NEAR-OPTIMUM SYNCHRONISERS FOR DPSK MODULATION SCHEMES

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Abstract:
In this paper, simplified forms of Maximum - A Posteriori (MAP) synchronizers are developed for the DPSK scheme. It is shown these circuits are simple enough to be realized only with digital ICs.

1. Introduction:

Symbol synchronization is an important operation used at the receiving end of synchronous communication systems, for providing a reference symbol clock to the receiver. In some DPSK demodulator realizations, the symbol clock is essential for demodulation it self. But in most synchronous demodulators the symbol clock is required for cleaning up the demodulated data and to provide an output with uniform data widths. Figs. 1 and 2 show block diagram of the timing relationships of the synchronizer.

Let (ε + n. T) be the modulation instants, unknown to the receiver, where n is a running integer and T is the symbol period. Then, symbol synchronization is essentially an estimation problem, whereby ε has to be estimated based on the DPSK signal v(t) with a minimum estimation error.

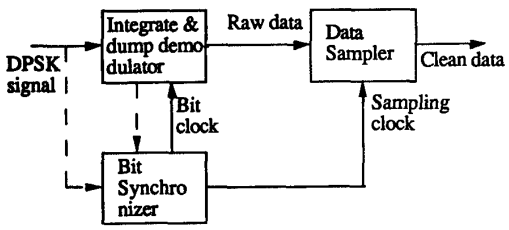

Fig. 1: DPSK demodulator and symbol synchronizer

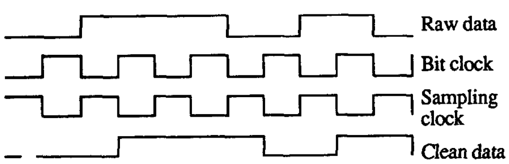

Fig. 2: Synchronizer waveforms

In Maximum-A Posteriori (MAP) estimation, ε is assumed to be a random variable taking a value in the range -T/2 to +T/2 and its probability density function (PDF) is assumed to be conditioned on v(t). Then, estimating ε involves choosing ε such that, this conditional PDF is maximized which in turn minimizes estimation error [1]. In Maximum Likelihood (ML) estimation, ε is assumed to be an unknown constant in the range -T/2 to +T/2 and ε is estimated. If ε is assumed to be uniformly distributed in the range -T/2 to +T/2, then MAP and ML estimation are identical [1]. In the following sections, MAP estimator topologies and its variants are used to develop basic synchronizers for the DPSK scheme, which are then used to derive simplified synchronizer circuits for the DPSK scheme.

2. The MAP estimator:

Fig. 3 shows the received DPSK signal v(t) over a duration of τ sec. (known as the estimation time), during which estimation of ε is carried out. In this paper, only 2-level modulation is considered for simplicity. The instant at which the estimation is started is the reference time 0 and the signal is observed from that instant onwards for τ seconds for estimating ε.

Signal v(t) to the synchronizer can be written as,

$$v(t) = s(t\,;\,\varepsilon) + z(t) \qquad (1)$$

where, $s(t\,;\,\varepsilon)$ is the DPSK signal component and z(t) is the channel noise. It is assumed to be a filtered white Gaussian noise with a zero mean and a variance of σ_z^2.

Let N+1 bits, numbered 0 to N be observed for estimating the symbol clock. Then the k th interval of the input signal is given by:

$$(k-1)\,T + \varepsilon < t \le k.T + \varepsilon \text{ and } 0 < t \le N.\,T, \text{ as } \tau = NT$$

For the k th interval, the DPSK signal can be written as,

$$s(t\,;\,k,\varepsilon) = g\{t - (k-1).T - \varepsilon\}.\,\mathrm{Cos}\,\{\omega_c\,(t-\varepsilon) + \phi_{nk}\} \qquad (2)$$

where, ϕ_{nk} is the differential phase modulation during the k th interval. $g\{t - (k-1).T - \varepsilon\}$ is the base-band pulse waveform defined in the interval $(k-1).T + \varepsilon < t < k.T + \varepsilon$: The base-band waveform has the desired wave shape in this interval and is zero elsewhere.

