Sensitivity Analysis of Failure-Prone Flexible Manufacturing Systems

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Abstract

Performance evaluation of failure-prone manufacturing systems is of significant practical interest. In this paper, we consider the parametric sensitivity analysis of manufacturing systems producing multiple part types. This involves transient analysis of the Markov model of the system and computation of the derivative of the cumulative throughput with respect to the varying parameters. Such an analysis would be useful in studying the effects of various not exactly known parameters on the system performance, idenitifying the bottlenecks and hot spots and in system optimization studies.

In this paper, we derive expressions for the sensitivity of the first two moments of the throughput-related performability with respect to failure and repair rates of various subsystems such as machines, guided vehicles, robots, etc. We also present two examples to illustrate the theoretical results of this paper.

1 Introduction

1.1 Manufacturing Systems

A typical manufacturing system is an interconnection of failure-prone subsystems such as numerically controlled machines, assembly stations, automated guided vehicles (AGVs), robots, conveyors and computer control systems. Such systems have a degree of flexibility due to the versatality of the equipment. The flexibility provides a certain degree of fault-tolerance and the manufacturing system operates at a degraded level of performance during the times of repair and reconfiguration.

Throughput, the number of parts produced in unit time and manufacturing lead time, the amount of time the workpiece resides on the factory floor, are two important performance measures in a manufacturing system. Discrete event simulation, Markov chains, queuing networks and stochastic petri nets are used to compute these performance measures. Reliability, availability, cumulative operation time are also performance measures of interest in manufacturing system studies. The above modeling tools could be used for computing these measures as well.

It is also possible to conduct performability analysis i.e. combined performance-dependability analysis. However, an exact monolithic model incorporating processing, setups, failures and repairs would be very large and stiff. The stiffness arises because of large difference in the frequency of occurrences of various events. Typically part processing rates are orders of magnitude higher than the failure rates. Time-scale decomposition is used to obtain near accurate and computationally efficient soultions. Here the exact model is decomposed into (1) a higher level (slow time-scale) structure state process describing the failure-repair behaviour. (2) a lower level (fast-time scale) performance model describing part movement and processing activities. Several studies were reported in the literature on performability analysis of manufacturing systems using time-scale decomposition [8] [10] [9]. In this paper, we are interested in developing models for sensitivity analysis. In particular, we derive expressions for the changes in first and second moments of the throughput due to the changes in the rate matrix.

Parametric sensitivity analysis refers to the study of the system behaviour due to changes in the parameters. In this paper we are concerned with computing the effect of changes in the rate constants of the structure state model or performance model on the throughput. Such analysis helps (a) guide system optimization (b) find the reliability, performance and the performability bottlenecks in the system and (c) identify the model parameters that could produce significant errors.

In principle, it is possible to perform sensitivity analysis using dependability model alone, the performance model alone and using the composite model. For example, we could consider the effect of a machine failure rate on the mean or variance of the throughput or lead time using the composite performance-dependability model, the effect of machiine processing times on the variability of the throughput of a failure-free manufacturing system or the effect of repair rate of a guided vehicle on the system availability. In our analysis here,we are concerned with smooth variations in the parameters of the rate matrix and their effect on system performance and not with large changes such as adding one or more AGVs or an NC machine or

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changes in scheduling strategies and so on.

1.2 Earlier Work

Performability studies of manufacturing systems have attracted a lot of interest in the recent past. In [8], Viswanadham *et al.* describe the formulation of dependability and performance models and the evaluation of performability measures for manufacturing systems. In [10, 5], it was shown that the cumulative performability distribution function satisfies a linear hyperbolic partial differential equation and a methodology is presented for the evaluation of moments of the distribution function for manufacturing systems producing multiple part types. Efficient algorithms based on uniformization were presented for the computaion of the first two moments of performability distribution function in [9].

Literature exists on the sensitivity analysis of performability models of fault-tolerant computing systems. Blake *et al.* [1] study the sensitivity of reliability and performance measures of a multiprocessor system prone to failures using Markov models. Efficient algorithm based on uniformization was presented [3] for the computation of sensitivity, the derivative, of the transient solution of a continuos time Markov chain with respect to a parameter of a rate transition matrix. Muppala [4] extended these results to reward models based on generalised stochastic Petri nets (GSPNs) along the lines of uniformization proposed in [3].

