}{\partial^+}[\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)]\xi + mg\xi^\dagger(\tilde\sigma^\perp\cdot A^\perp)\frac{1}{\partial^+}\xi \\ + mg(\frac{1}{\partial^+}\xi^\dagger)(\tilde\sigma^\perp\cdot A^\perp)\xi - mg\xi^\dagger\frac{1}{\partial^+}[(\tilde\sigma^\perp\cdot A^\perp)\xi],\end{aligned} \tag{B12}$$

$$\mathcal{H}_{ggg} = 2gf^{abc}\Big[\partial^i A^j_a A^i_b A^j_c + (\partial^i A^i_a)\frac{1}{\partial^+}(A^j_b\partial^+ A^j_c)\Big], \tag{B13}$$

$$\begin{aligned}\mathcal{H}_{qqgg} = g^2\Big[\xi^\dagger(\tilde\sigma^\perp.A^\perp)\frac{1}{i\partial^+}(\tilde\sigma^\perp.A^\perp)\xi - \frac{1}{i\partial^+}(\xi^\dagger\tilde\sigma^\perp.A^\perp)\tilde\sigma^\perp.A^\perp\xi \\ + 4\frac{1}{\partial^+}(f^{abc}A^i_b\partial^+ A^i_c)\frac{1}{\partial^+}(\xi^\dagger T^a\xi)\Big],\end{aligned} \tag{B14}$$

$$\mathcal{H}_{qqqq} = 4g^2\frac{1}{\partial^+}(\xi^\dagger T^a\xi)\frac{1}{\partial^+}(\xi^\dagger T^a\xi), \tag{B15}$$

$$\begin{aligned}\mathcal{H}_{gggg} = \frac{g^2}{2}f^{abc}f^{ade}\Big[A^i_b A^j_c A^i_d A^j_e \\ + 2\frac{1}{\partial^+}(A^i_b\partial^+ A^i_c)\frac{1}{\partial^+}(A^j_d\partial^+ A^j_e)\Big].\end{aligned} \tag{B16}$$

APPENDIX C: TRANSVERSE SPIN IN FREE FERMION FIELD THEORY

1. Poincare Generators: Operator Forms

The symmetric energy momentum tensor

$$\Theta^{\mu\nu} = \Big[\overline{\psi}\gamma^{\mu}\frac{1}{4}\, i\overleftrightarrow{\partial}^{\nu}\, \psi + \overline{\psi}\gamma^{\nu}\frac{1}{4}\, i\overleftrightarrow{\partial}^{\mu}\, \psi\Big]. \tag{C1}$$

The momentum operators are given by

$$\begin{aligned} P^{+} &= \frac{1}{2}\int dx^{-}d^2x^{\perp}\overline{\psi}\gamma^{+}\frac{1}{2}\, i\overleftrightarrow{\partial}^{+}\, \psi \\ &= \frac{1}{2}\int dx^{-}d^2x^{\perp}[\xi^{\dagger}i\partial^{+} - (i\partial^{+}\xi^{\dagger})]\xi. \end{aligned} \tag{C2}$$

$$\begin{aligned} P^{i} &= \frac{1}{2}\int dx^{-}d^2x^{\perp}\Big[\overline{\psi}\Big\{\gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{i} + \gamma^{i}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+}\Big\}\psi\Big] \\ &= \frac{1}{2}\int dx^{-}d^2x^{\perp}[\xi^{\dagger}i\partial^{i} - (i\partial^{i}\xi^{\dagger})]\xi. \end{aligned} \tag{C3}$$

The Hamiltonian operator is

$$\begin{aligned} P^{-} &= \frac{1}{2}\int dx^{-}d^2x^{\perp}\overline{\psi}\Big[\gamma^{-}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+} + \gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{-}\Big]\psi \\ &= \frac{1}{2}\int dx^{-}d^2x^{\perp}\Big[\xi^{\dagger}\frac{1}{i\partial^{+}}[m_F^2 - (\partial^{\perp})^2] - (\frac{1}{i\partial^{+}}[m_F^2 - (\partial^{\perp})^2]\xi^{\dagger})\Big]\xi. \end{aligned} \tag{C4}$$

The longitudinal scaling operator (at $x^{+} = 0$) is

$$\begin{aligned} K^{3} &= -\frac{1}{2}\int dx^{-}d^2x^{\perp}x^{-}\Big[\overline{\psi}\gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+}\, \psi\Big] \\ &= -\frac{i}{4}\int dx^{-}d^2x^{\perp}x^{-}[\xi^{\dagger}\partial^{+}\xi - (\partial^{+}\xi^{\dagger})\xi]. \end{aligned} \tag{C5}$$

