AXIAL VECTOR CURRENT MATRIX ELEMENTS AND QCD SUM RULES

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The matrix element of the isoscalar axial vector current, $\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d$, between nucleon states is computed using the external field QCD sum rule method. The external field induced correlator, $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle$, is calculated from the spectrum of the isoscalar axial vector meson states. Since it is difficult to ascertain, from QCD sum rule for hyperons, the accuracy of validity of flavour SU(3) symmetry in hyperon decays when strange quark mass is taken into account, we rely on the empirical validity of Cabbibo theory to dertermine the matrix element $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s$ between nucleon states. Combining with our calculation of $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$ and the well known nucleon β-decay constant allows us to determine $\langle p,s|\,\frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{9}\bar{d}\gamma_\mu\gamma_5 d + \frac{1}{9}\bar{s}\gamma_\mu\gamma_5 s\,|p,s\rangle$ occuring in the Bjorken sum rule. The result is in reasonable agreement with experiment. We also discuss the role of the anomaly in maintaining flavour symmetry and validity of OZI rule.

PACS numbers:

I. INTRODUCTION

The determination of flavour singlet and non-singlet axial vector matrix elements between nucleon states is of considerable interest theoretically and experimentally. While non-singlet currents appear in nucleon and hyperon beta decays, the linear combination

$$\langle p,s|\,\frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{9}\bar{d}\gamma_\mu\gamma_5 d + \frac{1}{9}\bar{s}\gamma_\mu\gamma_5 s\,|p,s\rangle = -s_\mu G_{Bj} \tag{1}$$

appears in the integral of the first moment of the polarised structure function $g_1(x)$ in the sumrule of Bjorken [1]. Here $|p,s\rangle$ denotes proton state of momentum p and polarisation s_μ. We have introduced the constant G_{Bj} to denote the linear combination of axial vector current with coefficients equal to square of quark charges. Nearly three decades ago Ellis and Jaffe [2] using a simple minded application of the OZI rule, set the strange quark current matrix element between proton states to be zero, which immediately enabled them to write

$$G_{\rm Bj} = \frac{1}{6}G_A + \frac{5}{18}G_8 \tag{2}$$

where, G_A is the isovector axial vector coupling occuring in nucleon beta decay and G_8 is the octet current coupling. Eqn.(2) is in disagreement with experiment [3] if SU(3) flavor symmetry is a good symmetry [4]. Already in 1979 Gross, Trieman and Wilczek [5] had pointed out that because the light quark masses are unequal, i.e.

$$\frac{m_d - m_u}{m_u + m_d} = O(1),$$

one should expect large violations of Isospin symmetry in the Bjorken sumrule if anomaly is neglected. The matrix elements of the anomaly between vacuum and Goldstone states $|\pi\rangle$ and $|\eta\rangle$ are not zero and play a crucial role in maintaining flavour symmetry. As we shall discuss further in Section II, this at once implies that Ellis-Jaffe assumptions of simulataneous validity of SU(3) flavour symmetry and OZI rule are mutually incompatible. The octet current has no anomaly while the singlet does.

The computation of matrix elements of the axial current in Eqn.(1) is clearly a problem in QCD. This can be addressed using the external field method introduced in the context of QCD sumrules by Ioffe and Smilga [6] and Balitsky and Yung [7] in 1983 to calculate the magnetic moments and which has since then been used for numerous other matrix elements as well. In fact computation of the axial vector current matrix elements has been done using these methods in ref.[8]-[15].

The reasons for reconsidering the earlier QCD sum rule determination of G_{Bj} are as follows.

1. Besides the usual Lorentz invariant chiral and gluon condensates present in QCD vacuum, additional non-invariant correlators $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle$ induced by the external field, enter the sumrules. If in Eqn.(1), on the left hand side, we had only flavour non-singlet currents, the corresponding external field induced correlator of dimension three,

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|0\rangle|_{\rm Ext.\ Field} \quad \text{and} \tag{3}$$

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s|0\rangle|_{\rm Ext.\ Field} \tag{4}$$

for example, can be found using the ward identity and PCAC.

The determination of either the isoscalar current or SU(3) singlet current correlators

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d|0\rangle|_{\rm Ext.\ Field} \tag{5}$$

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|0\rangle|_{\rm Ext.\ Field} \tag{6}$$

with the help of Ward identities are no longer simple since they involve the gluon anomaly. In earlier works some authors [10, 11, 12] used simply the values of the non-singlet current induced correlators, as an expedient measure, while Ioffe and Khodjamirian [13] found inconsistencies with SU(3)

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flavor symmetry in their calculations of the correlator in Eqn.(6). Indeed later in ref.[15], the correlator Eqn.(6) was treated as an unknown free parameter and its value fixed by using the experimental value of G_{Bj}. In the present work we follow a different procedure. The external field induced correlator corresponding to the isoscalar current Eqn.(5) is determined directly from the axial vector meson spectrum and we verify that similar determination of the nonsinglet isovector current induced correlator Eqn.(3) indeed yields a value consistent with Ward identity and PCAC.

2. We also clarify the differences in the calculations of the Wilson coefficients between ref.[9] and [11] on one hand and ref. [10] on the other.

3. In the analysis of sumrules, we use a value of the QCD scale parameter Λ consistent with present data, while earlier works used a significantly lower value.

This paper is organised as follows. In the next section we briefly recapitulate the arguments of Gross, Treiman and Wilczek, regarding the role of the anomaly in maintaining the flavour symmetry. Although the anomaly is superficially a flavor singlet its matrix elements between the vacuum and the Goldstone state $|\pi^0\rangle$ and $|\eta\rangle$, are not zero. We also make a brief digression on the OZI rule. In section III we give a summary of the external field method, and an appendix explains the differences between ref. [9] and [10] in the computation of the Wilson coefficients. In Section IV we outline the determination of the external field induced vacuum correlators which is used in Section V to determine the isoscalar matrix element and we end with a brief discussion.

