

A CALCULATION OF HIGGS MASS IN THE STANDARD MODEL

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Abstract

The assumption that the ratio of the Higgs self-coupling to the square of its yukawa coupling to the top is (almost) independent of the renormalization scale fixes the Higgs mass within narrow limits at $m_H = 160 GeV$ using only the values of gauge couplings and top mass .

The suggestion that the mass of the Higgs may be symbiotically related to the mass of the top quark arose from analogy with BCS like theories which predict the existence of a scalar bound state at nearly twice the mass of the quasi particle in the weak-coupling limit [1]. Indeed top-condensate models[2] given certain approximations do lead to a close relation between the top and the Higgs mass. On the other hand it was pointed out long ago point by Cabibbo et.al.[3] that the assumption that no new physics appears until energies typical of Grand unified theories, combined with the use of the renormalisation group [RG] equations at the one loop level leads to both an upper bound and a lower bound on the Higgs mass . As explained by them , these bounds come about due to the sensitive dependence of the solution of the RG equations on the initial value of the Higgs self coupling $\lambda(t)$ at some low energy scale. These studies were further extended by Beg et.al[4] , who considered energy scales up to the Landau pole , and by Lindner[5] who extended it to lower energies [6].

At the present time all details of the RG equations are known except for the integration constant of the equation for $\lambda(t)$. The question then arises whether there is some way to determine this constant which we take to be the value of $\lambda(t)$, at the Z -mass , in terms of other known parameters of the standard model.

I now make the assumption that $\lambda(t)$ and the square of the Higgs coupling to the top , $g_t^2(t) = G(t)$ have the same t-dependence . Here t is defined as

$$t = \frac{1}{2} \log\left(\frac{\mu^2}{m_Z^2}\right) \quad (1)$$

with μ as the renormalization scale . Implementing this over a modest range of values is sufficient to fix the value of $\lambda(0)$ within narrow limits and hence the Higgs mass.

Consider the RG equations [7] for $G(t)$ and $\lambda(t)$

$$\frac{dG(t)}{dt} = \frac{1}{8\pi^2} \frac{9}{2} G(t) \left\{ G(t) - \frac{17}{54} g_1^2(t) - \frac{1}{2} g_2^2(t) - \frac{16}{9} g_3^2(t) \right\} \quad (2)$$

$$\begin{aligned} \frac{d\lambda}{dt} = \frac{1}{8\pi^2} .6 \{ \lambda^2(t) + \lambda(t) [G(t) - \frac{1}{4} g_1^2(t) - \frac{3}{4} g_2^2(t)] - \\ G^2(t) + \frac{1}{16} g_1^4(t) + \frac{1}{8} g_1^2(t) g_2^2(t) + \frac{3}{16} g_2^4(t) \} \end{aligned} \quad (3)$$

Here $g_1, g_2,$ and g_3 are respectively the U(1) , SU(2) and SU(3) gauge couplings . Defining v to be

$$v = (2\sqrt{2} * G_F)^{-1/2} \quad (4)$$

the on-shell masses of the top and Higgs are related to v by

$$m_t^2 = G(t = \log\left(\frac{m_t}{m_Z}\right)) * v^2 \quad (5)$$

$$m_H^2 = 2 * \lambda(t = \log\left(\frac{m_H}{m_Z}\right)) * v^2 \quad (6)$$

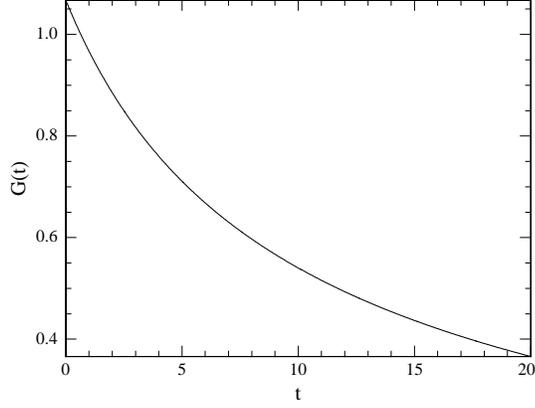


Figure 1: The solution $G(t)$ obtained from solving eq.(2).