The transverse boost operators are

$$\begin{aligned} E^{i} &= -\frac{1}{4}\int dx^{-}d^2x^{\perp}x^{i}\Big[\overline{\psi}\gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+}\, \psi\Big] \\ &= -\frac{1}{4}\int dx^{-}d^2x^{\perp}x^{i}[\xi^{\dagger}i\partial^{+} - (i\partial^{+}\xi^{\dagger})]\xi. \end{aligned} \tag{C6}$$

The generators of rotations are

$$\begin{aligned} J^{3} &= \frac{1}{2}\int dx^{-}d^2x^{\perp}\Big\{x^{1}\Big[\overline{\psi}\Big\{\gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{2} + \gamma^{2}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+}\Big\}\psi\Big] \\ &\quad - x^{2}\Big[\overline{\psi}\Big\{\gamma^{+}\frac{1}{4}\, i\overleftrightarrow{\partial}^{1} + \gamma^{1}\frac{1}{4}\, i\overleftrightarrow{\partial}^{+}\Big\}\psi\Big]\Big\} \\ &= \int dx^{-}d^2x^{\perp}\Big[\xi^{\dagger}[\frac{i}{2}(x^{1}\overrightarrow{\partial}^{2} - x^{2}\overrightarrow{\partial}^{1})\xi - [\frac{i}{2}(x^{1}\overrightarrow{\partial}^{2} - x^{2}\overrightarrow{\partial}^{1})\xi^{\dagger}]\xi \\ &\quad + \xi^{\dagger}\frac{\sigma_3}{2}\xi]. \end{aligned} \tag{C7}$$

and

$$F^i = \frac{1}{2}\int dx^- d^2x^\perp \Big\{ x^- \Big[\overline{\psi}\Big\{ \gamma^+ \frac{1}{4}\ i\overleftrightarrow{\partial}^i + \gamma^i \frac{1}{4}\ i\overleftrightarrow{\partial}^+ \Big\}\psi \Big]$$

$$- x^i \Big[\overline{\psi}\Big\{ \gamma^+ \frac{1}{4}\ i\overleftrightarrow{\partial}^- + \frac{1}{4}\gamma^-\ i\overleftrightarrow{\partial}^+ \Big\}\psi \Big]\Big\}$$

$$= \frac{i}{2}\int dx^- d^2x^\perp \xi^\dagger \Big[x^i (m^2 - (\partial^\perp)^2)\frac{1}{\partial^+} - x^- \frac{\partial}{\partial x^i}$$

$$+ \frac{1}{\partial^+}\Big\{ -\frac{\partial}{\partial x^i} - i\epsilon^{ij}\sigma^3 \frac{\partial}{\partial x^j} + \epsilon^{ij} m \sigma^j \Big\}\Big]\xi$$

$$- \frac{i}{2}\int dx^- d^2x^\perp \Big[[x^i (m^2 - (\partial^\perp)^2)\frac{1}{\partial^+} - x^- \frac{\partial}{\partial x^i}$$

$$+ \frac{1}{\partial^+}\Big\{ -\frac{\partial}{\partial x^i} + i\epsilon^{ij}\sigma^3 \frac{\partial}{\partial x^j} + \epsilon^{ij} m \sigma^j \Big\}\xi^\dagger]\Big]\xi. \qquad \text{(C8)}$$

2. Fock Representation

Free spin-half field operator is

$$\xi(x) = \sum_\lambda \chi_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3\sqrt{k^+}} [b(k,\lambda)e^{-ik.x} + d^\dagger(k,-\lambda)e^{ik.x}]. \qquad \text{(C9)}$$

In terms of Fock space operators

$$P^+ = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^+ \sum_\lambda [b^\dagger(k,\lambda)b(k,\lambda) + d^\dagger(k,-\lambda)d(k,-\lambda)]. \qquad \text{(C10)}$$

$$P^i = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^i \sum_\lambda [b^\dagger(k,\lambda)b(k,\lambda) + d^\dagger(k,-\lambda)d(k,-\lambda)]. \qquad \text{(C11)}$$

$$P^- = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \frac{m_F^2 + (k^\perp)^2}{k^+} \sum_\lambda [b^\dagger(k,\lambda)b(k,\lambda) + d^\dagger(k,-\lambda)d(k,-\lambda)]. \qquad \text{(C12)}$$

$$K^3 = -\frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^+ \sum_\lambda \Big([\frac{\partial b^\dagger(k,\lambda)}{\partial k^+} b(k,\lambda) + \frac{\partial d^\dagger(k,-\lambda)}{\partial k^+} d(k,-\lambda)]$$