II. FLAVOUR SYMMETRY, OZI, AND THE ANOMALY

We briefly recall the argument of ref [5]. If we ignore the anomaly then we have, for the divergence of isoscalar current

$$\begin{aligned}\partial^\mu\left[\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d\right] &= i(m_u+m_d)\left[\bar{u}\gamma_5 u+\bar{d}\gamma_5 d\right]\\ &+ i(m_u-m_d)\left[\bar{u}\gamma_5 u-\bar{d}\gamma_5 d\right]\end{aligned} \tag{7}$$

We also have, for the isovector

$$\begin{aligned}F_\pi m_\pi^2\phi_{\pi 0} &= \partial^\mu\left[\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d\right]\\ &= i(m_u+m_d)\left[\bar{u}\gamma_5 u-\bar{d}\gamma_5 d\right]\\ &+ i(m_u-m_d)\left[\bar{u}\gamma_5 u+\bar{d}\gamma_5 d\right]\end{aligned} \tag{8}$$

Combining these using PCAC one gets

$$\begin{aligned}&\langle N|\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d)|N\rangle_{I=1} =\\ &\frac{m_u-m_d}{m_u+m_d}\langle N|\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d)|N\rangle_{I=1}\end{aligned} \tag{9}$$

where the subscript $I=1$ on the nucleon states denotes the difference between proton and neutron matrix elements

$$\langle N|\mathcal{O}|N\rangle_{I=1} = \langle p|\mathcal{O}|p\rangle - \langle n|\mathcal{O}|n\rangle.$$

Eqn.(9) implies a large violation of isospin in Bjorken sum-rule since

$$\frac{m_d-m_u}{m_u+m_d} = O(1).$$

This conclusion is avoided by noting that one has ignored the anomaly. In Eqn.(7) one should write

$$\begin{aligned}\partial^\mu\left[\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d\right] &= i(m_u+m_d)\left[\bar{u}\gamma_5 u+\bar{d}\gamma_5 d\right]\\ &+ i(m_u-m_d)\left[\bar{u}\gamma_5 u-\bar{d}\gamma_5 d\right]\\ &+ 2\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\end{aligned} \tag{10}$$

where $\tilde{G}^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}$. Using a Sutherland type argument it is derived in ref [5]

$$i\langle 0|(m_u\bar{u}\gamma_5 u+m_d\bar{d}\gamma_5 d)|\pi^0\rangle+\langle 0|\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\pi^0\rangle = 0 \tag{11}$$

and

$$\langle 0|2i(m_q\bar{q}\gamma_5 q)|\pi^0\rangle+\langle 0|\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\pi^0\rangle = 0 \tag{12}$$

where, $q=s,\ c,...$ etc. Using again a PCAC argument they[5] obtain

$$\begin{aligned}2i\langle 0|m_u\ \bar{u}\gamma_5 u+m_d\ \bar{d}\gamma_5 d|\pi^0\rangle &= \frac{m_u-m_d}{m_u+m_d}\ F_\pi\ m_\pi^2\sqrt{2}\\ &= -2\langle 0|\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\pi^0\rangle\\ &= 4i\langle 0|m_s\ \bar{s}\gamma_5 s|\pi^0\rangle\\ &= 4i\langle 0|m_c\ \bar{c}\gamma_5 c|\pi^0\rangle\\ &= 4i\langle 0|m_b\ \bar{b}\gamma_5 b|\pi^0\rangle\ ...(13)\end{aligned}$$

In other words the matrix elements of the anomaly are far from being a flavour singlet. It was pointed out by Novikov et.al [16] that the matrix elements of the anomaly between vacuum and $|\eta\rangle$ is again not zero.

Writing

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s|\eta\rangle = i\sqrt{6}F_\pi k_\mu, \tag{14}$$

and taking its diveregence one gets

$$\langle 0|-4im_s\bar{s}\gamma_5 s|\eta\rangle = \sqrt{6}F_\pi m_\eta^2, \tag{15}$$

where we have ignored m_u and m_d in comparison with m_s. For the singlet current one may write

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s|\eta\rangle = if_1 k_\mu \tag{16}$$

If SU(3) flavour symmetry were exact, as happens when all light quark masses are neglected, $f_1=0$. On the other hand F_π remains finite in the chiral limit of massless quarks. We can expect then $F_\pi >> f_1$. Setting $f_1 = 0$, it is then easy to obtain from Eqns. (15) and (16) cf.[16],

$$\langle 0| \frac{3\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta\rangle = \sqrt{\frac{3}{2}} F_\pi m^2_\eta \tag{17}$$

We learn from Eqns.(13) and (17) above that the matrix elements of the anomaly between the vacuum and non-singlet goldston boson $|\pi^0\rangle$ and $|\eta\rangle$ is not zero but proportional to quark mass differences.

We note that Ioffe and Shifman [17] using Eqns. (13) and (17) above obtained the result

$$\begin{aligned} r &= \frac{\Gamma(\psi(2s) \to J/\psi(1s)\pi^0)}{\Gamma(\psi(2s) \to J/\psi(1s)\eta)} \\ &= 3\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \left(\frac{m_\pi}{m_\eta}\right)^4 \left(\frac{p_\pi}{p_\eta}\right)^3 \end{aligned} \tag{18}$$

Experimentally one has $r = (3.07 \pm 0.70) \times 10^{-2}$. We use this to find m_u/m_d and obtain

$$\frac{m_u}{m_d} = 0.44 \pm 0.07$$

which is consistent with values obtained using entirely different inputs; Gao [18] $m_u/m_d = 0.44$ and Leutwyler [19] $m_u/m_d = 0.553 \pm 0.043$.

