SURFACE RAY CONTRIBUTION TO BISTATIC RADAR CROSS SECTION OF A GENERAL PARABOLOID OF REVOLUTION

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ABSTRACT

A comprehensive method has been proposed for the determination of the RCS of general paraboloid of revolution utilizing the Geodesic Constant Method (GCM) developed by the authors. The analytical expressions have been obtained for all the surface ray parameters required in the bistatic RCS computation in one-parameter form. The inclusion of the diffraction due to higher-order geodesics in the analysis increases the accuracy of the theoretical RCS predictions.

Introduction

The radar cross section (RCS) is an important parameter for consideration in the design of aerospace bodies. The monostatic and bistatic RCS essentially depend on the shapes of the aerospace vehicles which are often quite complex. However, these structures are essentially hybrids of simple canonical shapes such as the quadrics like the cone, cylinder, sphere and ellipsoid etc.

The theoretical study of the bistatic RCS in the high frequency domain has been an active area of research due to its potential applications. While the simpler quadrics like the right circular cylinder [1] and cone [2] have been extensively studied, no theoretical study has been carried out for the RCS of a general paraboloid of revolution (GPOR). A GPOR is an important surface from aerospace structures point of view and often constitutes the leading sections such as radomes in aircraft, nose cone in satellite launch vehicles and missiles.

In this paper, we present a method to treat the problem of RCS of a general paraboloid of revolution based on the Geodesic Constant Method (GCM) in conjunction with the UTD.

Bistatic Radar Cross Section of GPOR

A general paraboloid of revolution (GPOR) is described (Fig. 1) by a set of parametric equation [3]

$$x = au \cos \phi \quad y = au \sin \phi \quad z = -u^2 \quad (1)$$

in which "a" is a shaping parameter which controls the sharpness of the GPOR.

In contrast to the cone, a GPOR does not exhibit the tip singularity, so that a tip-diffraction treatment is not required for the RCS solution. However, just like a finite cone, a GPOR frustum also has a circular rim from which edge-

diffraction would take place. Marhefka et al [2] have developed a method for predicting the edge-diffracted field components which may be readily applied to this case as well. Consequently, from the high-frequency scattering point of view, in this paper we concentrate on the surface-diffracted component of the EM field.

The UTD developed by Pathak et al [4] predicts the surface diffracted field on an arbitrary convex surface, in terms of its surface ray geometric parameters. Since, a GPOR frustum retains its overall convexity, the UTD can be readily applied to calculate the surface diffraction from a GPOR, once all the surface ray geometric parameters are known. The ray geometric parameters for a GPOR are presented here in an explicit form.

In the high frequency limit, the bistatic RCS scenario can be described (Fig. 1) as the propagation of the EM field originating from the source, launching on the surface of the GPOR, and finally arriving at the observation point after being shed in space. This arrival could either be by the phenomenon of reflection or by the generation of surface waves which creep along the surface of the GPOR which shed in free space. The extended Fermat's principle requires that these paths be extremal. Hence, while the ray propagation in free space is along straight lines, the surface propagation on the GPOR is along the geodesics.

It may be observed that the feasible surface ray paths could be both in anti-clockwise (right geodesics) and clockwise (left geodesics) directions. It has been shown by the authors that in certain situations the contribution of the higher-order geodesics (i.e., geodesics with multiple encirclements round the GPOR) could be comparable to those by the primary geodesics [5]. Hence a complete analysis must incorporate the higher-order geodesics as well which could be deleted only if their contribution to the total surface ray field has been found to be negligible.

The analysis presented here is for the m -th order right geodesic denoted by the subscript "rm". Using the geodesic coordinate system, the equation of the m -th order right geodesic is given as

$$\frac{d\phi}{du} = \frac{\pm h_{rm}(4u^2+a^2)^{1/2}}{au(a^2u^2-h_{rm}^2)^{1/2}} \quad (2)$$

which has been integrated as [3]

$$\phi = \frac{\pm h_{rm}}{a^2} \ln \frac{a[4u^2+a^2]^{1/2} + 2[a^2u^2-h_{rm}^2]^{1/2}}{a[4u^2+a^2]^{1/2} - 2[a^2u^2-h_{rm}^2]^{1/2}} + \sin^{-1} \left| \frac{a[a^2u^2-h_{rm}^2]^{1/2}}{u[a^4+4h_{rm}^2]^{1/2}} \right| + h'_{rm} \quad (3)$$

The First Geodesic Constant h_{rm} is obtained in terms of the launching and shedding points (e.g. S_1 and P_1) by a simple univariate search. It is possible to express all the surface ray geometric parameters required for the EM field computation by the UTD analysis, in terms of the First Geodesic Constant h_{rm} alone, leading to the name of this method as the Geodesic Constant Method (GCM). We present some of the key parameters without derivation. The surface normal vector $\mathbf{N}$ is expressed as

$$\hat{\mathbf{N}} = \frac{2u \cos \phi}{(a^2 + 4u^2)^{1/2}} \hat{\mathbf{i}} + \frac{2u \sin \phi}{(a^2 + 4u^2)^{1/2}} \hat{\mathbf{j}} + \frac{a}{(a^2 + 4u^2)^{1/2}} \hat{\mathbf{k}} \quad (4)$$

Similarly, the unit tangent vector along the right m-th order geodesic is given as

$$\hat{\mathbf{t}} = t_x \hat{\mathbf{i}} + t_y \hat{\mathbf{j}} + t_z \hat{\mathbf{k}} \quad (5)$$

where

$$t_x = \frac{a(a^2 u^2 - h_{mm}^2)^{1/2} \cos \phi - h_{mm}(a^2 + 4u^2)^{1/2} \sin \phi}{au(a^2 + 4u^2)^{1/2}}$$

$$t_y = \frac{a(a^2 u^2 - h_{mm}^2)^{1/2} \sin \phi + h_{mm}(a^2 + 4u^2)^{1/2} \cos \phi}{au(a^2 + 4u^2)^{1/2}}$$

$$t_z = \frac{-2u(a^2 u^2 - h_{mm}^2)^{1/2}}{au(a^2 + 4u^2)^{1/2}}$$

The radius of curvature ρ_g along the geodesic on the GPOR is derived as

$$\rho_g = \frac{-a^3(a^2 + 4u^2)^{3/2}}{2(a^4 + 4h_{mm}^2)} \quad (6)$$

On the other hand, the torsion of the m-th order geodesic τ_{rm} is found to be

$$\tau_{rm} = \frac{-8h_{mm}(a^2 u^2 - h_{mm}^2)^{1/2}}{a^3(a^2 + 4u^2)^{3/2}} \quad (7)$$

Equations (6) and (7) are required in obtaining several interaction-dependent ray geometric parameters of which the blendings functions are examples. The arc length and the generalized Fock parameter for m-th order right geodesic have already been derived in Ref. [3]. Finally, the specular point for the reflected ray and the individual launching and shedding points for the surface rays are determined by the algorithm outlined by Bach in Ref. [6]. Once these ray geometric parameters are available, the RCS of the general paraboloid of revolution can be readily determined.

In conclusion, a theoretically comprehensive method for the bistatic radar cross section calculation of a general paraboloid of revolution (GPOR) frustum has been presented in this paper along with the closed-form solutions for the surface geometric parameters needed in these calculations.

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Fig. 1 Bistatic radar cross section (RCS) of a general paraboloid of revolution (GPOR) frustum in the high frequency domain. Only the right (anti-clockwise) 1st and second order geodesics have been shown. The complete solution includes the left (clockwise) geodesics of all feasible orders. The bistatic RCS of GPOR may also contain the specular reflection contributions.