$$- [b^\dagger(k,\lambda)\frac{\partial b(k,\lambda)}{\partial k^+} + d^\dagger(k,-\lambda)\frac{\partial d(k,-\lambda)}{\partial k^+}]\Big). \qquad \text{(C13)}$$

$$E^i = \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \sum_\lambda k^+ \Big([\frac{\partial b^\dagger(k,\lambda)}{\partial k^i} b(k,\lambda) + \frac{\partial d^\dagger(k,-\lambda)}{\partial k^i} d(k,-\lambda)]$$

$$- [b^\dagger(k,\lambda)\frac{\partial b(k,\lambda)}{\partial k^i} + d^\dagger(k,-\lambda)\frac{\partial d(k,-\lambda)}{\partial k^i}]\Big). \qquad \text{(C14)}$$

$$J^3 = \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\sum_\lambda [\Big([k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1}]b^\dagger(k,\lambda)\Big)b(k,\lambda) - b^\dagger(k,\lambda)[k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1}]b(k,\lambda)$$
$$+ \Big([k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1}]d^\dagger(k,-\lambda)]\Big)d(k,-\lambda) - d^\dagger(k,-\lambda)[k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1}]d(k,-\lambda)]$$
$$+ \frac{1}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\sum_\lambda \lambda[b^\dagger(k,\lambda)b(k,\lambda) + d^\dagger(k,\lambda)d(k,\lambda)] \quad \text{(C15)}$$

with $\lambda = \pm 1$.

$$F^i = i\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}k^i \sum_\lambda \Big(\Big[\frac{\partial b^\dagger(k,\lambda)}{\partial k^+}b(k,\lambda) + \frac{\partial d^\dagger(k,-\lambda)}{\partial k^+}d(k,-\lambda)\Big]$$
$$- \Big[b^\dagger(k,\lambda)\frac{\partial b(k,\lambda)}{\partial k^+} + d^\dagger(k,-\lambda)\frac{\partial d(k,-\lambda)}{\partial k^+}\Big]\Big)$$
$$+ \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\frac{m_F^2 + (k^\perp)^2}{k^+}\sum_\lambda \Big(\Big[\frac{\partial b^\dagger(k,\lambda)}{\partial k^i}b(k,\lambda) + \frac{\partial d^\dagger(k,-\lambda)}{\partial k^i}d(k,-\lambda)\Big]$$
$$- \Big[b^\dagger(k,\lambda)\frac{\partial b(k,\lambda)}{\partial k^i} + d^\dagger(k,-\lambda)\frac{\partial d(k,-\lambda)}{\partial k^i}\Big]\Big)$$
$$- \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\frac{\epsilon^{ij}}{k^+}k^j \sum_{\lambda\lambda'}\sigma^3_{\lambda\lambda'}\Big[b^\dagger(k,\lambda)b(k,\lambda') - d^\dagger(k,-\lambda')d(k,-\lambda)\Big]$$
$$- \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\frac{\epsilon^{ij}}{k^+}m_F \sum_{\lambda\lambda'}\sigma^j_{\lambda\lambda'}\Big[b^\dagger(k,\lambda)b(k,\lambda') + d^\dagger(k,-\lambda)d(k,-\lambda')\Big]. \quad \text{(C16)}$$

3. Transverse Spin of a Single Fermion

For a single fermion of mass m and momenta $(k^+, k^\perp)$, we have,

$$P^+ \mid k\lambda\rangle = k^+ \mid k\lambda\rangle,\ P^1 \mid k\lambda\rangle = k^1 \mid k\lambda\rangle,\ P^2 \mid k\lambda\rangle = k^2 \mid k\lambda\rangle,$$
$$P^- \mid k\lambda\rangle = \frac{(k^\perp)^2 + m^2}{k^+} \mid k\lambda\rangle,\ \mathcal{J}^3 \mid k\lambda\rangle = \frac{1}{2}\lambda \mid k\lambda\rangle,$$
$$K^3 \mid k\lambda\rangle = -ik^+\frac{\partial}{\partial k^+} \mid k\lambda\rangle,\ E^1 \mid k\lambda\rangle = ik^+\frac{\partial}{\partial k^1} \mid k\lambda\rangle,\ E^2 \mid k\lambda\rangle = ik^+\frac{\partial}{\partial k^2} \mid k\lambda\rangle,$$
$$F^1 \mid k\lambda\rangle = \left(2ik^1\frac{\partial}{\partial k^+} + i\frac{(k^\perp)^2 + m^2}{k^+}\frac{\partial}{\partial k^1} - \frac{k^2}{k^+}\lambda\right) \mid k\lambda\rangle - \frac{m}{k^+}\sum_{\lambda'}\sigma^2_{\lambda'\lambda} \mid k\lambda'\rangle,$$
$$F^2 \mid k\lambda\rangle = \left(2ik^2\frac{\partial}{\partial k^+} + i\frac{(k^\perp)^2 + m^2}{k^+}\frac{\partial}{\partial k^2} + \frac{k^1}{k^+}\lambda\right) \mid k\lambda\rangle + \frac{m}{k^+}\sum_{\lambda'}\sigma^1_{\lambda'\lambda} \mid k\lambda'\rangle. \quad \text{(C17)}$$