1.3 Outline of the Paper

In section 2, we review the notions of structure state process and accumulated rewards and also model the performability distribution function and its moments. In section 3, we derive the sensitivity for the mean and second moment of performability and provide recurrence equations for their efficient evaluation. Section 4 considers two manufacturing systems one producing single part type and the other producing multiple part types and their sensitivity results are presented. The results on the sensitivities of second moments are omitted for the sake of brevity; they may be found in [2].

2 Composite Dependability and Performability Models

In this section, we briefly review the concepts and results that are necessary to present our main results on sensitivity analysis.

2.1 Structure State Process

Definition 1 : *Given a manufacturing system, its* **structure state** *is a vector whose components describe the condition of individual subsystems due to failures, repairs, and reconfigurations.*

The structure state of the system evolves with time. The dynamics of the state transitions is captured via the structure state process defined below.

Definition 2 : *Let $Z(u)$ be the structure state of a manufacturing system at time $u \geq 0$. Then the family of random variables $\{Z(u) : u \geq 0\}$ is called the* **structure state process** *(SSP) of the manufacturing system.*

Let S be the state space of the SSP of cardinality N. For each part type $j \in \{1, 2, \ldots, P\}$, S can be partioned into two disjoint sets $S_{O,j}$ and $S_{F,j}$: $S_{O,j}$ is the set of operational states in which part type j can be produced and $S_{F,j}$ is the set of failed or nonoperational states in which part type j cannot be produced.

Example 1a:

Consider a flexible cell with two identical machines M_1 and M_2 and an AGV. Let the state of M_1, M_2 and AGV be designated as 1 when they are up and 0 if they are down. The state space of the SSP has six states :

$$S = \{(21), (11), (01), (20), (10), (00)\}$$

We assume that no work is done when AGV fails or when both machines fail. Thus work progresses only in (21) and (11) states. If the failure and repair times are assumed exponential, then S is a Markov chain. Suppose we produce two part types 1 and 2 at this machine shop according to the following scheduling policy : (i) Produce both 1 and 2 if both machines are up. (ii) Produce 1 only if only one machine is up. Then we have

$$S_{O,1} = \{(21), (11)\} \; ; \; S_{F,1} = \{(01), (20), (10), (00)\}$$

$$S_{O,2} = \{(21)\} \; ; \; S_{F,2} = \{(11), (01), (20), (10), (00)\}$$

Suppose that the structure state process $\{Z(u), u \geq 0\}$ represented by S is a homogeneous finite-state continuous time Markov chain with infinitesimal generator matrix Q. Let $p_i(t)$ be the unconditional probability that the Markov chain is in state i at time t, then $\mathbf{p}(t)$ represents the state probability vector of the Markov chain. Then,

$$\dot{\mathbf{p}}(t) = \mathbf{p}(t)Q; \quad \mathbf{p}(0) = \mathbf{p}_0$$

where $\mathbf{p}_0$ is the initial probability vector.

2.2 Dependability Measures

We now define two important measures, reliability and availability. For this, we define the indicator random variables for each $j = 1, 2, \ldots, P$

$$I_j(u) = \begin{cases} 1 & \text{if } Z(u) \in S_{O,j} \\ 0 & \text{if } Z(u) \in S_{F,j} \end{cases}$$

Also define

$$I(u) = \begin{cases} 1 & \text{if } I_j(u) = 1 \text{ for some } j = 1, 2, \ldots, P \\ 0 & \text{otherwise} \end{cases}$$

Definition 3 : *The probability of producing the ith part type, $R_i(t)$ is given by*

$$R_i(t) = P\{I_i(u) = 1, \forall u \in [0, t]\}$$

Further the probability of system functioning properly is given by

$$R(t) = P\{I(u) = 1, \forall u \in [0, t]\}$$

Example 1b:

For the system in example 1a, we have

$$R_1(t) = p_{21}(t) + p_{11}(t) \; ; \; R_2(t) = p_{21}(t)$$

$$R(t) = p_{21}(t) + p_{11}(t)$$

where $p_{ij}(t)$ is the probability that we can find the system in state (ij).

