# REDUCTION OF SEARCH ORDER IN SURFACE RAY ANALYSIS FOR A CLASS OF NONDEVELOPABLE SATELLITE LAUNCH VEHICLES

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## ABSTRACT

A Geodesic Constant Method (GCM) developed by the authors is applied to the high frequency EM problems for a nondevelopable satellite launch vehicle (SLV) modeled by a general paraboloid of revolution and a right circular cylinder. The junction transition point is obtained by the application of the Hertz's principle of Particle Dynamics for the first time in EM theory. The GCM is also applicable to cone-cylinder type of SLV where the junction transition point cannot be located analytically in spite of the developability of the individual surfaces. Actual mutual coupling results have been presented for a pair of arbitrarily placed slot antennas on an SLV.

## Introduction

In this paper, the generic class of hybrid quadric surfaces of revolution (h-QUASORs) has been treated for the high frequency EM analysis. It is possible to effectively model a very wide class of rotationally symmetric bodies with truncated finite sections of QUASORs. Satellite launch vehicles (SLVs) and subsonic missiles, for example, can often be accurately modeled by a combination of a right circular cylinder and a general paraboloid of revolution (GPOR). An efficient method based on a more general property of surface-diffracted ray paths, derived from Hertz's principle in Particle Dynamics, has been presented in this paper, to determine the ray geometric parameters for h-QUASORs in general for the high frequency EM applications [1,2]. The specific example chosen is that of an SLV modeled by a non-developable h-QUASOR structure.

## Ray Tracing on a Satellite Launch Vehicle

An SLV is an example of an h-QUASOR (Fig. 1)which may be described by the two sets of hybrid parametric equations [3]

$$x = au\cos\phi \qquad y = au\sin\phi \qquad z = -u^2 \quad \text{with } |u| \leq u_t \tag{1}$$

where a is the shaping parameter of the truncated GPOR and

$$x = \rho\cos\phi \qquad y = \rho\sin\phi \qquad z = z \quad \text{with } -z_c \leq z \leq -u_t^2 \tag{2}$$

where $\rho$ is the radius of the cylinder.

The main difficulty in the application of the GTD to such problems arises from the fact that the h-QUASORs in general are not developable. Hence it is not possible to make use of the developability property along with the extended

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Fermat's principle to determine the transition point. Even in those cases where the individual surfaces of an h-QUASOR are developable, such as a cone-cylinder model of an SLV, the equation for the transition point is nonlinear whose solution cannot be obtained in the closed form.

For non-developable h-QUASORs, we invoke a more general property that when an extremal path crosses the junction between two adjacent surfaces, the angle with respect to a smooth junction line is preserved. This property of the geodesic follows from the Hertz's principle in Particle Dynamics [4]. Hence in Fig. 1, the angles and that the geodesic makes with the junction line of the two surfaces, are equal. *Since the angle invariance property is a local one, it does* not impose global requirements such as the developability of the surface. A satisfactory starting point for $\phi_t$ is provided here by applying the arc $PP_1$ onto the tangent plane of the cylinder at $P_1$ [3]

$$\phi = \phi_s + \frac{(\phi_f - \phi_s)(z_t - z_s)}{PP_1 + z_t - z_s} \tag{3}$$

$$\text{where } PP_1 = \frac{[u_f(a^2+4u_f^2)^{1/2} - u_t(a^2+4u_t^2]^{1/2}}{2} + \frac{a^2}{4} \ln \frac{2u_f+(a^2+4u_f^2)^{1/2}}{2u_t+(a^2+4u_t^2)^{1/2}} \tag{4}$$

Equation (3) leads to a quick convergence of the solution. For those cases where S is away from the junction edge while P is close to it, the starting point (3), *for all practical purposes, may be used to replace* $\phi_t$, *thereby* obviating the need for numerical search for $\phi_t$.

## Numerical Results and Discussion

The height-to-radius ratio $H_t/R$ has been used to characterize SLVs in the following discussion. A realistic SLV diameter of 10.0 $\lambda$ is chosen, for which the ray-theoretic assumptions are valid. The representative $H_t/R$ ratio is kept as 5.0.

Since the SLV is described in two rotational (parabolic and circular-cylinder) coordinate systems there are three geodesic directions viz. u, (on GPOR) and z (for circular cylinder) along with the common geodesic parameter $\phi$. Mutual coupling results have been presented to show the variation along all these three principal geodesic directions of the h-QUASORs. These results represent the first ever attempt at ray tracing on a convex nondevelopable satellite launch vehicle, and demonstrate the power of the GCM as a general technique for surface ray tracing.