We arrive at

$$m\mathcal{J}^1 \mid k\lambda\rangle = \left(\frac{1}{2}F^2P^+ + K^3P^2 - \frac{1}{2}E^2P^- - P^1\mathcal{J}^3\right) \mid k\lambda\rangle$$
$$= m\sum_{\lambda'}\frac{\sigma^1_{\lambda'\lambda}}{2} \mid k\lambda'\rangle, \quad \text{(C18)}$$

$$m\mathcal{J}^2 \mid k\lambda\rangle = \Big(-\frac{1}{2}F^1 P^+ - K^3 P^1 + \frac{1}{2}E^1 P^- - P^2 \mathcal{J}^3\Big) \mid k\lambda\rangle$$
$$= m\sum_{\lambda'} \frac{\sigma^2_{\lambda'\lambda}}{2} \mid k\lambda'\rangle. \tag{C19}$$

APPENDIX D: TRANSVERSE SPIN IN FREE MASSLESS SPIN ONE FIELD THEORY

1. Poincare Generators: Operator Forms

The symmetric gauge invariant energy momentum tensor

$$\Theta^{\mu\nu} = F^{\lambda\mu} F^{\nu}_{\lambda} - g^{\mu\nu}\mathcal{L}. \tag{D1}$$

where the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{D2}$$

with

$$F^{\mu\nu} = \partial^{\nu} A^{\mu} - \partial^{\mu} A^{\nu}. \tag{D3}$$

We choose $A^+ = 0$ gauge. Only the transverse fields A^i are dynamical variables. The momentum operators are given by

$$P^+ = \frac{1}{2}\int dx^- d^2x^\perp \partial^+ A^j \partial^+ A^j, \tag{D4}$$

$$P^i = \frac{1}{2}\int dx^- d^2x^\perp \Big(A^j \partial^+ \partial^j A^i - A^i \partial^+ \partial^j A^j - A^j \partial^+ \partial^i A^j\Big). \tag{D5}$$

The Hamiltonian operator is

$$P^- = \frac{1}{2}\int dx^- d^2x^\perp \Big[\frac{1}{4}(\partial^+ A^-)^2 + \frac{1}{2}F^{ij}F_{ij}\Big]$$
$$= \frac{1}{2}\int dx^- d^2x^\perp \partial^i A^j \partial^i A^j = -\frac{1}{2}\int dx^- d^2x^\perp A^j (\partial^i)^2 A^j \tag{D6}$$

The longitudinal scale generator (at $x^+ = 0$) is

$$K^3 = -\frac{1}{2}\int dx^- d^2x^\perp x^- \partial^+ A^j \partial^+ A^j. \tag{D7}$$

The transverse boost generators are

$$E^i = -\frac{1}{2}\int dx^- d^2x^\perp x^i \partial^+ A^j \partial^+ A^j. \tag{D8}$$

The generators of rotations are

$$J^3 = \frac{1}{2}\int dx^- d^2x^\perp \Big(x^1[\partial^+ A^2 \partial^i A^i + \partial^+ A^1(\partial^2 A^1 - \partial^1 A^2)]$$
$$-x^2[\partial^+ A^1 \partial^i A^i + \partial^+ A^2(-\partial^2 A^1 + \partial^1 A^2)]\Big)$$
$$= \frac{1}{2}\int dx^- d^2x^\perp \Big(x^1[\partial^+ A^1 \partial^2 A^1 + \partial^+ A^2 \partial^2 A^2] - x^2[\partial^+ A^1 \partial^1 A^1 + \partial^+ A^2 \partial^1 A^2]\Big)$$
$$+ \frac{1}{2}\int dx^- d^2x^\perp [A^1 \partial^+ A^2 - A^2 \partial^+ A^1]. \tag{D9}$$

and

$$F^i = \frac{1}{2}\int dx^- d^2x^\perp \Big(x^- \Big(A^{ja}\partial^+\partial^j A^i - A^i \partial^+ \partial^j A^j - A^j \partial^+ \partial^i A^j \Big)$$
$$-x^i[A^k(\partial^j)^2 A^k]\Big) - 2\int dx^- d^2x^\perp A^i \eta^{ij}\partial^j A^j, \quad \text{no summation over } i,j \; , \tag{D10}$$

with $\eta^{12} = \eta^{21} = 1$, $\eta^{11} = \eta^{22} = 0$.