Encouraged by the agreement between Eqn.(17) and experiments let us consider the matrix element

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta\rangle = i f_s k_\mu \tag{19}$$

Taking the divergence

$$\begin{aligned} \langle 0|\partial^\mu \bar{s}\gamma_\mu\gamma_5 s|\eta\rangle &= \langle 0|2im\bar{s}\gamma_5 s + \frac{\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta\rangle \\ &= f_s m^2_\eta \end{aligned} \tag{20}$$

Using Eqns.(15) and (17) we have

$$f_s = \frac{-\sqrt{6}}{3} F_\pi \tag{21}$$

Now it is known that the Goldberger-Treiman relation

$$\sqrt{2} m_N G_A = g_{\pi N} F_\pi \tag{22}$$

is accurate to a few percent and is exact in the chiral limit of massless quarks. Corrections to it have the structure [20]

$$1 - \frac{\sqrt{2} m_N G_A}{g_{\pi N} F_\pi} = \Delta = C_1 m^2_\pi + C_2 m^4_\pi ln(m^2_\pi) + .. \tag{23}$$

In other words, GT relation is obtained by retaining only the Goldstone pole and discarding the continuum contribution in the dispersion integrands. In the chiral limit, equations analogous to Eqns.(22) and (23) also hold good for the nucleon matrix element of the octet current with G_A replaced by $G_U + G_D - 2G_S$ and $g_{\pi N}$ replaced by $g_{\eta N}$ etc. with leading corrections proportional to quark mass as in Eqn(23). If we now naively extend these dispersion relation considerations to the nucleon matrix element of the strange quark current $\bar{s}\gamma_\mu\gamma_5 s$, retaining only the η-pole and discard the continuum as well as η' pole we immediately obtain from Eqn.(14), (19) and (21)

$$G_S = \frac{-1}{3}(G_U + G_D - 2G_S) \tag{24}$$

or

$$G_U + G_D + G_S = 0 \tag{25}$$

which is same as the Skyrme model result [21].

Our main point in this section is that OZI cannot be applied naively. The anomaly is important to avoid gross violations of SU(3) flavour symmetry and matrix elements of the anomaly are not flavour symmetric.

It is worth emphasising that OZI rule violates unitarity, a cardinal property of all S-matrix elements. Corrections to OZI rule can be estimated using unitarity and they are very much process dependent [22]. Charmonium decays illustrate the point. For example,

$$\begin{aligned} \text{B.R.}(J/\psi(1s) \to \rho\pi) &= 1.27 \times 10^{-2}, \qquad \text{while} \\ \text{B.R.}(\psi(2s) \to \rho\pi) &\leq 8.3 \times 10^{-5} \end{aligned}$$

despite the fact $\Gamma_{tot}(\psi(2s)) = 277$ keV is just a factor of three larger than $\Gamma_{tot}(J/\psi(1s)) = 87$ keV. Again in the decay of $J/\psi(1s)$ into light mesons, SU(3) flavor symmetry works better in pseudoscalar vector decays than in vector tensor decays. Also the decay mode $J/\psi(1s) \to \phi\pi\pi$ is not doubly suppressed as one might naively expect. These emphasise the point that corrections to OZI rule are to be studied individually for each matrix element [22] and there is no universal principle which tells a priori how good is the OZI rule for any specific matrix element.

III. QCD SUMRULE WITH AN EXTERNAL FIELD

We shall follow closely the notations of Ioffe [14]. We consider the nucleon correlator in an external field

$$\Pi(p, A_\mu) = i \int d^4x\ e^{ip.x} \langle 0|T\{\eta(x), \bar{\eta}(0)\}|0\rangle \Big|_{A_\mu} \tag{26}$$

where $\eta(x)$ is the nucleon current

$$\eta(x) = \epsilon^{abc} u^a(x)\ C\gamma_\mu u^b(x)\gamma^\mu\gamma_5 d^c(x) \tag{27}$$

with proton quantum numbers; u^a, d^b are quark fields and a, b, c are color indices. A_μ refers to constant external field. To compute the matrix element of the current j^5_μ between a proton state $\langle p|j^5_\mu|p\rangle$ one adds a term

$$\Delta\mathcal{L} = j^5_\mu A^\mu \tag{28}$$

to the Lagrangian and evaluates $\Pi(p, A_\mu)$ in Eqn.(26) upto terms linear in A_μ. In ref [8]-[13] the following four different currents were considered.

$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d \qquad \text{isovector} \tag{29}$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d \qquad \text{isoscalar} \tag{30}$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s \qquad \text{octet} \tag{31}$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s \quad \text{SU(3) singlet} \tag{32}$$

Bearing in mind that the corrections to OZI rule are a priori unknown and can be large, we shall consider only the isoscalar current Eqn.(30) here, since the nucleon current of Ioffe in Eqns.(26-27) above has only up and down quark fields, and therefore couples to the $\bar{s}\gamma_\mu\gamma_5 s$ term in the octet current, Eqn.(31), or singlet, Eqn.(32),only through gluonic corrections that is by corrections to OZI rule.

In deriving the QCD sumrules one must take into account, external field induced correlators . We define

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle|_{A_\mu} = F A_\mu \tag{33}$$
$$\langle 0|g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\rho\mu}\gamma^\rho q|0\rangle|_{A_\mu} = H A_\mu \tag{34}$$

It is clear that the constants F and H are in general different corresponding to the different $\Delta\mathcal{L}$ introduced in Eq.(28-32). The constant F can be obtained from

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle|_{A_\mu} = i\int d^4x\langle 0|T(\Delta\mathcal{L}, \bar{q}\gamma_\mu\gamma_5 q)|0\rangle \tag{35}$$

and the constant H from

$$\langle 0|g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\rho\mu}\gamma^\rho q|0\rangle|_{A_\mu} = i\int d^4x\langle 0|T(\Delta\mathcal{L}, g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\rho\mu}\gamma^\rho q)|0\rangle \tag{36}$$

Complete details of the calculation of $\Pi(p, A_\mu)$ in Eqn.(26) can be found in ref.[8, 9, 10, 11, 12, 13] for both the isovector and isoscalar currents. For the isoscalar axial vector matrix element the sum rule then reads

$$\frac{-M^6}{L^{4/9}}E_2 + \frac{16\pi^2}{3}\ F\ \frac{M^4}{L^{4/9}}E_1 + \frac{b}{4}\frac{M^2}{L^{4/9}}E_0 + \frac{16\pi^2}{3}\ H\ \frac{M^2}{L^{8/9}}E_0 - \frac{4}{9}a^2 L^{4/9} = \tilde{\lambda}^2_N(G + AM^2)exp[-m^2_N/M^2] \tag{37}$$