2.3 Markov Reward models

Let $\{Z(u) : u \geq 0\}$ be the structure state process of a manufacturing system. In each structure state, the system can be associated with a performance measure which may be manufacturing lead time, throughput, work in progress, machine utilization, etc. In the most general case, the chosen performance index is a random variable.

Definition 4 : *Let $\{Z(u), u \geq 0\}$ be the structure state process of a manufacturing system with associated state space S. Let r_{ij} be the reward in structure state i for part type j, and $\mathbf{r}_i$ be the row vector $[r_{i1}, \ldots, r_{iP}]$. Then we have*

Instantaneous reward *at time t for part type j*

$$X_j(t) = r_{Z(t)j} = r_{ij} \quad \text{if } Z(t) = i; \; j = 1, \ldots, P$$

The **instantaneous reward vector** *at time t, $X(t)$, is defined as*

$$\mathbf{X}(t) = [X_1(t), X_2(t), \ldots, X_P(t)]$$

Also

$$\mathbf{X}(t) = \mathbf{r}_i \quad \text{if } Z(t) = i$$

Accumulated reward over time *interval* $[0, t]$ *for part type j*

$$Y_j(t) = \int_0^t X_j(u)du = \int_0^t r_{Z(u)j}du$$

Accumulated reward vector *at time t, $\mathbf{Y}(t)$, is defined as*

$$\mathbf{Y}(t) = [Y_1(t), Y_2(t), \ldots, Y_P(t)] = \int_0^t \mathbf{r}_{z(u)}du$$

since r_{ij}'s are nonnegative, $\{Y_j(u); j = 1, 2, \ldots, P\}$ are nonnegative monotonically nondecreasing function of time t.

2.4 Distribution of $\mathbf{Y}(t)$

The evolution of the composite process $\{Z(u), \mathbf{Y}(u), u \geq 0\}$ is completely described by the $N \times N$ matrix $F(\mathbf{y}, t)$ defined by

$$F_{ij}(\mathbf{y}, t) = Pr\{Z(t) = j, \mathbf{Y}(t) \leq \mathbf{y} \mid Z(0) = i\}$$

Evidently, $F_{ij}(\mathbf{y}, t)$ is the cumulative distribution function of $\{\mathbf{Y}(t) \leq \mathbf{y}, Z(t) = j\}$, given that the system starts in the initial structure state i. We note that since $r_{ij} \geq 0$, for any $y_j < 0$, $F(\mathbf{y}, t) = \mathbf{0}$.

The forward partial differential equations (PDEs) that characterize the evolution of the composite process $\{ Z(u), \mathbf{Y}(u), u \geq 0 \}$ are given by the following theorems.

Theorem 1 : The joint distribution matrix $F(\mathbf{y}, t)$ satisfies the following linear hyperbolic partial differential equation

$$\frac{\partial F(\mathbf{y}, t)}{\partial t} = -\sum_{j=1}^{P} \frac{\partial F(\mathbf{y}, t)}{\partial y_j} R_j + F(\mathbf{y}, t)Q$$

where $R_j = Diag\{r_{1j}, r_{2j}, \ldots, r_{Nj}\}$ and $Diag$ denotes a diagonal matrix. The boundary conditions are $F(\mathbf{0}, t) = \mathbf{0}, t \geq 0$ and $F(\mathbf{y}, 0) = \prod_{p=1}^{P} U(y_p) I_N$, where I_N is an $N \times N$ identity matrix.

2.5 Moment Recursions

The solution of partial differential equations satisfied by $F(y, t)$ is computationally cumbersome for large systems ([6],[7]). In the following, we state recursive ordinary differential equations describing the evolution of the moments of performability. Specifically, the diffrential equation describing nth moment will be expressed in terms of $(n-1)$th moment of performability. Formally, let $\mathbf{m}_n^j(t)$ be the column vector denoting the nth conditional moment vector for part type j, defined by

$$\begin{aligned} m_{n,i}^j(t) &= E\left[\mathbf{Y}_j^n(t) \mid Z(0) = i\right]; i = 1, 2, \ldots, N; \\ & \quad j = 1, 2, \ldots, P \\ \mathbf{m}_n^j(t) &= \left[m_{n,1}^j, m_{n,2}^j, \ldots, m_{n,N}^j\right]^T \end{aligned}$$

It is clear that $\mathbf{m}_0^j(t) = \mathbf{e}$; $\mathbf{m}_n^j(0) = \mathbf{0}$ for $n \geq 1$. In addition, the nth moment of cumulative reward for part type j is given by $E[(Y(t))^n] = \mathbf{p}(0)\mathbf{m}_n^j(t)$.

