A TENSOR APPROACH TO UNDERSTAND DYNAMIC INSTABILITY IN A POWER SYSTEM AND DESIGN A STABILIZER

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ABSTRACT

This paper presents a physical explanation of dynamic instability that afflicts the present-day power systems. Tensor analysis has been used to compute the components of torque generated in various windings and controller of a synchronous generator. The identification of the source of instability is utilized to design a stabilizer which ensures stable operation at all operating conditions.

KEYWORDS

Dynamic stability, Tensor analysis, Power system stabilizers

INTRODUCTION

A synchronous generator operating in an inter-connected power system has been afflicted by various kinds of instability over the last sixty years. Ingenious solutions have been found to stabilize generators and the power systems. The solutions have been, by and large effective, leading to the operation of massive generators in extended grids with greater efficiency. On the other hand, attempts to solve one problem have at times given rise to other problems.

Although, there must have been qualitative understanding of the causes leading to a certain type of instability, a quantitative assessment of the forces or torques of de-stabilization have not always been possible. Hunting or small sustained oscillations of rotors in generators was fairly common in early thirties. Damper windings were used and the hunting could be controlled. Steady state stability which limited the transfer of bulk power across long lines leading to monotonic pull out of the generator had to be contended with. High gain automatic voltage regulators intended to maintain the terminal voltage within limits were found to improve steady state stability. Transient instability following large disturbances is presently being controlled by fast fault clearance, field forcing with high gain low time constant voltage regulators, fast valving and resistor braking. These AVR's have, on the other hand, led to dynamic instability, where even small disturbances cause the generator to pull out of synchronism in an oscillatory mode. Power System Stabilizers (PSS) are now commonly used to overcome dynamic instability. The choice of the PSS-gain and time constants is somewhat arbitrary in view of the inadequate understanding of the physical processes that lead to dynamic instability. A large number of papers have been published in recent years proposing adaptive or self-tuning stabilizers. Modern control theory is commonly used in the design of stabilizers based on input/output or state variable models. No attempt is made to understand why and how a power system becomes unstable, much less attempt to design a PSS based on this understanding.

This paper presents a brief account of the findings spread over a number of years which have ultimately led to the understanding of the physical processes leading to instability and the design of a simple, robust, self tuning power system stabilizer. The analysis has been based essentially on exact quantification of various torque components controlling steady state and dynamic instability. This has been possible by utilizing the properties of tensor invariance.

ELECTRODYNAMICS OF A SYNCHRONOUS GENERATOR WITH AVR CONTROL

A synchronous generator with AVR and governor control, connected to an infinite busbar through a tie-line has been considered so as not to confuse the basic issues involved. The PSS however, has been tested on both multi-machine and single machine configurations.

The dynamics of the synchronous generator is represented by the well known equation

$$Mp^2\Delta\delta + T_D p\Delta\delta + T_S\Delta\delta = 0 \qquad (1)$$

Negative or zero synchronizing torque (T_S) leads to the loss of steady-state instability. Negative or zero damping torque lead to dynamic instability. Operating conditions, line impedance, voltage regulator gain and time constant etc. affect both T_D and T_S. Fig 1 computed by SenGupta et.al. indicates how AVR gain improves steady state stability and then leads to dynamic instability as T_S acquires positive value and T_D becomes negative. Computation of eigen values which is the most common practice for ascertaining stability may help to determine the sign and the magnitude of the total T_D and T_S. What this computation does not provide is a physical explanation of why and how these torque coefficients change with operating and system conditions.

In order to obtain the physical explanation, it was considered necessary to trace these torque coefficients to their sources of origin.

Damping is a dissipative process and it has been correctly guessed even more than fifty years ago to be caused by the incremental copper loss in various windings when currents in these windings oscillate due to rotor oscillations.

Synchronizing torque on the other hand is a spring action which is present during steady state operation and is supplemented by changes in reactive power caused by oscillating currents. Attempts to relate the copper loss in windings ($\Sigma\Delta i^2R$) and T_D computed from eigen values or by other means even in the absence of an AVR were unsuccessful. So were attempts to compute T_S from reactive power components until Gabriel Kron explained the causes of these discrepancies [1]. The role of the AVR was not understood very clearly either. DeMello and Concordia [2] were able to quantify the damping and synchronizing contributions of an AVR with the help of the simple Heffron-Phillips model [3] of a generator.