Here m_N is the nucleon mass and M^2 is the Borel mass variable. In the right hand side, $\tilde{\lambda}_N$ is defined by

$$\langle 0|\eta(x)|p\rangle = \lambda_N v(p) \qquad \text{and} \qquad \tilde{\lambda}^2_N = \frac{\lambda^2_N}{32\pi^2}$$

$G_U + G_D = G$ is the isoscalar axial vector current matrix element. The term AM^2 arises from the fact that in the presence of external field there are non-diagonal transition between nucleon and excited states. In the left hand side

$$L = \frac{ln(M/\Lambda)}{ln(\mu/\Lambda)},\ a = -(2\pi)^2\langle\bar{q}q\rangle,\ b = \langle g^2_s G^a_{\mu\nu}G^a_{\mu\nu}\rangle,$$
$$E_0 = 1 - e^{-W^2/M^2},$$
$$E_1 = 1 - (1 + W^2/M^2)\ e^{-W^2/M^2},$$
$$E_2 = 1 - (1 + W^2/M^2 + W^4/2M^4)\ e^{-W^2/M^2}$$

μ is the renormalization scale, which we take to be 1 GeV, and Λ is the QCD scale which for 3 flavor case is 247 MeV [30].

Although the sum rule has been written down earlier by several authors the purpose of reconsidering it here are the following.

1. It is clear the constants F and H should be determined from other sumrules or different considerations before we can use it to find G in Eqn.(37). As will be explained in detail in the next section, for the isovector case, it is relatively easy to determine them using PCAC. For the singlet currents one must take into account the anomaly. A decade ago Ioffe and Khodjamirian [13] attempted to compute F using the Ward identity and sum rules for the divergence of the SU(3) singlet current. They found inconsistency with SU(3) flavour symmetry chiefly because of the large difference between the strange quark mass and the up or down quark mass. In later works [14, 15] this lead Ioffe and his collaborators to use the sumrule, Eqn.(37), to find F using the experimental value of G_{Bj} in Eqn.(1). In contrast here for the isoscalar matrix element we shall determine the constant F from the spectrum of isoscalar axial vector mesons, and use that value to determine from Eqn.(37) the value of G and therefore G_{Bj}.

2. The external field Lagrangian, modifies the propagation of the current $\eta(x)$, in the QCD vacuum in two ways. One in which the external field directly couples to the fields in $\eta(x)$. The other, in which the external field modifies the vacuum as represented by the induced correlators F and H, which appear in the second and fourth terms in the sum rule, Eqn.(37). Now consider the difference between the three $\Delta\mathcal{L}$ in Eqns.(30), (31), and (32). As long as gluon loops are not included, the additional term $\bar{s}\gamma_\mu\gamma_5 s A^\mu$ will not couple to the u and d fields in $\eta(x)$. The strange quark term will only affect the constants F and H in Eqn.(35) and (36). Putting it differently with this assumption, the same sum rule, Eqn.(37), is valid for all the three currents, Eqn.(30)-(32) namely, the isoscalar, octet and SU(3) singlet with only the external field induced correlators F and H being different. Correspondingly if we were to compute only the matrix element of $\langle p, s|\bar{s}\gamma_\mu\gamma_5 s|p, s\rangle$ then we would consider, in place of Eqn.(28), the external field Lagrangian

$$\Delta\mathcal{L} = \bar{s}\gamma_\mu\gamma_5 s A^\mu$$

and the corresponding sum rule for the strange quark matrix element in Eqn.(37) will have, in its left hand side, only the second and fourth terms due to modifications of the vacuum and all other terms will be set to zero. In the light of our discussion of OZI rule, one may then expect that the sum rule Eqn.(37) will work better for the isoscalar matrix element than for the octet or SU(3) singlet.

3. The sum rule, Eqn.(37), differs from those of ref.[10, 15] also in the coefficient of the third and fourth terms due to difference in the calculation of Wilson coefficients. This point is elaborated in the Appendix.

IV. EXTERNAL FIELD INDUCED CORRELATORS

We now turn to the computation of the constants F and H defined in Eqns.(33-36). First consider the isovector case with the current defined in Eqn.(29) and the corresponding correlator

$$\begin{aligned}\Pi^{I=1}_{\mu\nu} &= \frac{i}{2}\int d^4x e^{iq.x} \\ &\langle 0|T(\bar{u}(x)\gamma_\mu\gamma_5 u(x) - \bar{d}(x)\gamma_\mu\gamma_5 d(x), \\ &\bar{u}(0)\gamma_\nu\gamma_5 u(0) - \bar{d}(0)\gamma_\nu\gamma_5 d(0))|0\rangle \qquad (38)\end{aligned}$$

One can write

$$\Pi^{I=1}_{\mu\nu} = -\Pi^{I=1}_1(q^2)g_{\mu\nu} + \Pi^{I=1}_2(q^2)q_\mu q_\nu$$

where the coefficient of $g_{\mu\nu}$, Π_1, gets contribution only from spin 1^+ states while the coefficient of $q_\mu q_\nu$, Π_2, has contributions form both 1^+ and 0^- states.