Theorem 2 : The moment vector $\mathbf{m}_n^j(t)$, $n \geq 1$, $j = 1, 2, \ldots, P$ evolves according to the equation

$$\frac{d\mathbf{m}_n^j(t)}{dt} = Q\mathbf{m}_n^j(t) + nR_j\mathbf{m}_{n-1}^j(t); \;\; \mathbf{m}_0^j(t) = \mathbf{e}$$

In [9], computaional methods for efficiently solving for $m_n^j(t)$ as well as several examples in the area of manufacturing systems were presented.

3 Sensitivity Analysis

3.1 Computational Model for Sensitivity Analysis of AMSs

In section 2, we have discussed methods to solve the CTMC and obtain the solution for the performability distribution and its various moments. In this section we extend this study and derive the equations for calculating the sensitivity of the moments and propose an efficient solution based on uniformization. In sections 3.2.1 and 3.2.2, we derive expressions for the sensitivities of the mean and second moment of performability with respect to any variations in the parameters of the dependability model, assuming that the parameters of the performance model remain fixed.

3.2 Sensitivity analysis using Markov models

Let θ be the uncertain input parameter in the manufacturing system. Typically, θ could be the failure rate or the repair rate of a machine or an AGV. Then $\{Z(u,\theta), u \geq 0\}$ represents structure state process of the manufacturing system. Let $Q(\theta)$ represent the infinitesimal generator matrix of the homogeneous finite state continuous-time Markov chain modeling the structure-state process and $\mathbf{p}(\theta,t)$ be the state probability vector. We assume that $\frac{\partial Q(\theta)}{\partial \theta}$ exists. Then the time dependent behaviour of the CTMC can be described by

$$\dot{\mathbf{p}}(\theta,t) = \mathbf{p}(\theta,t)Q(\theta)\ ,\ \mathbf{p}(\theta,0) = \mathbf{p}_0$$

In the above equation, we assume that the initial probability vector is independent of θ. The solution of the above equation can be obtained using uniformization.

$$\mathbf{p}(\theta,t) = \sum_{n=0}^{\infty} \boldsymbol{\pi}_n(\theta) e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

where $\boldsymbol{\pi}_n(0)$ satisfies the recurrence relations,

$$\boldsymbol{\pi}_n(\theta) = \boldsymbol{\pi}_{n-1}(\theta)P(\theta)\ ;\ \boldsymbol{\pi}_0(\theta) = \mathbf{p}_0$$

with $P(\theta) = I + \frac{Q(\theta)}{\lambda}$ and $\lambda \geq max(|q_{ii}(\theta)|)$.

The derivatives of the probability vector $\mathbf{p}(\theta)$ can also be obtained using uniformization

$$\frac{\partial \mathbf{p}(\theta,t)}{\partial \theta} = \sum_{n=0}^{\infty} \mathbf{d}_n(\theta) e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

where $\mathbf{d}_n(\theta)$ is computed using the recurrence relations

$$\mathbf{d}_n(\theta) = \mathbf{d}_{n-1}(\theta)P(\theta) + \boldsymbol{\pi}_{n-1}(\theta)\frac{\partial P(\theta)}{\partial \theta}\ ;\ \mathbf{d}_0(\theta) = \mathbf{0}$$

Once the probabilities and their derivatives are known, it is straight forward to compute the sensitivities of various performance measures with respect to θ. For example, one can define $R_j(\theta,t)$ be the reliability of producing part type j, then we know that

$$R_j(\theta,t) = \sum_{i \in S_{O,j}} \mathbf{p}_i(\theta,t)$$

One can compute the sensitivity of $R_j(\theta,t)$ with respect to θ using the method described above.