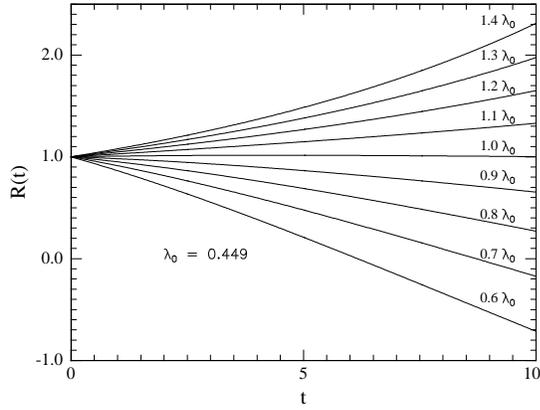


Figure 2: The ratio $R(t) = \frac{\lambda(t)}{G(t)}$ corresponding to different initial values of $\lambda(0)$ expressed in units of λ_0 .

Eq.(2) which gives the running of the top coupling is homogeneous and is easily solved. I use

$$[\alpha]^{-1} = 127.9; \quad \text{Sin}^2\theta_W = 0.231; \quad \text{and} \quad \alpha_3 = 0.119 \quad (7)$$

at $t=0$ and take the on-shell value of $G(t)$ to be unity which corresponds to $m_t = v$ cf.eq.(5). Solving eq(2) with these inputs, and inserting the resulting $G(t)$ in eqn (3), the latter is solved numerically [8] in the range $0 < t < 10$ using a range of values of $\lambda(0)$ as the initial value at $t=0$.

In Fig. 1, the solution of Eq.(2) giving the square of the top coupling $G(t)$ is plotted. It is a decreasing function and drops from a value of 1.068 at $t=0$

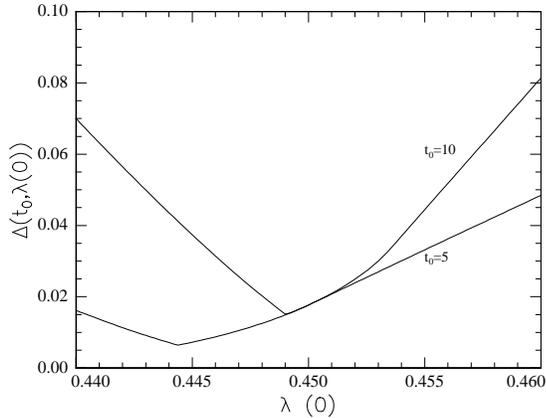


Figure 3: The variation of $\Delta(t_0, \lambda_0)$ with $\lambda(0)$ for $t_0 = 10$ and $t_0 = 5$.

to 0.711 at $t=5$ and further to 0.540 at $t=10$. To find which value of $\lambda(0)$ will make the solution $\lambda(t)$ have the same t dependence as $G(t)$ I consider the ratio

$$R(t) = \frac{\lambda(t)}{G(t)} * \frac{G(0)}{\lambda(0)}$$

which is plotted in Fig.2. It is seen that the flow of $R(t)$ is sensitively dependent on $\lambda(0)$. To find the best value of $\lambda(0)$ which satisfies the scale independence assumption we determine for different initial values of $\lambda(0)$ the values $\Delta(t_0, \lambda(0))$ which is defined as the difference between the maximum and minimum value of $R(t)$ in the interval $0 < t < t_0$. This is displayed in Fig.3 for the two intervals $t_0 = 5$ and 10. We take the value of $\lambda(0)$ corresponding to the minimum of $\Delta(t_0, \lambda(0))$, to be the best estimate for $\lambda(0)$ and denote it by λ_0 . If we use the interval upto $t_0 = 10$, we get the value 0.449 and if we reduce the search interval down to $t_0 = 5$ we get 0.444. Returning to fig.2 we see that there is a clear separation of trajectories $R(t)$ and even a ten percent departure in the value of $\lambda(0)$ from λ_0 distinctly fails to meet the criterion of almost constant behaviour for $R(t)$. The origin of the various lower and upper bounds on the Higgs mass as a function of μ i.e. the energy scale upto which we like the theory to be valid can be seen to follow from how far out in t one wants to go.