## REFERENCES

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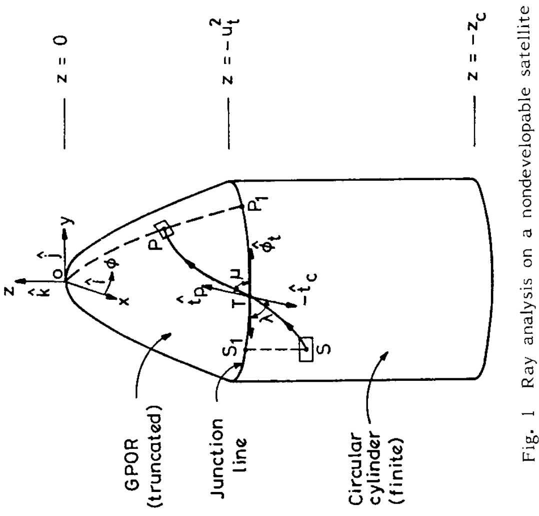

Fig. 1 Ray analysis on a nondevelopable satellite launch vehicle.

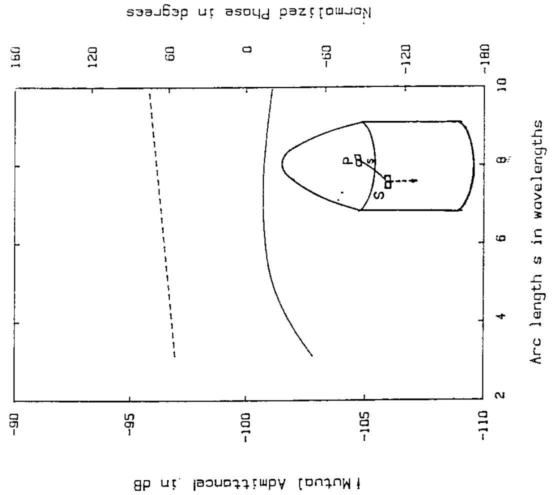

**Fig.2** Mutual Admittance vs. $z_s$ locations between two rect. slots (0.5λ x 0.2λ) on a SLV of junc. dia = 10λ and $H_t/R$ = 5.0. $\phi_s$ = 0.0, $u_f$ = 4.899, $\phi_f$ = 15° (const.). Magnitude (——), Phase (----).

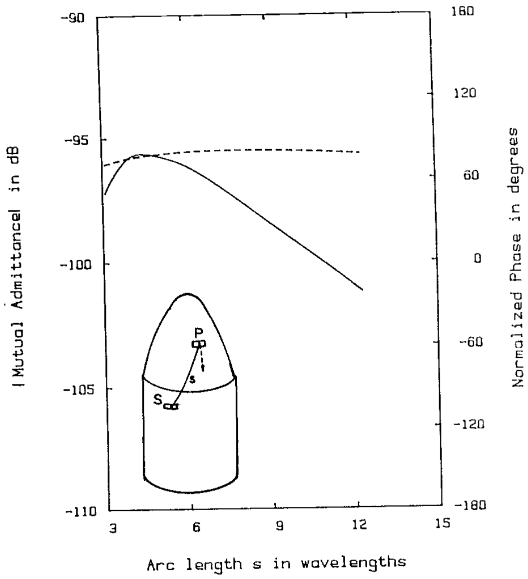

Fig. 3 Mutual Admittance vs. $u_f$ locations between two rect. slots $(0.5\lambda \times 0.2\lambda)$ on a SLV of junc. dia = $10\lambda$ and $H_t/R = 5.0$. $z_s = -26.0$, $\phi_s = 0.0$, $\phi_f = 15°$ (const.). Magnitude (——), Phase (-----).

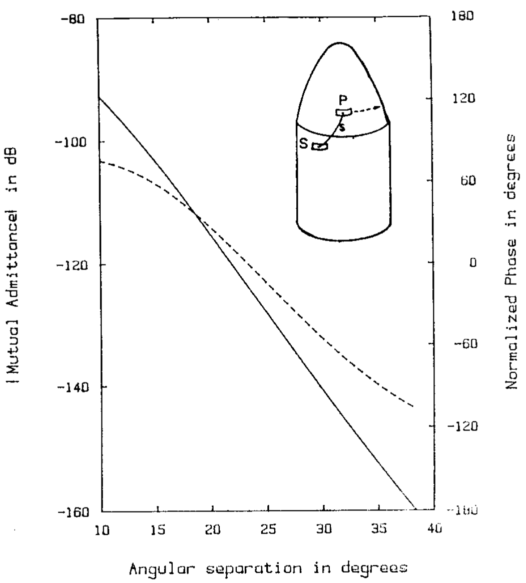

Fig. 4 Mutual Admittance vs. angular separation between two rect. slots $(0.5\lambda \times 0.2\lambda)$ on a SLV of junction dia $=10\lambda$ and $H_t/R = 5.0$. $z_s = -26.0$, $\phi_s = 0.0$ and $u_f = 4.899$. Magnitude (——), Phase (-----).