Example 1c:

For the system in example 1a, let us assume that λ_m and λ_a represent the failure rates of the machines and AGV respectively. We also assume that no repair is possible for any failed subsystem. Then the probabilties $p_{21}(t)$ and $p_{11}(t)$ are given by

$$p_{21}(t) = e^{-(2\lambda_m+\lambda_a)t}\ ;\ p_{11}(t) = 2\left[e^{-(\lambda_m+\lambda_a)t} - e^{-(2\lambda_m+\lambda_a)t}\right]$$

The sensitivity of $R_2(t)$ with respect to λ_a is given by

$$R_2(\lambda_a,t) = \frac{dp_{21}(t)}{d\lambda_a} = -te^{-(2\lambda_m+\lambda_a)t}$$

In the rest of this section, we concentrate on deriving recurrence relations for computing the first and second moments of the throughput related performability.

3.2.1 Sensitivity of first moment of performability

Consider the moment recursion diffrential equations for the first moment (obtained from Theorem 2)

$$\frac{d\mathbf{m}_1^j(\theta,t)}{dt} = Q(\theta)\mathbf{m}_1^j(\theta,t) + \mathbf{r}_j\ where\ \mathbf{r}_j = R_j\mathbf{e}$$

As above, efficient solution of the above equation can be obtained using uniformization in [9]. Infact, $\mathbf{m}_1(\theta,t)$ can be expressed as

$$\mathbf{m}_1(\theta,t) = \sum_{m=0}^{\infty} \alpha_m(t)\gamma_m(\theta) \tag{1}$$

where, $P(\theta)$ and λ are as defined before and

$$\alpha_m(t) = \frac{e^{-\lambda t}(\lambda t)^{m+1}}{(m+1)!}$$

$$\gamma_m(\theta) = \sum_{n=0}^{m} P(\theta)^n \mathbf{r}_j$$

The following recurrence equations can be used to describe α_m and γ_m

$$\alpha_m(t) = \alpha_{m-1}(t)\frac{\lambda t}{(m+1)}\ ;\ \alpha_0(t) = \lambda t e^{-\lambda t} \tag{2}$$

$$\gamma_m(\theta) = P\gamma_{m-1}(\theta) + \frac{\mathbf{r}_j}{\lambda}\ ;\ \gamma_0(\theta) = \frac{\mathbf{r}_j}{\lambda} \tag{3}$$

Thus $\mathbf{m}_1(\theta, t)$ could be recursively computed using equations (1), (2) and (3). Now let $\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}$ represent the sensitivity of $\mathbf{m}_1(\theta, t)$ with respect to θ. By straightforward differentiation, we obtain

$$\left.\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}\right|_{\infty} = \sum_{m=0}^{\infty} \alpha_m(t) \frac{\partial \gamma_m}{\partial \theta} \qquad (4)$$

Let $\mathbf{b}_m$ represent the vector $\frac{\partial \gamma_m}{\partial \theta}$ and it can be written as

$$\mathbf{b}_m = \frac{\partial \gamma_m}{\partial \theta} = \frac{\partial P}{\partial \theta}\gamma_{m-1} + P\frac{\partial \gamma_{m-1}}{\partial \theta}$$

Now $\mathbf{b}_m$ can be calculated using the recurrence equation

$$\mathbf{b}_m = \frac{\partial P(\theta)}{\partial \theta}\gamma_{m-1} + P\mathbf{b}_{m-1} \; ; \; \mathbf{b}_0 = \mathbf{0} \qquad (5)$$

Similarly, the sensitivity $\mathbf{m}_1(\theta, t)$ can be computed using the recurrence equation

$$\left.\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}\right|_{k} = \left.\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}\right|_{k-1} + \alpha_k(t)\mathbf{b}_k \; ; \qquad (6)$$
$$\left.\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}\right|_{0} = \mathbf{0}$$

Thus the sensitivity of $\mathbf{m}_1(\theta, t)$, $\frac{\partial \mathbf{m}_1(\theta,t)}{\partial \theta}$ could be recursively computed using equations (6), (5), (3) and (2).