It is easy to see, using the gauge condition, $A_\mu q^\mu = 0$ and Eqns.(28), (32) and (34) that

$$F(I=1) = -\Pi^{I=1}_1(q^2=0) \qquad (39)$$

This can be easily evaluated using the ward-identity which reads

$$\begin{aligned}&- \ \Pi^{I=1}_1(q^2)\ q^2 + \Pi^{I=1}_2(q^2)\ q^4 \\ &= \ (m_u+m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle \\ &\quad -i(m_u+m_d)^2\int d^4x e^{iq.x}\langle 0|T(\bar{d}\gamma_5 u(x), \bar{u}\gamma_5 d(0))|0\rangle \qquad (40)\end{aligned}$$

Isolating the pion pole in $\Pi_2(q^2)$ and pseudoscalar correlation matrix element in the r.h.s. we can write near the pion pole

$$-\Pi^{I=1}_1(q^2)q^2 + \frac{F_\pi^2 q^4}{(m_\pi^2 - q^2)} \approx -F_\pi^2 m_\pi^2 + \frac{F_\pi^2 m_\pi^4}{(m_\pi^2 - q^2)} \qquad (41)$$

From which we get

$$\Pi^{I=1}_1(0) = -F_\pi^2 \qquad (42)$$

For the isoscalar case we need to consider the analogue Eqn(38)

$$\begin{aligned}\Pi^{I=0}_{\mu\nu} &= \frac{i}{2}\int d^4x e^{iq.x} \\ &\langle 0|T(\bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x), \\ &\bar{u}(0)\gamma_\nu\gamma_5 u(0) + \bar{d}(0)\gamma_\nu\gamma_5 d(0))|0\rangle \qquad (43)\end{aligned}$$

with

$$F(I=0) = -\Pi^{I=0}_1(q^2=0) \qquad (44)$$

However, unlike the isovector case the ward identities are no longer simple since it involves anomaly Eqn.(10) and Eqn.(40) is replaced, in the right hand side, by considerably more complex set of terms including the anomaly. We shall therefore not use the ward identity to find $\Pi^{I=0}_1(0)$.

On the other hand $\Pi^{I=0}_1(q^2)$ satisfies a dispersion relation. Using the operator product expansion for $\Pi^{I=0}_1(q^2)$ the value of $\Pi^{I=0}_1(0)$ can be found by the QCD sum rule method. Analogous procedure of course can be used for the isovector case as well. This procedure is well known and complete details can be found in ref.[23]. Here we shall simply write the final result. Denoting by $\hat{L}_{M^2}$ the Borel transform [31] we have from

$$\begin{aligned}\Pi_1(q^2) &= \frac{1}{\pi}\int \frac{\Im\Pi_1(s)\ ds}{s-q^2} + \text{subtractions} \qquad (45) \\ \hat{L}_{M^2}\Pi_1(q^2) &= \frac{1}{\pi M^2}\int \Im\Pi_1(s)\ e^{-s/M^2}\ ds \qquad (46)\end{aligned}$$

where $\Im\Pi_1$ denotes imaginary part of Π_1. Eqn.(46) is usually used to compute the mass of the f_1 state when isoscalar correlator is considered and A_1 when isovector correlator is considered [24]. Similarly from

$$\frac{\Pi_1(q^2) - \Pi_1(0)}{q^2} = \frac{1}{\pi}\int \frac{\Im\Pi_1(s)\ ds}{s(s-q^2)} + \text{subtractions} \qquad (47)$$

we obtain,

$$\hat{L}_{M^2}\left[\frac{\Pi_1(q^2)}{q^2}\right] - \frac{\Pi_1(0)}{M^2} = \frac{1}{\pi M^2}\int \frac{\Im\Pi_1(s)\ e^{-s/M^2}\ ds}{s} \qquad (48)$$

which can be used to calculate $\Pi_1(0)$. From the OPE expansion for the isovector current correlator Eqn.(38), we have

$$\begin{aligned}\frac{\Pi_1(q^2)}{q^2} &= -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ ln\frac{Q^2}{\mu^2} - \frac{2}{Q^4}\ \hat{m}\langle\bar{q}q\rangle \\ &+ \ \frac{\alpha_s}{12\pi Q^4}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle + \left(\frac{32}{9}+\frac{64}{81}\right)\frac{\alpha_s\pi}{Q^6}\langle\bar{q}q\rangle^2 \qquad (49)\end{aligned}$$

where $\hat{m} = (m_u+m_d)/2$. The above Eqn.(49) has also been used in the analysis of τ decay [25]. In the right

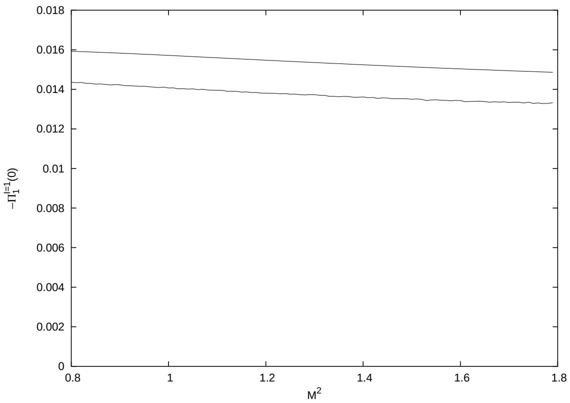

FIG. 1: The plot show $-\Pi_1^{I=1}(0)$ obtained from Eqn.(48) in the interval $0.8 \leq M^2 \leq 1.8(GeV)^2$. The upper curve is obtained for model A, cf. Eqn.(48), and the lower curve is for model B. See text.

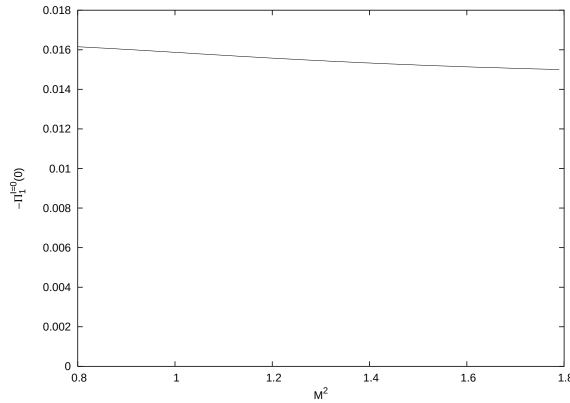

FIG. 2: $\Pi_1^{I=0}(0)$ obtained from Eqn.(48) using $f_1(1285)$ and ignoring its width as in model A.

