TOMOGRAPHIC APPROACH TO ACOUSTIC NOISE MAPPING IN SHALLOW SEA

Vinayak Dutt and G.V. Anand

Department of Electrical Communication Engineering
Indian Institute of Science
Bangalore 560 012, INDIA

ABSTRACT

This paper presents a method for the acoustic noise mapping of a shallow ocean using a tomographic approach. The output of a planar array of hydrophones in an ocean is expressed as a weighted line integral of the acoustic field. A set of measurements of the array output for different steering directions and several array locations can be inverted using algebraic reconstruction technique to reconstruct the acoustic field.

INTRODUCTION

The possibility of using a tomographic technique for noise mapping of the ocean was first suggested by Rockmore [1]. One important application of noise mapping is in ocean surveillance to detect and localise discrete acoustic sources which would stand out as peaks in the noise map. Rockmore has formulated the noise mapping problem for an unbounded medium. In this paper we have extended the formulation to shallow ocean by taking into account the multipath effects due to reflections from the top and the bottom surfaces. For this purpose, it is necessary to replace linear arrays by planar arrays. The acoustic field due to a source can be expressed as a sum of normal modes. Each vertical column of a planar array is used as a mode filter to extract the signal due to a single normal mode. Each vertical column is designed to select the same (say, the mth mode). The mode-filtering eliminates the multipath effect, and the planar array output is seen to be a family of fan-beam projections of the depth-averaged noise intensity distribution when the array is steered in different directions. Projections from several arrays can be inverted by tomographic methods to reconstruct the map of depth-averaged noise intensity. We have used an algebraic reconstruction technique [2] for performing the tomographic inversion. Computer simulation results are presented for two different noise intensity distributions over a region of finite support.

FORMULATION OF THE PROBLEM

The normal-mode representation for acoustic field at a receiver located at (x_o, y_o, z_o) due to a source of amplitude $A(x,y,z)$ located (x,y,z) is given by [3],

$$p(x,y,z,t) = \pi j \sum_{m=1}^{M} A(x,y,z)\,\psi_m(z_o)\,\psi_m(z)\,H'_o(\xi_m r) \cdot \exp(-\alpha_m r - jwt) \qquad (1)$$

where w is the angular frequency of the source, α_m is the attenuation coefficient of the mth mode, M is the total number of propagating modes, ξ_m is the horizontal component of the wave number of the mth mode, $\psi_m(z)$ is the eigen function for the mth mode, $H'_o(\xi_m r)$ is the Hankel function of the first kind and

$$r = [(x-x_o)^2 + (y-y_o)^2]^{1/2} \qquad (2)$$

Hence, the output of a horizontal line array of (2N+1) elements with inter-element spacing as d, steered in the direction θ, centred at (x_o, y_o, z_o), is given by

$$p(x,y,z,t,\theta) = \pi j \sum_{m=1}^{M} A(x,y,z)\,\psi_m(z)\,H'_o(\xi_m r)\psi_m(z_o) \cdot D_m(\theta,\gamma)\exp(-\alpha_m r - jwt), \qquad (3)$$

where

$$D_m(\theta,\gamma) = \frac{\sin[(2N+1)(d/2)(\xi_m \cos\gamma - k\cos\theta)]}{\sin[(d/2)(\xi_m \cos\gamma - k\cos\theta)]} \qquad (4)$$

is the directivity function of the array, $k = w/c$ is the wave number; and

$$\gamma = \tan^{-1}[(y-y_o)/(x-x_o)] \qquad (5)$$

is the bearing of the source.

In order to avoid the multi-mode field at the array, we replace each element of the horizontal array by a vertical column of (2M-1) hydrophones. Each column is designed as a null-steering mode filter [4] to select the first mode. A normal mode is a standing wave pattern formed by the interference of two quasi-plane waves travelling at angles $\pm\theta_m$ with respect to the horizontal, where,

$$\theta_m = \cos^{-1}(\xi_m/k), \qquad m = 1, \ldots, M. \qquad (6)$$

Mode filtering is achieved by steering nulls in the directions of arrival of all unwanted modes. To reject the mth mode, a 3-elemental vertical array with the weighting coefficient sequence

$$w_m = \{1,\ -2\cos[(k^2 - \xi_m^2)^{1/2}\Delta],\ 1\} \qquad (7)$$

is required, where Δ is the inter-element spacing of the vertical array. All modes higher than the first can be rejected using a (2M-1) element vertical array with weighting coefficient sequence,

$$W = w_2 * w_2 * \ldots * w_M \qquad (8)$$

where * denotes the convolution. It is important to note that the weighting coefficients are independent of the source bearing angle and the array depth. The weighting coefficients are also real and symmetric, which is an added advantage during implementation.