3.2.2 Sensitivity of second moment of performability

Consider the differential equation for the second moment

$$\frac{d\mathbf{m}_2^j(\theta,t)}{dt} = Q(\theta)\mathbf{m}_2^j(\theta,t) + 2R_j\mathbf{m}_1^j(\theta,t)$$

The above diffrential equation can be solved using uniformization based on the following recurrence equations [9], where $s_2(\theta, t) = \mathbf{p}_0\mathbf{m}_2(\theta, t)$.

$$s_2(\theta,t,k) = \frac{2}{\lambda}\sum_{n=0}^{k}\sum_{m=0}^{k} \boldsymbol{\pi}_n R\gamma_m(\theta)\alpha_{n+m+1}(t) \qquad (7)$$

$$\boldsymbol{\pi}_k = \boldsymbol{\pi}_{k-1}P(\theta) \; ; \; \boldsymbol{\pi}_0 = \mathbf{p}_0 \qquad (8)$$

The recurssions for $\alpha_m(t)$ and $\gamma_m(\theta)$ are given above. Now, $s_2(\theta, t, k) \longrightarrow s_2(\theta, t)$ as $k \longrightarrow \infty$. The sensitivity of the second moment w.r.t. θ can be written as

$$\left.\frac{\partial s_2(\theta,t)}{\partial \theta}\right|_{k} = \frac{2}{\lambda}\sum_{n=0}^{k}\sum_{m=0}^{k}\left[\boldsymbol{\pi}_n R\frac{\partial \gamma_m}{\partial \theta}\alpha_{n+m+1} + \frac{\partial \boldsymbol{\pi}_n}{\partial \theta}R\gamma_m\alpha_{n+m+1}\right] \qquad (9)$$

Parameter rate(per hour)	*Sensitivity of mean throughput at 100 hours*
Failure of Mcs.	-804.3292
Failure of AGV	-415.9707
Repair of Mcs.	34.5448
Repair of AGV	61.5705

Table 1: *Sensitivities of throughput of machine center*

Let $\mathbf{d}_n = \frac{\partial \boldsymbol{\pi}_n}{\partial \theta}$. Now from (8), we have

$$\mathbf{d}_n = \mathbf{d}_{n-1}P + \boldsymbol{\pi}_{n-1}\frac{\partial P}{\partial \theta} \; ; \; \mathbf{d}_0 = \mathbf{0} \qquad (10)$$

The sensitivity of second moment can be written as

$$\left.\frac{\partial s_2(\theta,t)}{\partial \theta}\right|_{k} = \left.\frac{\partial s_2(\theta,t)}{\partial \theta}\right|_{k-1} + \frac{2}{\lambda}\left[\sum_{m=0}^{k}[\boldsymbol{\pi}_k R\mathbf{b}_m\alpha_{m+k+1} + \mathbf{d}_k R\gamma_m\alpha_{m+k+1}] + \sum_{n=0}^{k-1}[\boldsymbol{\pi}_n R\mathbf{b}_k\alpha_{n+k+1} + \mathbf{d}_n R\gamma_k\alpha_{n+k+1}]\right] \qquad (11)$$

Thus the sensitivity of the second moment can be recursively computed to the desired accuracy using equations (8 - 11).

4 Numerical Results

In this section we present two manufacturing examples to illustrate the theory discussed in earlier sections and discuss the results.

Example 2:

We consider a machine center having two identical machines, and an AGV. The AGV transports parts between the machines and the load/unload station. The failure rates of the machine and AGV are 0.02 per hour and 0.1 per hour respectively. A single repair facility is available which can repair the failed machines at a rate 2.0/hour and AGV at a rate 1.0/hour. The repair of AGV is given higher priority over the machine. We assume that the machine center produces parts of single type at a rate of 12 parts per hour. The performance measures of the machine center in various structure states are calculated using petri net based performance model. The sensitivities of mean throughput of the combined performability model with respect to various parameters are calculted and are tabulated in table 1.