hand side of Eqn.(46) and Eqn.(48) we use for the A_1, the central mass value from experiments and adopt two models

$$\text{model A} \ : \ \Im\Pi_1(s) = \pi h_A^2 m_{A_1}^4 \ \delta(s - m_{A_1}^2) \tag{50}$$

$$\text{model B} \ : \ \Im\Pi_1(s) = \frac{K \ \Theta(s - (m_\rho + m_\pi)^2)}{(s - m_{A_1}^2)^2 + \Gamma^2 m_{A_1}^2} \tag{51}$$

with $\Gamma = 300$ MeV. Following Zyablyuk [25] we use the values

$$g_s^2 \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle = 0.5 \ (\text{GeV})^4 \tag{52}$$

and for the four quark term

$$\frac{64\pi}{9} \alpha_s \langle \bar{q} q \rangle^2 = 3 \times 10^{-3} \ (\text{GeV})^6 \tag{53}$$

Eqn.(53) takes into account the renormalization corrections to the 4 quark operator computed by Adam and Chetyrkin [26]. We note that the estimate in eqn.(53) is smaller than the value used in the original work by Shiffman, Vainshtein and Zakharov [24].

Using the above we find (cf. Fig.1)

$$\Pi_1(0) = -0.0154 \ (GeV)^2 \qquad \text{(model A)} \tag{54}$$

$$\Pi_1(0) = -0.0138 \ (GeV)^2 \qquad \text{(model B)} \tag{55}$$

which is to be compared with the ward identity value Eqn.(40) with $F_\pi^2 = 0.017 \ (GeV)^2$. We like to stress that the above calculation does not use ward identity and PCAC.

Consider now the isoscalar case

$$\Pi_{\mu\nu}^{I=0} = \frac{i}{2} \int d^4x e^{iq.x} \langle 0 | T\{ \bar{u}\gamma_\mu\gamma_5 u(x) + \bar{d}\gamma_\mu\gamma_5 d(x), \bar{u}\gamma_\nu\gamma_5 u(0) + \bar{d}\gamma_\nu\gamma_5 d(0)\} | 0 \rangle \tag{56}$$

with

$$\Pi_{\mu\nu}^{I=0} = -\Pi_1^{I=0}(q^2) g_{\mu\nu} + \Pi_2^{I=0}(q^2) q_\mu q_\nu .$$

Before proceeding with the calculations we note the interesting similarities between the vector states $\rho(770)$, $\omega(780)$, $\phi(1020)$ and the axial vector states $A_1(1235), f_1(1285), f_1(1420)$. Both ρ and A_1 are broad resonances and are nearly degenerate with their $I = 0$ partners ω and f_1 respectively. Morever the decays of $\phi(1020), f_1(1420)$ are dominated by strange mesons. To evaluate $\Pi_1^{I=0}(0)$ we can proceed in an analogous manner as for the isovector case above. First we note that in the physical side or the r.h.s. of the sum rules, Eqns.(46) and (48), $f_1(1285)$ will replace A_1. The higher mass state $f_1(1420)$ is effectively included in the sum over higher mass states which in the usual QCD sumrule approach are represented by the quark loop contributions with an effective threshold W^2. We should stress here that the details of the decay modes of $f_1(1285)$ and $f_1(1420)$ are irrelevant and do not enter the sum rules.

Now turning to the OPE it is clear, from Eqn.(38) and Eqn.(43), that as long as quark anihilation diagrams are neglected the OPE for $I = 1$ and $I = 0$ currents will be identical. Consider the first term in Eqn.(49). This arises from the single quark loop and is the same for the isovector and isoscalar case. However in the isoscalar case we must include corrections that can arise from a three loop diagram in which the initial quark current loop annihilates to a two gluon intermediate followed by materialisation to the final current quark loop. These diagrams are expected to contribute with coefficients like α_s^2/π^2 and for that reason we neglect them here.

The calculation of $\Pi_1^{I=0}(0)$ is then straight forward and we obtain (cf. Fig.2)

$$\Pi_1^{I=0}(0) = -0.0152 \ (GeV)^2 \tag{57}$$

In the analysis of the sumrule Eqn.(37) we shall therefore use $F = 0.0152 \ (GeV)^2$ as the central value and study the effect of variation around it.

We now turn to the determination of the dimension 5 correlator or the constant H. For the isovector case this has been computed by Novikov et.al. [27], using the sumrule for Π_2 in

$$
\begin{aligned}
i & \int d^4 x e^{iq.x} \langle 0|T\{\bar{u}\gamma_\mu\gamma_5 d(x), g_s\bar{d}\frac{\lambda^a}{2}\tilde{G}^a_{\beta\nu}\gamma_\beta u|0\rangle \\
&= -\Pi_1(q^2)g_{\mu\nu} + \Pi_2(q^2)q_\mu q_\nu \qquad (58)
\end{aligned}
$$

defining

$$
\langle 0|g_s\bar{d}\frac{\lambda^a}{2}\tilde{G}^a_{\beta\alpha}\gamma_\beta u(x)|\pi\rangle = -\delta^2 F_\pi k_\alpha. \qquad (59)
$$

They obtained $\delta^2 = 0.21(GeV)^2$. However they had used a somewhat high value for the four quark correlator. This has been reanalysed by Ioffe and Oganesian [15] using more upto date values of various paramters. They obtain $\delta^2 = 0.16(GeV)^2$. We have independently reanalysed the sum rule of Novikov et.al. [27] and agree with Ioffe and Oganesian [15]. From Eqn(58) and Eqn(59) one gets the value $H(I=1) = \delta^2 F_\pi^2$.