After the mode filtering, only the contribution of the first mode remains. Hence, the total field due to source distribution throughout the volume can be expressed in the integral form as,

$$p(\theta) = \pi j \int_0^{\infty} r\,dr \int_0^{2\pi} d\gamma \int_0^{h} dz\, A(x_o + r\cos\gamma,\ y_o + r\sin\gamma, z) \cdot \psi_1(z_o)\,\psi_1(z)\,H'_o(\xi_1 r) \cdot D_1(\theta,\gamma)\exp(-\alpha_1 r - jwt) \qquad (9)$$

where h is the ocean depth. Separating the integral with respect to z as

$$F(x_o + r\cos\gamma,\ y_o + r\sin\gamma) = \int_0^{h} A(r_o + r\cos\gamma,\ y_o + r\sin\gamma,\ z)\,\psi_1(z)\,dz \qquad (10)$$

we get,

$$p(\theta) = \pi j \int_0^\infty r dr \int_0^{2\pi} d\gamma \, F(x_o + r\cos\gamma, \, y_o + r\sin\gamma) \cdot D_1(\theta,\gamma)\, \Psi_1(z_o)\, H'_o(\xi_1 r)\, \exp(-\alpha_1 r - jwt) \quad (11)$$

The power output of the array is

$$I(\theta) = E[\, |p(\theta)|^2] \quad (12)$$

where E is the expectation operator.

Assuming that the noise distribution is uncorrelated and has slow variations along the depth, representing the depth-averaged noise intensity as

$$\sigma^2(x_o + r\cos\gamma, \, y_o + r\sin\gamma) = 1/h \int_0^h |A(x_o + r\cos\gamma, \, y_o + r\sin\gamma, \, z)|^2 dz \quad (13)$$

and using the far-field approximation for the Hankel function,

$$|H'_o(\xi_1 r)|^2 \simeq (2/\pi\xi_1 r) \quad (14)$$

equation (12) can be written as

$$I(\theta) = 2\pi(\Psi_1^2(z_o)/\xi_1) \int_0^{2\pi} D_1^2(\theta,\gamma)\, d\gamma \, [\int_0^\infty \sigma^2(x_o + r\cos\gamma, \, y_o + r\sin\gamma)\, \exp(-2\alpha_1 r) r\, dr] \quad (15)$$

Now if the inner integral of equation (15) is represented as,

$$S(\gamma) = \int_0^\infty \sigma^2(x_o + r\cos\gamma, \, y_o + r\sin\gamma) \cdot \exp(-2\alpha_1 r) r\, dr, \quad (16)$$

we get,

$$I(\theta) = 2\pi(\Psi_1^2(z_o)/\xi_1) \int_0^{2\pi} S(\gamma)\, D_1^2(\theta,\gamma)\, d\gamma \quad (17)$$

Defining $\alpha = \cos\gamma$ and $\beta = (k/\xi_m)\cos\theta$, (18)

we can write,

$$I(\theta)\Big|_{\theta = \cos^{-1}(\xi_1\beta/k)} = 2\pi(\Psi_1^2(z_o)/\xi_1) \cdot \int_{-1}^{+1} (S(\cos^{-1}(\alpha))/\sqrt{1-\alpha^2}) \cdot G(\beta - \alpha)\, d\alpha \quad (19)$$

where

$$G(x) = \sin^2[((2N+1)/2)\, d.x.]/\sin^2[d.x/2] \quad (20)$$

is the directivity function. Hence, from equation (19) it can be seen that $I(\theta)$ is a convolution integral. So $S(\gamma)$ can be obtained from $I(\theta)$ by deconvolution as $G(\alpha)$ is a known function. Equation (16) indicates that $S(\gamma)$ is a family of fan-beam projections of the function $\sigma^2(x,y)$, the vertex of the fan being at (x_o, y_o). Several families of projectioons can be obtained by deploying arrays at several locations. From these families of projections, the depth-averaged sound intensity distribution $\sigma^2(x,y)$ can be reconstructed using the algebraic reconstruction technique [2].

ALGEBRAIC RECONSTRUCTION TECHNIQUE

Let the deconvolved mode power $S(\gamma)$ obtained for various receiver locations be denoted by S_j where subscript j denotes the deconvolved power for jth bearing γ_j and receiver at (x_j, y_j). Let the total number of such measurements be N_p.