We can observe that the sensitivity values with respect to failure rates are negative indicating that an increase in failure rate will result in loss of throughput. In particular, the effect of failure and repair rates of AGV are comparitively lesser than the ones for the machines. This

Machine	$M1$	$M2$
Part. No.	*Processing Time in minutes*	
$P1$	10	-
$P2$	-	8
$P3$	5	10

Table 2: *Processing times of part types in flexible cell*

Parameter	*Sensitivity of mean and second moment of throughput at 100 hours*		
	Part1	Part 2	Part 3
Failure of $M1$	-871.7995 (-380349.61)	634.7389 (251218.87)	-507.7983 (-132123.53)
Failure of $M2$	183.3142 (81068.37)	-755.2045 (-300067.57)	-527.0830 (-136940.67)
Repair rate	30.5406 (12931.28)	-10.0553 (-4591.86)	30.2081 (7695.72)

Table 3: *Sensitivities of the mean and second moment of throughput*

is because the AGV's utilisation is much less compared to that of the machines.

Example 3:

Consider a flexible cell having two machines $M1$ and $M2$ and producing three part types $P1$, $P2$ and $P3$. The average machining times for the production of parts are exponentially distributed and are given in the table 2:

We assume that the machines can fail independently with rates (per hour) 0.02 and 0.01 and there is a single repair facility which can repair the machines at the rate of 0.5 per hour. We assume the availability of 3 fixtures for part types $P1$ and $P2$ and 4 fixtures for part type $P3$. The performance model is solved using MVA analysis and reward rates for different part types in various structure states are obtained. Then the performability model is solved using the methodology discussed in section 3. Table 3 lists the sensitivities of mean throughput and second moment after 100 hours of production, with respect to failure rates and repair rates. The values in the second line, enclosed in the brackets, indicate the sensitivities of the second moment.

We can observe that an increase in failure rate of machine $M1$ will result in loss of production of part types 1 and 3 but increases the throughput of part type 2. This is because machine 2 is left free during this failure time for producing parts of type 2.

5 Conclusions

The performability analysis of failure-prone flexible manufacturing systems facilitates the evaluation of performance measures such as mean throughput and its variation. The recurrence relations for the evaluation of sensitivities of the first and second moments of the cumulative performability to parameters such as failure and repair rates are given. Two manufacturing examples are presented to illustrate the application of these results.

References

[1] J.T. Blake, A. L. Reibman, and K. S. Trivedi. Sensitivity analysis of reliability and performability measures for multiprocessor systems. In *Proceedings of the 1988 ACM SIGMETRICS Conference in Measurement and Modeling of Computer Systems, Santa Fe, New Mexico*. ACM, 1988.

[2] V. Gopalakrishna. *Performability evaluation of Automated Manufacturing Systems.* PhD thesis, Dept. of Computer Science and Automation, Indian Institute of Science, Bangalore, March 1994(Under Preperation).

[3] P. Heidelberger and A. Goyal. Sensitivity analysis of continuous time Markov chains using uniformization. In G. Iazeolla, P. J. Courtois, and O. J. Boxma, editors, *Computer Performance and Reliability*, pages 93–104. Elsevier Science Publishers B.V., 1988.

[4] Jogesh K. Muppala. *Performance and Dependability modeling using Stochastic Reward Nets.* PhD thesis, Dept. of Electrical Engineering, Duke University, Durham, April 1991.

[5] K. R. Pattipati, V. Gopalakrishna, Ranga Mallubhatla, and N. Viswanadham. Markov reward models and hyperbolic systems. In *Proceedings of second international workshop on performability modelling of Computer and Communication systems, Le Mont Saint-Michel, France*, 1993.

[6] K. R. Pattipati, Yong Li, and Henk A. P. Blom. A united framework for the performability evaluation of fault-tolerant computer systems. *IEEE Transactions on Computers*, March 1993.

[7] K. R. Pattipati and Samir A. Shah. On the computational aspects of performability models of fault-tolerant computer systems. *IEEE Transactions on Computers*, 39(6):832–836, June 1990.

[8] N. Viswanadham, Y. Narahari, and R. Ram. Performability of automated manufacturing systems. *Control and Dynamic Systems*, 47:77–120, 1991.

[9] N. Viswanadham, K. R. Pattipati, and V. Gopalakrishna. Performability studies of AMSs with multiple part types. *Submitted to IEEE Transactions on Robotics and Automation.*

[10] N. Viswanadham, K. R. Pattipati, and V. Gopalakrishna. Performability studies of AMSs with multiple part types. *invited paper at the 1993 IEEE Robotics and Automation conference at Atlanta, GA.* IEEE, May 1993.