To find the value H in the isoscalar case we proceed as follows. Ioffe and Khodjamirian [13] have computed

$$
\langle 0|g_s\sum_q \bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\beta\alpha}\gamma_\beta q(x)|0\rangle|_{SU(3)\ singlet} = 3h_0 A_\alpha \qquad (60)
$$

and find $h_0 \approx 3\times 10^{-4}$ (GeV)^4. Since the matrix elements in Eqns.(59) and (60) are quite small we can use SU(3) flavour symmetry and write

$$
\begin{aligned}
& \langle 0|g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\nu\mu}\gamma_\nu q|0\rangle|_{Iso-singlet} \\
&= \frac{2}{3}\langle 0|g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\nu\mu}\gamma_\nu q|0\rangle|_{SU(3)\ singlet} \\
&+ \frac{1}{3}\langle 0|g_s\bar{q}\frac{\lambda^a}{2}\tilde{G}^a_{\nu\mu}\gamma_\nu q|0\rangle|_{Octet} \qquad (61)
\end{aligned}
$$

From which we obtain

$$
\begin{aligned}
H &= \frac{2}{3}h_0 + \frac{1}{3}\delta^2 F_\pi^2 \\
&\approx 1.14\times 10^{-3}\ (GeV)^4 \qquad (62)
\end{aligned}
$$

We shall use this as the central value in the our analysis of the sumrule Eqn.(37). The above term is far less significant in the determinations of G in Eqn.(37) than the dimension 3 correlator constant F calculated in the earlier part of this section.

V. DETERMINATION OF $G_U + G_D$ FROM THE SUMRULE

We can use the values of F and H determined in the previous section in Eqn.(37) to find $G_U + G_D$. But first we need to determine $\tilde{\lambda}_N^2$ which can be obtained from Ioffe's sumrule for the nucleon mass.

$$
\begin{aligned}
\frac{M^6\ E_2}{L^{4/9}} + \frac{bM^2\ E_0}{4L^{4/9}} &+ \frac{4a^2L^{4/9}}{3} - \frac{a^2m_0^2}{3M^2} \\
&= \tilde{\lambda}_N^2\ e^{-m_N^2/M^2} \qquad (63)
\end{aligned}
$$

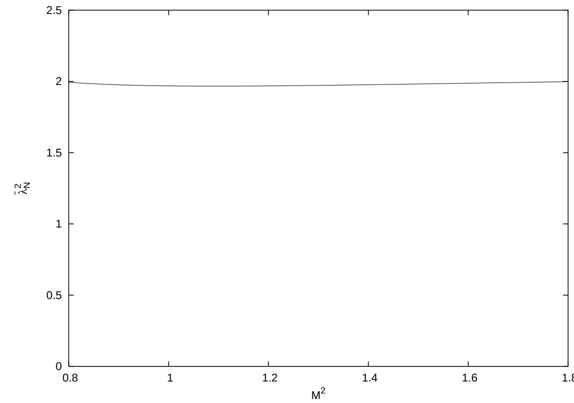

FIG. 3: $\tilde{\lambda}_N^2$ obtained from Eqn.(63) is shown in the Borel mass interval $0.8 \leq M^2 \leq 1.8$ $(GeV)^2$.

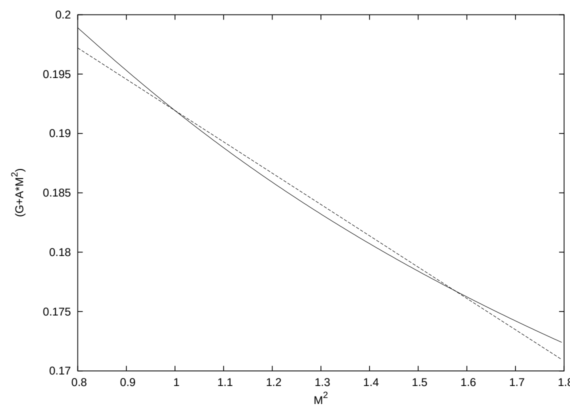

FIG. 4: $G + AM^2$ obtained from Eqn.(37) is shown in the Borel mass interval $0.8 \leq M^2 \leq 1.8$ $(GeV)^2$. A straight line fit, dotted line gives $G = 0.22$ and $A = -0.026$.

Experimentally we have $\Lambda_{QCD} = 247$ MeV [30], which corresponds to $\alpha_s(1\text{ GeV}) = 0.5$. To find $\tilde{\lambda}_N^2$ we shall use the experimental value $m_N = 0.938$ GeV and we fix $W^2 = 2.22$ by looking for the best fit in the least square sense in the Borel mass interval 0.8 GeV$^2 < M^2 < 1.8$ GeV^2, we find (cf. Fig.3)

$$
\tilde{\lambda}_N^2 = 1.975\ GeV^6
$$

This can now be used in the sumrule Eqn.(37), along with the values $F = 0.0152\ (GeV)^2$ and $H = 1.14\times 10^{-3}\ (GeV)^4$. We find fitting the sum rule with $W^2 = 2.22$ and the same interval for M^2 as used in Eqn.(63) (cf. Fig.4)

$$
G = G_U + G_D = 0.22 \text{ and } A = -0.026\ (GeV)^{-2} \qquad (64)
$$

Since our calculation of F and H in the previous section are not exact we have varied them by 20%. An numerical increase of F by 20% yields a value $G = 0.34$ and $A = -0.011(GeV)^{-2}$, while a decrease by 20% gives a value $G = 0.10$ and $A = -0.041(GeV)^{-2}$. Compared to this a change of value of H by 20% barely changes the result by 2%.

VI. CONCLUSION

To evaluate the linear combination $\frac{4}{9}G_U + \frac{1}{9}G_D + \frac{1}{9}G_S$ occuring in the Bjorken sum rule we can proceed as follows. We write

$$G_{Bj} = \frac{1}{6}(G_U - G_D) + \frac{1}{3}(G_U + G_D) - \frac{1}{18}(G_U + G_D - 2G_S) \quad (65)$$

First term is known from neutron β decay and we have

$$G_U - G_D = 1.267 \quad (66)$$

The last term in the Eqn.(59) is also known from hyperon β decay assuming the validity of flavour SU(3) symmetry. We have [4]

$$G_U + G_D - 2G_S = 0.585 \quad (67)$$

We can now use the value of $G_U + G_D$ determined in Sec.V Eqn.(64) to get

$$G_{Bj}(\mu^2 = 1\ GeV^2) = 0.32 \quad (68)$$

to be compared with experimental value [3]

$$G_{Bj}(\mu^2 = 5\ GeV^2) \approx 0.28, \quad (69)$$

which increases slightly when μ^2 is decreased from 5 GeV^2 to 1 GeV^2. Taking into account the μ dependence of the singlet matrix element [28, 29] the experimental number, Eqn.(69), increases by a few percent to 0.29 at $\mu^2 = 1GeV^2$.