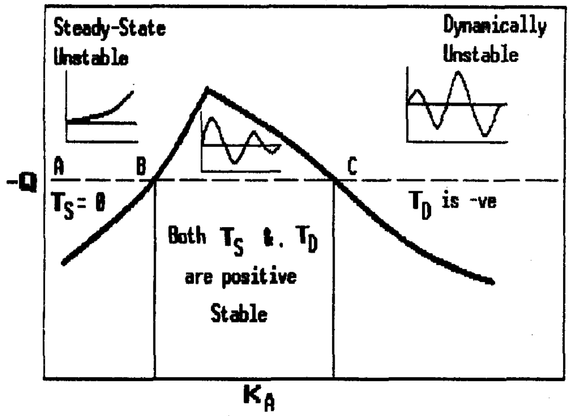

Fig.1 Dynamic and Steadystate Stability Limits

The discrepancy Kron pointed out has been presented by a simple diagram in reference 4, and is restated in Fig. 2. It is apparent from this figure that the observer on Park's axes which are rigidly connected to the rotor oscillates with the rotor. If the armature current i changes from P to P', the changes in the d-q axes component observed are B_1B_2 and A_1A_2 respectively. But $\overline{B_1B_2}+\overline{A_1A_2} \neq \overline{PP'}$. From Kron's axes which rotates synchronously the changes observed are BB' and AA' and $\overline{BB'}+\overline{AA'}=\overline{PP'}$. Covariant differentials transform as tensors provide absolute changes instead of apparent changes. Covariant differentials expressed as, $\delta i^{\pi} = \Delta i^{\pi} + \Gamma^{\alpha}_{\alpha b} i^{\alpha} \Delta x^{\beta}$ (2)

assume a simple form, $\delta i = \Delta i + \varrho i \Delta\Theta$ (3)
where the rotation tensor ϱ takes care of the observer's rotation and is given by

$$\varrho = \begin{bmatrix} & -1 \\ +1 & \end{bmatrix} \quad (4)$$

It is needless to add, that since the field and amortisseur windings rotate with the observer, apparent and absolute changes in these windings are the same.

In the case of the changes in currents and voltages in the armature windings, the absolute changes and apparent changes are not the same. That is why $T_D \neq \Sigma(\Delta i)^2 R/h^2$. Where Δi represents the observed changes in currents in different windings in the absence of AVR and h is the per-unit frequency of oscillation. As stated earlier in tensor analysis, the relative movement between the observer and the observed is taken into account by defining covariant differentials (δi). In this system $\delta i = \Delta i + \varrho i \Delta\Theta$
or

$$\begin{bmatrix} \delta i_d \\ \delta i_q \end{bmatrix} = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} + \begin{bmatrix} & -1 \\ +1 & \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \Delta\Theta \quad (5)$$

As soon as, the correct values of the incremental currents were computed using δi and not Δi, damping torque T_D, was found to be given by $\Sigma(\delta i)^2 R/h^2$ with an accuracy better than eight places of decimals.

Extensive computation was carried out at various operating conditions and $T_D - \Sigma(\delta i)^2 R/h^2$ was found to be $<10^{-8}$.

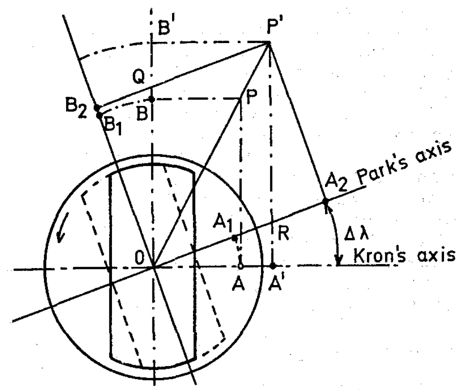

Fig. 2 Changes in Armature Current as seen from Park's and Kron's axes

COPPER LOSS	PARK'S AXIS	KRON'S AXIS
Field	0.04866611	0.04866611
Armature	-0.02422853	-0.04027642
Total	0.02443758	0.00838969
Total/h	1.13136908	0.38841160
(h=0.0216)	Total/h $\neq T_D$	Total/h = T_D
T_D (Computed)	0.38841160	0.38841160

TABLE 1 Computations of Damping Contributions

Table 1 represents a typical computation. It may be noted that the armature and line resistance contribute a small amount of negative damping. In early days in the absence of amortisseur windings positive damping was produced only in the field and could be cancelled by the negative damping of long lines. This would lead to hunting.