2. Fock Representation

The dynamical components of the free massless spin field operator in $A^+ = 0$ gauge are

$$A^i(x) = \sum_{\lambda=1}^{2} \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \delta^{i\lambda}[a(k,\lambda)e^{-ik.x} + a^\dagger(k,\lambda)e^{ik.x}]. \tag{D11}$$

In terms of Fock space operators, we have,

$$P^+ = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^+ \sum_\lambda a^\dagger(k,\lambda)a(k,\lambda). \tag{D12}$$

$$P^i = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^i \sum_\lambda a^\dagger(k,\lambda)a(k,\lambda). \tag{D13}$$

$$H = \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \frac{{k^\perp}^2}{k^+} \sum_\lambda a^\dagger(k,\lambda)a(k,\lambda). \tag{D14}$$

$$K^3 = -\frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^+ \sum_\lambda \Big[(\frac{\partial a^\dagger(k,\lambda)}{\partial k^+})a(k,\lambda) - a^\dagger(k,\lambda)\frac{\partial a(k,\lambda)}{\partial k^+})\Big]. \tag{D15}$$

$$E^i = \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} k^+ \sum_\lambda [(\frac{\partial a^\dagger(k,\lambda)}{\partial k^i})a(k,\lambda) - a^\dagger(k,\lambda)\frac{\partial a(k,\lambda)}{\partial k^i})]. \tag{D16}$$

$$J^3 = \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+} \sum_\lambda [\Big((k^1 \frac{\partial}{\partial k^2} - k^2 \frac{\partial}{\partial k^1})a^\dagger(k,\lambda)\Big)a(k,\lambda) - a^\dagger(k,\lambda)(k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1})a(k,\lambda)]$$
$$+ i\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\Big(a^\dagger(k,2)a(k,1) - a^\dagger(k,1)a(k,2)\Big). \tag{D17}$$

Introduce creation and annihilation operators

$$a(k,\uparrow) = \frac{-1}{\sqrt{2}}[a(k,1) - ia(k,2)], a(k,\downarrow) = \frac{1}{\sqrt{2}}[a(k,1) + ia(k,2)]. \tag{D18}$$

Then

$$J^3 = \frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\sum_\lambda [\Big((k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1})a^\dagger(k,\lambda)\Big)a(k,\lambda) - a^\dagger(k,\lambda)(k^1\frac{\partial}{\partial k^2} - k^2\frac{\partial}{\partial k^1})a(k,\lambda)]$$
$$+ \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\sum_\lambda \lambda a^\dagger(k,\lambda)a(k,\lambda). \tag{D19}$$

where λ now denotes circular polarization.

$$F^i = i\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}k^i \sum_\lambda (\frac{\partial a^\dagger(k,\lambda)}{\partial k^+}a(k,\lambda) - a^\dagger(k,\lambda)\frac{\partial a(k,\lambda)}{\partial k^+})$$
$$+\frac{i}{2}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\frac{(k^\perp)^2}{k^+}\sum_\lambda (\frac{\partial a^\dagger(k,\lambda)}{\partial k^i}a(k,\lambda) - a^\dagger(k,\lambda)\frac{\partial a(k,\lambda)}{\partial k^i})$$
$$-2\epsilon^{ij}\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\frac{k^j}{k^+}\sum_\lambda \lambda a^\dagger(k,\lambda)a(k,\lambda). \tag{D20}$$

3. Transverse Spin

Using the explicit form of the operators, we get for a state of momentum $k(k^+, k^\perp)$ and helicity λ,

$$\mathcal{J}^3 \mid k\lambda\rangle = \frac{W^+}{P^+} \mid k\lambda\rangle = \lambda \mid k\lambda\rangle,$$
$$W^1 \mid k\lambda\rangle = k^1\lambda \mid k\lambda\rangle,$$
$$W^2 \mid k\lambda\rangle = k^2\lambda \mid k\lambda\rangle,$$
$$W^- \mid k\lambda\rangle = \frac{(k^\perp)^2}{k^+}\lambda \mid k\lambda\rangle. \tag{D21}$$

$$\mathcal{J}^i \mid k\lambda\rangle = 0. \tag{D22}$$

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