In arriving at Eqn.(68) we have used SU(3) flavour symmetry via Eqn.(67) in Eqn.(65). Returning to Cabibbo theory, all the octet current matrix elements between the octet of baryon states are expressible in terms of two irreducible matrix elements F and D, and one has

$$G_U - G_D = F + D \quad (70)$$
$$G_U + G_D - 2G_S = 3F - D \quad (71)$$

In QCD sumrule calculations of the various octet current matrix elements between baryon states one first considers the limit of massless quarks which is of course automatically SU(3) symmetric. The constants F and D can be determined in a variety of ways. For example the matrix elements of the isovector current $\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$ between nucleon states gives $F + D$ while, $\Sigma \to \Sigma$ is proportional to F, $\Sigma \to \Lambda$ is proprtional to D and $\Xi \to \Xi$ to $D - F$. Details can be found in ref.[8, 9]. In ref.[9], using a different Lorentz structure invariant for the propagator in Eq.(26), the relation $7F \approx 5D$ was found which is not far from experiment. To incorporate the effect of quark masses one expands in quark mass and retains terms linear in them and also take into account the difference between the strange quark condensate and up or down quark condensate. We first note that since up and down quark masses are negligible, the isovector current matrix element in the nucleon is unaffected. In a recent update of the earlier calculation in ref.[8], Ioffe and Oganiesen [15] find

$$G_U - G_D = 1.37 \pm 0.01$$

Finding D, F and $D - F$ from the hyperon matrix elements, when quark masses are included, is sensitively dependent on the Borel mass region as was found by authors of [9]. For this reason it is difficult to decide how accurately Cabbibo theory is satisfied by using QCD sum rules. We have therefore relied on experiment which is reflected in the use of Eqn.(67) to compute G_{Bj} in Eqn.(65). Nevertheless, it is worth emphasising that QCD sumrules provide, though not precise, a quantitative explanation of G_{Bj}, without any arbitrary parameter and use only the value of vacuum condensates already obtained through other hadron properties.

APPENDIX A: WILSON COEFFICIENTS

There are two differences between the coefficients of the M^2 term in Eqn.(37), namely, the coefficient of the gluon condensate term, b, and the coefficient of the external field induced correlator, H, between ref.[9, 11, 12] on the one hand and ref.[10, 15] on the other. Full detail of the calculation can be found in the ref.[9]. These calcualtions have since been verified again by the original authors of ref.[9] themselves and authors of ref.[11] and ref.[12]. In ref.[9] the nucleon current correlator, Eqn(26), is calculated for a generic $\Delta\mathcal{L}$

$$\Delta\mathcal{L} = (g_u \bar{u}\gamma_\mu\gamma_5 u + g_d \bar{d}\gamma_\mu\gamma_5 d)A^\mu$$

with g_u and g_d as arbitray constants. According to Table 1 in ref. [9], the contribution of Fig (5) and (6) in ref.[9] have coefficient $-(2g_d + 10g_u/3)$ and $2(g_u + g_d)$. In the computation of G_A, $g_u = -g_d$ so that Fig (6) gives zero and there is complete agreement between ref.[9, 12] and ref.[10]. However for the isoscalar current, for which $g_u = g_d = 1$, the results are different. Ref. [9, 12] have

$$-(2g_d + \frac{10}{3}g_u) + 2(g_u + g_d) = -\frac{4}{3}g_u \qquad (g_u = g_d) \quad \text{(A1)}$$

Reversing the sign of Fig (6) gives

$$-(2g_d + \frac{10}{3}g_u) - 2(g_u + g_d) = -\frac{28}{3}g_u \qquad (g_u = g_d) \quad \text{(A2)}$$

which is result of ref.[10]. Thus ref.[9] and [10] agree numerically in the case $g_u = -g_d$ but different in the isoscalar case. In Eqn.(37) the coeffcient used corresponds to Eqn.(A1).

We now turn to the coefficient of the gluon condensate $\langle g_s^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle$. Again details of the calculations are given in ref.[9] in their Table 1 which uses their eqn(2.12) and Fig.2 which leads to coefficient $-g_u/4$. On the other hand ref.[10] seem to have $g_d/4$ for this coefficient. Thus ref.[9] and [10] agree numerically in the case $g_u = -g_d$ but have opposite signs in the isoscalar case. Interestingly enough if one tries to obtain the Wilson coefficients for $T\{\eta(x)\bar{\eta}(0)\}|_{A_\nu}$ by using a chiral rotation of the quark fields given by

$$u \longrightarrow e^{ig_u\ A.x\ \gamma_5} u$$
$$d \longrightarrow e^{ig_d\ A.x\ \gamma_5} d$$

in the propagator $\langle 0|T\{\eta(x)\bar{\eta}(0)\}|0\rangle$ in absence of the external field, which occurs in the mass sum rule and assuming the validity of chiral invariance of the vacuum to obtain $\langle 0|T\{\eta(x)\bar{\eta}(0)\}|_{A_\nu}|0\rangle$ one obtains $g_d/4$ for the coefficient of $\langle g_s^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle$. On the other hand it is well recognised that presence of non-perturbative gluon fileds in the vacuum state leads to breakdown of chiral symmetry, which would seem to invalidate such a calculation. Again in Eqn.(37) we have used the result $-g_u/4$ found in ref.[9] and ref.[12]. Fortuitiously enough these two differences in the coefficients of third and fourth terms in Eqn.(37) between ref.[9] and [10] numerically tend to compensate. Also we have seen the dimension three term characterised by F which occures with M^4 in Eqn.(37), is far more significant than the M^2 term in obtaining $G_U + G_D$. Our main point has been that F can be computed from the spectrum of axial mesons and we get a sensible answer for $G_U + G_D$ and therefore G_{Bj}.

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