From the optimal value λ_0 , we can use the mass-shell condition eq.(6) to obtain the Higgs mass. Using the value 0.449 at $t=0$ corresponds to the mass value

$$m_H = 160 GeV$$

Since the Higgs mass has only square-root dependence on λ , even a ten percent error in determining λ_0 is accurate enough to fix m_H within say 10

GeV of the value given above .

This is consistent with bounds obtained from an analysis of precision data [9] and is quite close to the estimate of D’Agostini and Degrassi [10] who using constrained analysis of direct search and precision measurement measurements find an expected value of 160-170 Gev for the Higgs mass as the central value with a standard deviation of 50 to 60 GeV.

We can find an estimate of λ_0 without having to solve eq.(3) as follows. If we assume $R(t)$ is a constant then we can set its derivative to be equal to zero. Implementing it at $t=0$ we have

$$\lambda(0) * \frac{d}{dt}G(t)|_{t=0} = G(0) * \frac{d}{dt}\lambda(t)|_{t=0} \quad (8)$$

With the use of eqns (2) and (3) this reduces to a quadratic equation for $\lambda(0)$ in terms of $G(0)$ and the gauge couplings[11]. It is instructive to consider three cases . a) $g_1 = g_2 = g_3 = 0$ b) $g_1 = g_2 = 0$ c) all have their experimental values.

Case a) $\lambda(0) = G(0)\frac{\sqrt{17}-1/2}{4}$. If we had adopted m_t instead of m_Z in defining t in eq.(1) , $G(0)$ is the on shell value. Using eq.(6) then one gets $m_H/m_t = 1.36$. Case b) Using $G(0)= 1.068$ as noted earlier and eq.(6) one gets $\lambda(0)= 0.425$ which indicates the important role of g_3 . Case (c) One gets $\lambda(0) = 0.433$ which is close to the optimal values of λ_0 obtained from fig.3. The discrepancy is not just a matter of numerics. Although $R(t)$ remains close to unity over a wide range it is not a constant. Computing the derivative $\frac{d}{dt}R(t)$ using the numerical solution to eq.(3) we check that it is zero at $\lambda(0)=0.433$ and has a non-zero but tiny slope of value = 0.0035 if $\lambda(0) = 0.449$ and 0.0016 if $\lambda(0) = 0.444$, which also explains the shift in λ_0 for the two cases considered in Fig.3. These little variations can easily be changed by shifting the $t=0$ point from m_Z .

It would be interesting to study the effect of higher order terms in the RG equations which are important at lower energies and see whether one can bootstrap the values of $G(0)$ and $\lambda(0)$.

Acknowledgements: I am grateful to V.Srinivasan for valuable discussions on quasi-fermion boundstates in BCS type models and T.Ferbel about experimental program to discover the Higgs. I thank the High energy group at Syracuse for their support and helpful discussions. I am very grateful to Abhay Pasupathy for his generous help with numerical computation and to him and Ankur Mathur for their help in preparing the manuscript.

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7. The normalisations of the couplings used here is same as used by Lindner[5]
8. After solving eq.(1) for $G(t)$, Lindner [5] expresses the integrating factor in terms of simple functions which is an excellent approximation. This is very useful to speed up the numerical solution of eq.(3). I have adapted his procedure , taking into account the small difference in gauge couplings used here as compared to his values.
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11. The procedure here is similar to the work of J.Kubo, K.Sibold and W.Zimmerman, Nucl.Phys. **B259**, (1985) 331 ; Phys.Lett **B220** 185 (1989) who use the reduction of coupling constants due to W.Zimmerman, Commun.Math.Phys. **97** (1985) 211 : R.Oehme and W.Zimmerman Commun.Math.Phys. **97** (1985) 569. There are some differences. First I do not attempt to relate the gauge couplings to the Yukawa coupling as these authors do , secondly the simplified procedure of eqn(8) is used after having verified the near constancy of $R(t)$ as otherwise setting the derivative equal to zero at some t value would be without justification.