Then,

$$S_j = \int_0^\infty \sigma^2(x_j + r\cos\gamma_j, \, y_j + r\sin\gamma_j)\, \exp(-2\alpha_1 r)\, r\, dr; \quad j = 1, \ldots, N_p \quad (21)$$

For the reconstruction of the noise intensity map, the scanned area is divided into a uniform grid, with the reconstruction being performed to estimate the noise intensity at these grid points (see Fig. 1). The value of σ^2 at other points is computed by interpolation of the values at the grid points. Now approximating the integral in the equation (21) by a summation,

$$S_j = \sum_{k=1}^{M_j} \sigma^2(x_{jk}, y_{jk})\, \exp(-2\alpha_1 r_{jk})\, r_{jk} \Delta r; \quad j = 1, \ldots, N_p \quad (22)$$

where M_j is the number of steps in the jth summation, and $\sigma^2(x_{jk}, y_{jk})$ is the estimate of the noise intensity at (x_{jk}, y_{jk}) computed by interpolation of the grid point noise intensity values, g_i $(i = 1, \ldots, N_g$, where N_g = number of grid points),

$$\sigma^2(x_{jk}, y_{jk}) = \sum_{i=1}^{N_g} d_{ijk}\, g_i; \quad j=1, \ldots, N_p; \quad k=1, \ldots, M_j. \quad (23)$$

and d_{ijk} is the contribution of g_i to the estimate of σ^2 at (x_{jk}, y_{jk}). So equation (22) can be written as,

$$S_j = \sum_{i=1}^{N_g} [\sum_{k=1}^{M_j} \exp(-2\alpha_1 r_{jk})\, r_{jk}\, d_{ijk} \Delta r] . g_i; \quad j=1, \ldots, N_p \quad (24)$$

Let

$$a_{ij} = \sum_{k=1}^{J} d_{ijk}\, \exp(-2\alpha_1 r_{jk})\, r_{jk} \Delta r; \quad i=1, \ldots, N_g, \; j=1, \ldots, N \quad (25)$$

then,

$$S_j = \sum_{i=1}^{N_g} a_{ij}\, g_i; \quad j=1, \ldots, N_p \quad (26)$$

Equation (26) is a linear set of equations in g_i which can be recursively solved by the algebraic reconstruction technique (ART), where the ART iteration is given by [2]:

$$\bar{g}^{(k+1)} = \bar{g}^{(k)} + \bar{a}_j \frac{(S_j - \bar{a}_j^T \bar{g}^{(k)})}{\bar{a}_j^T \bar{a}_j} \quad (27)$$

SIMULATION RESULTS

Computer simulations were carried out for a Pekeris model of the ocean with the parameters, depth as 50m, speed of sound in the ocean and the bottom medium as 1500 m/s and 2000 m/s respectively, ratio of density of bottom medium to that of ocean water as 1.2 and $\xi_1 = 0.2426$ rad/m.

The area scanned was a square 4000m x 4000m with the coordinates of its corners as (0,0), (0,4000), *(4000,4000)* and (4000,0). The line array had 21 elements with inter-element spacing 2m. The four arrays were located at (0,1000), (0,2000) (0,3000) and (1000,0), centred at a depth of 13m. 60 scans were taken from each array.

In these simulations, actually there was no deconvolution done as a delta function approximation was made for the array directivity function during the reconstruction.

Figure 2 gives the results for the two source distributions taken for the simulations. The actual depth-averaged noise intensity is shown in Figs. 2(a) and 2(b) respectively. The intensities in the map are scaled linearly in the range *0* to *9*.

The result for noise intensity distribution given by Fig. 2(a) is not as good as that for the distribution given by Fig. 2(b). This is so because all the arrays are on one side of the scanned area, and the direction of the gradient of the noise intensity distribution is along the scan directions for this array configuration, which results in inaccuracies in the reconstruction.

CONCLUSIONS

The present work shows that the tomographic approach for noise intensity mapping seems to give fairly good results even though no deconvolution of the array output was done and a small number of arrays was used. Presently work is being carried out on deconvolution processing to see the improvement in the reconstruction by this extra computational effort.

REFERENCES

[1] A.J. Rockmore, 'A Tomographic Approach to Multiarray Ocean Surveillance', IEEE Journal of Ocean Engg., Vol. OE-17, pp 83-89, April 1982.

[2] Simon Haykin, Ed., Array Signal Processing, Prentice Hall 1985, Ch. 6, pp 408-419.

[3] L. Biekhovskikh and Yu. Lysanov, Fundamentals of Ocean Acoustics, Springer-Verlag 1982, Ch. 6, pp 120-122.

[4] H.M. Chouhan and G.V. Anand, 'A New Technique of Acoustic Mode Filtering in Shallow Sea', Submitted to the Journal of Acoustical Society of America for publication.