THE ROLE OF THE VOLTAGE REGULATOR

When a voltage regulator is included it is found that

$$T_D \neq \Sigma(\delta i)^2 R/h^2 \quad (6)$$

$$T_S \neq \Sigma(\delta i)^2 X/h + T_{SS} \quad (7)$$

T_{SS} being the steady-state component. This difference $T_D - \Sigma(\delta i)^2 R/h^2$, in the presence of an AVR and without governor control could only be attributed to the damping produced by the AVR. It has been shown by rigorous analysis [5] that

$$T_D - \Sigma(\delta i)^2 R/h^2 = -\mathrm{Real}[\Delta V_R \Delta i^*_f]/h^2 = T_{Dr} \quad (8)$$

and $T_S - (\Sigma(\delta i)^2 X/h + T_{SS}) = -\mathrm{Im}[\Delta V_R \Delta i^*_f]/h = T_{Sr}$ (9)

This relationship may be explained by a heuristic argument based on the properties of tensor invariance, which states that if A+B=C and if A and B are tensors, then C must also be a tensor, since a tensor if non-zero in one frame of reference has to be non-zero in all frames of reference.

The total damping torque coefficient T_D is independent of the observer same and can be represented by a tensor. At the same time $\delta i^2 R$ calculated in each winding provides exact "copper loss" in that winding and is independent of the observer's reference. It therefore follows that, $T_{Dr} = (T_D - \Sigma(\delta i)^2 R/h^2)$ is also a tensor to which a physical meaning may be ascribed and would be related to the power input into the field through the feedback loop of the AVR. For a propotional voltage regulator $\Delta V_R = -(K_A/(1+sT_A))\Delta V_t$, where ΔV_t is the change in terminal voltage. Neglecting T_A for the time being

$$T_{Dr} = \text{Real}[K_A \Delta V_t \Delta i^*_f]/h^2 = K_A \Delta V_t \Delta i_f \cos\psi/h^2 \quad (10)$$

$$T_{Sr} = \text{Im}[K_A \Delta V_t \Delta i^*_f]/h = K_A \Delta V_t \Delta i_f \sin\psi/h \quad (11)$$

Where ψ is the phase angle between ΔV_t and Δi_f. The damping and synchronizing torque contributions from the AVR is related to the active and the reactive power input from the generator terminal through the feedback loop.

The angle ψ is absolutely critical, since it determines whether the AVR should provide positive or negative damping torque. Since the field and amortisseur dampings are always positive and the armature contribution to damping is very small, dynamic stability is determined almost entirely by the AVR's contribution, which is negative when $\psi > 90°$ and positive when $\psi < 90°$.

Large gain (K_A) propotionately increases the AVR's contribution. When T_{Dr} tends to be negative ($\psi > 90°$) large gain amplifies the negative damping.

Fig. 3 shows how the angle ψ varies with reactive power for the line length represented by X_e, the line reactance. The model in this case is the one used by Heffron and Phillips in which the amortisseur windings are not included, although extensive computations have been carried out on rigorous models.

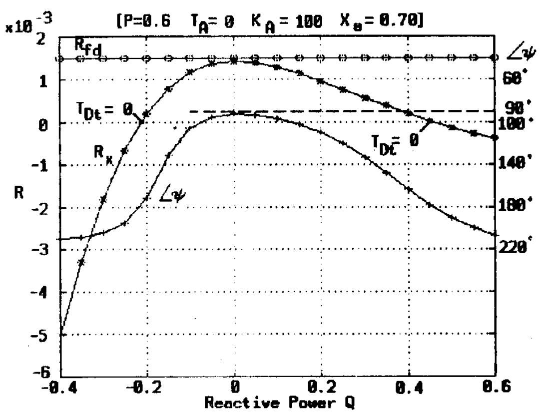

Fig. 3 Variations in R (field) and ψ with Q_t

It may be seen that $\psi > 90°$ for most operating conditions, particularly when X_e, the line reactance is large. This implies that the effective AVR/field resistance when viewed from the generator terminal is usually negative over a fairly wide operating range.

At sustained oscillations, $\Delta V_t \Delta i_f \cos\psi/h^2$ cancels $\Delta i^2_f R_f$ in the field winding and $T_D=0$, the armature/line contribution having been neglected. This is apparent from Fig. 3.

In the presence of amortisseur winding $\psi \neq 180°$ during sustained oscillations, but as long as $\psi > 90°$, ΔV_t, effectively "sees" negative resistance in the dynamically coupled electromagnetic closed loop that the AVR/field circuit represents.

If $\Sigma(\delta i)^2 R/h^2$ represents power dissipation and positive damping, $\Delta V_t \Delta i_f \cos\psi/h^2$ represents power generation or negative damping when $\psi > 90°$ or the "resistance" is negative.

MATHEMATICAL MODEL

Although all the studies were carried out using covariant differentials (δi) and Kron's hybrid reference frame [6], very simple relations can be derived based on PARK'S reference axes using apparent changes (Δi) alone if armature and line contributions are neglected.

Let $[\Delta V] = [Z].[\Delta i]$ represent the usual incremental equation of a synchronous generator in Park's axes. Substitute the p operator in the [Z] matrix by $j\omega h$ representing sustained oscillation,

$$[\Delta i] = [Z]^{-1}.[\Delta V]$$

where $[\Delta i] = [\Delta i_f, \Delta i_d, \Delta i_q, \Delta i_{kd}, \Delta i_{kq}]^T$

The following relations provide various damping torque contributions.

$T_{df} = \Delta i^2{}_f R_f/h^2$: from field

$T_{dkd} = \Delta i^2{}_{kd} R_{kd}/h^2$: from d-axis amortisseur

$T_{dkq} = \Delta i^2{}_{kq} R_{kq}/h^2$: from q-axis amortisseur

$T_{dr} = K_A \Delta V_t \Delta i_f \cos\psi/h^2$: from the AVR

Calculation of synchronizing torque components is slightly more complex [5] but is not discussed here since the objective is to determine and control dynamic instability.

A PSS BASED ON CANCELLATION OF NEGATIVE DAMPING CONTRIBUTION OF THE AVR

Since the AVR is the main source of negative damping, if a PSS is designed so as to exactly cancel the negative damping contribution of the AVR or in other words change the "effective resistance" from negative to positive, T_{Dr} becomes positive ensuring total positive damping [7].

This was done as follows. ΔV_t may be taken as caused by Δi_f, the change in the field current, $\Delta\omega$, the change in speed and $\Delta\dot{\omega}$, the change in acceleration. In other words

$$\Delta V_t = K_f \Delta i_f + K_\omega \Delta\omega + K_a \Delta\dot{\omega} \quad (12)$$

This relation is rigorously correct even in the presence of amortisseur windings where

$$K_f = [X_e X_{ad}(1 - X_{ad}/X_{kd})\cos\delta_t]/(X^*_d + X_e) \quad (13a)$$

$$K_a = \frac{V_b}{(\omega^2_o h^2 V_t)}\left[\frac{V_{qt} X^*_d \sin\delta_t}{(X^*_q + X_e)} - \frac{V_{dt} X^*_d \cos\delta_t}{(X^*_d + X_e)}\right] \quad (13b)$$

$$K_\omega = \frac{X_e}{(\omega_o V_t)}\left[\frac{V_{qt}}{(X^*_d+X_e)} \times \{E_{fd}-(X_d-X^*_d)i_d\} + \frac{V_{dt}(X_q-X^*_q)i_q}{(X^*_d+X_q)}\right] \quad (13c)$$

The coefficient K_f is positive for almost all normal operating conditions, since δ_t is seldom greater than 90°.

$$h^2T_{Dr} = \text{Real}[K_A \Delta V_t \Delta i^*_f] = \text{Real}[K_A(K_f\Delta i_f + K_\omega \Delta\omega + K_a \Delta\dot{\omega})\Delta i^*_f] \quad (14)$$

In the above expression, the term $K_A K_f \Delta i_f \Delta i^*_f$ is almost always positive and $\Delta i_f \Delta i^*_f$ is positive and K_f is positive for most operating conditions. If $K_\omega \Delta\omega$ and $K_a \Delta\dot{\omega}$, are cancelled by appropriate feedback, T_{Dr} is given by $\text{Real}[K_A K_f \Delta i_f \Delta i^*_f]$, which is positive. For cylindrical rotor turbo-alternators

$$K_a = CP_t/(V_t h^2 \omega^2_o) \quad (15)$$

$$K_\omega = (C/\omega_o)[V_t/X^*_d + Q_t/V_t] \quad (16)$$

where, $C = X^*_d Xe/(X^*_d+X_e)$

Extensive computation has shown the effectiveness of this PSS in damping out oscillations. As P_t, Q_t, V_t and h change with operating conditions, they may be easily measured at the terminal and equations 15-16 could be used to compute the gain K_ω and K_a with the help of suitable analog or digital circuits.