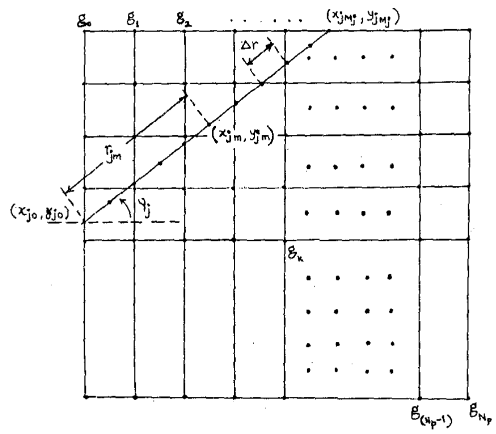

Fig. 1. Illustration of the noise intensity projection integral by a summation over a square reconstruction region.

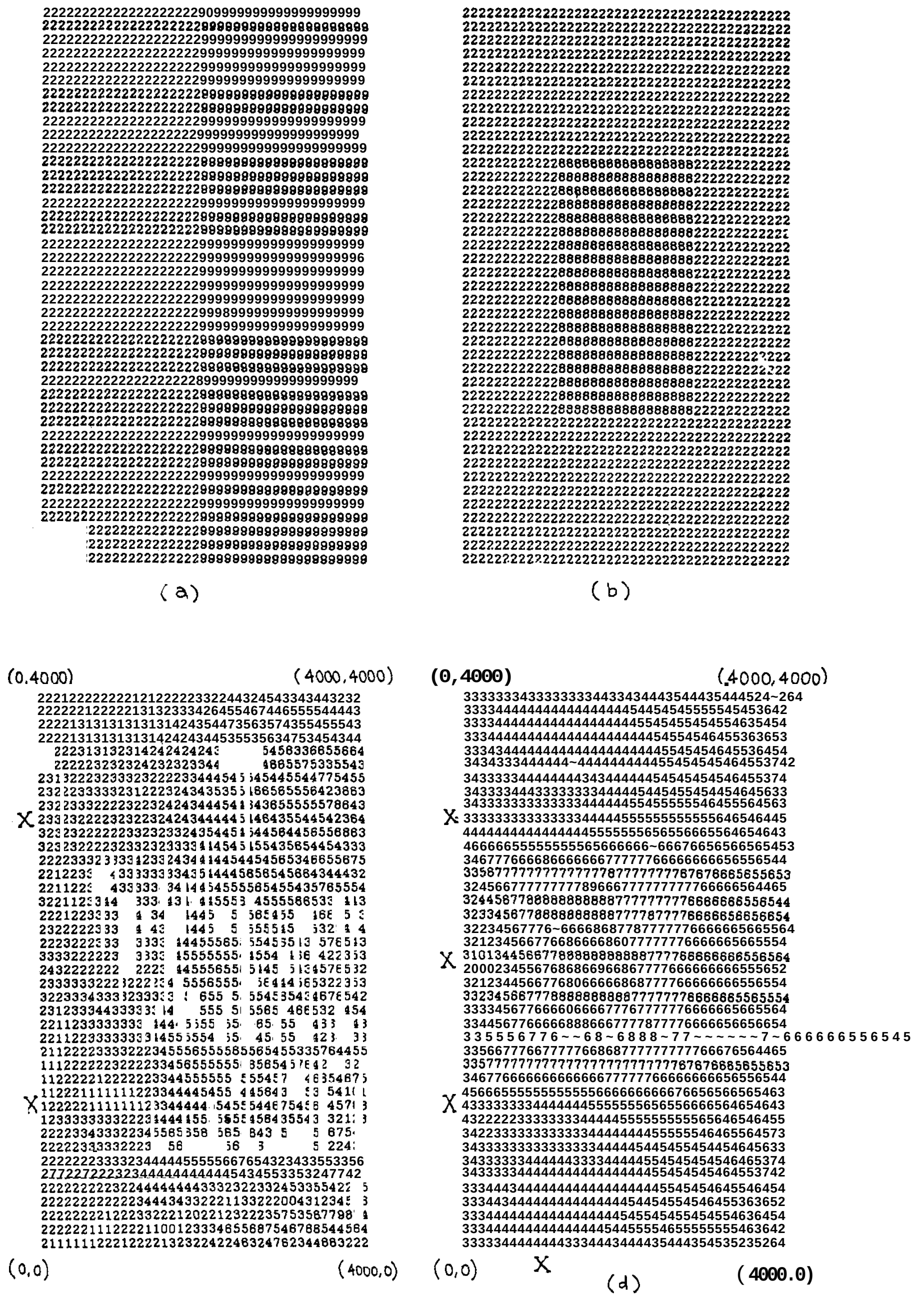

Fig. 2. Simulation results. (a) and (b) give the actual depth averaged noise intensity map used for simulations (c) and (d) give the reconstruction using the tomographic approach for (a) and (b) respectively. (X in part (c) and (d) give the positions of the arrays).