EXPERIMENTAL RESULTS

Extensive experiments were carried out on a micro-alternator to evaluate the performance of the proposed PSS [8]. The rotor oscillations were recorded at varied operating conditions, with and without the PSS. From these oscillograms the rotor mode eigen values ($\sigma+j\omega$) were computed and plotted as shown in Fig.4.

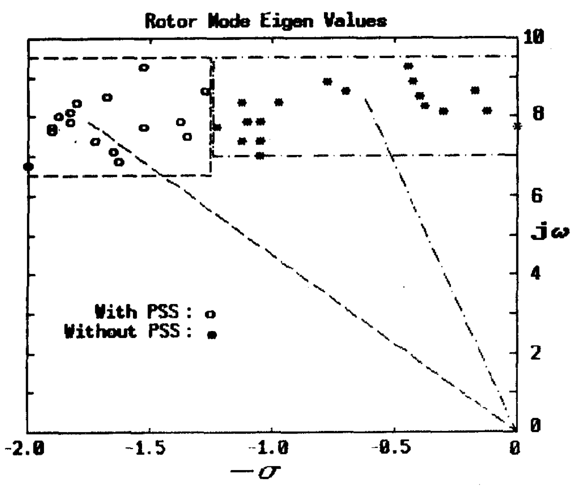

Fig.4 Plot of Eigen values with & without PSS

The adaptive PSS shifts all the poles deep into the negative plane indicating excellent damping with oscillations dying out within 2-3 cycles. What is important to note is that the frequency of oscillation is not reduced. (reduction of oscillating frequency indicates a reduction in synchronizing torque since $\omega h = \sqrt{(T_S/M)}$).

SUMMARY

a. Rigorous tensor analysis helped to establish the identity between damping and "copper loss" in various windings and synchronizing torque and reactive power.
b. The analysis led to the derivation of an expression for computing the AVR contribution to damping and synchronizing torques.
c. The AVR may inject into or absorb power from the system during small oscillations depending whether ΔV_t "sees" negative or positive resistance in the closed loop electrodynamic circuit.
d. The power expression for the AVR contribution led to the development of a power system stabilizer which cancells the negative contribution from the AVR. Indeed it is made to contribute positive damping. Since the updating of PSS gains is carried out with simple relations, the self-tuning PSS is simple and robust.

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REFERENCES

1. G. Kron, "A physical interpretation of the Riemann-Christoffel curvature tensor", The Tensor(New Series) 1955. vol.4
2. F.P. Demello and C. Concordia, "Concepts of synchronous machine stability as affected by excitation control", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-88, pp 189-202, 1969.
3. W.G. Heffron and R.A. Phillips, "Effect of modern amplidyne voltage regulators on underexcited operation of large turbine generators", AIEE Trans.(PAS), Vol.71,pp. 692-697, 1952.
4. D.P. Sen Gupta, N.V. Balasubramanium and J.W. Lynn, "Synchronizing and damping torques in synchronous machines", Proc. IEE, Vol. 114, pp 1451-1457, 1967.
5. D.P. Sen Gupta, N.G. Narahari and J.W. Lynn, "Damping and synchronizing torques contributed by a voltage regulator to a synchronous generator, a qualitative assessment", ibid., Vol. 124, pp 702-708, 1977.
6. D.P. Sen Gupta and J.W. Lynn, "Electrical Machine Dynamics", The MacMillan Press Ltd., London, 1980.
7. D.P.SenGupta, N.G.Narahari, I.Boyd and B.W.Hogg, " An adaptive power system stabilizer which cancels the negative damping torque of a synchronous generator", Proc. IEE, Vol.132, pp. 109-117, 1985.
8. D.P.SenGupta, E.Swidenbank and S.K.Sinha, "An adaptive stabilizer for excitation control: An experimental study", Proc. of Platinum Jubilee Conference on Systems and Signal Processing, pp.170-174,1986.