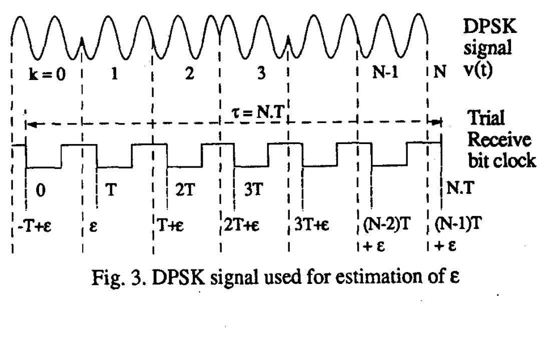

Fig. 3. DPSK signal used for estimation of ε

(2) can be further written as,

$s(t\,;k,\varepsilon) = g\{t-(k-1).T-\varepsilon\}.(\text{Cos}\{\omega_c(t-\varepsilon)\}.\text{Cos}\,\phi_{nk}$

$-\,\text{Sin}\{\omega_c(t-\varepsilon)\}.\text{Sin}\,\phi_{nk})$

For a 2-level DPSK signal ϕ_{nk} is either 0 or 180 deg. Hence,

$s(t\,;k,\varepsilon) = g\{t-(k-1).T-\varepsilon\}.(\text{Cos}\{\omega_c(t-\varepsilon)\}.\text{Cos}\,\phi_{nk})$

Let $a(k) = +1$ for $\phi_{nk} = 0$ & -1 for $\phi_{nk} = 180$ deg. and let the modified base-band pulse waveform be defined by:

$h(t-(k-1).T-\varepsilon) = g\{t-(k-1).T-\varepsilon\}.\text{Cos}\{\omega_c(t-\varepsilon)\}$

Then, $s(t,k,\varepsilon) = a(k).\,h(t-(k-1).T-\varepsilon)$ (3)

Using (3) in (1):

$$v(t) = \left\{\sum_{k=0}^{N} s(t\,;k,\varepsilon)\right\} + z(t) \qquad (4)$$

Let v(t) be expanded in terms of M arbitrary orthogonal functions $\psi_1(t), \psi_2(t), \ldots\ldots\ \psi_M(t)$.

$$\text{Then, } v(t) = \sum_{k=0}^{N}\sum_{i=1}^{M} v_{ik}.\,\psi_i(t) \qquad (5)$$

where, v_{ik} is given by: $v_{ik} = \int_{(k-1)T+\varepsilon}^{k.T+\varepsilon} v(t).\,\psi_i(t)\,dt$

v_{ik}, for i = 1 to M, are the coefficients of the series expansion of v(t), given by (5), during the k th interval.

Similarly, we define h_{ik} and z_{ik} for i = 1 to M as coefficients of the series expansion of base-band waveform and the channel noise during the k^{th} interval and are respectively given by:

$$h_{ik} = \int_{(k-1)T\varepsilon}^{k.T+\varepsilon} h(t-(k-1).T-\varepsilon).\,\psi_i(t)\,dt \qquad (6)$$

$$z_{ik} = \int_{(k-1)T+\varepsilon}^{k.T+\varepsilon} z(t).\,\psi_i(t)\,dt \qquad (7)$$

It may be noted that as $h(t-(k-1).T-\varepsilon)$ is a base-band waveform, h_{ik} is independent of k depends only on ε.

Using 4, 6 and 7 in 5, we get,

$$v(t) = \sum_{k=0}^{N}\{a(k).\sum_{i=1}^{M} h_{ik}.\,\psi_i(t)\} + \sum_{k=0}^{N} z_{ik}.\,\psi_i(t)) \qquad (8)$$

Let V_k, H_k, Z_k be row vectors of coefficients of their orthogonal coefficients during the k th interval. They can be represented as follows:

$V_k = [v_{1k}, v_{2k}, \ldots\ldots, v_{Mk}]$

$H_k = [h_{1k}, h_{2k}, \ldots\ldots, h_{Mk}]$

$Z_k = [z_{1k}, z_{2k}, \ldots\ldots, z_{Mk}]$

Then, using (5) and (8), these vectors are related by:

$$V_k = a(k).\,H_k + Z_k \qquad (9)$$

In (9) for a given ε, H_k is a vector with constant elements and Z_k is a vector whose elements are Gaussian distributed. Hence, V_k is a vector whose elements are Gaussian distributed with mean values given by the vector $\{a(k).\,H_k\}$ and having identical variance as those of the elements of Z_k.

Let A= {a(0), a(1), ..., a(N)] with a(i) = +/- 1 denote the (N+1) bits observed and let U = [$s(t;0,\varepsilon)$, $s(t;1,\varepsilon)$, ..., $s(t;N,\varepsilon)$] represent v(t) as a vector of (N+1) signal waveform segments during τ for k = 0, 1, ... N. Let $p(\varepsilon / U)$ be the conditional PDF of ε, conditioned on the input waveform U. The MAP estimator has to maximize this $p(\varepsilon / U)$ to obtain ε', the estimated value of ε. This maximization is carried out as follows: Using Bayes' rule on conditional probabilities we get,

$p(\varepsilon / U) = p(U / \varepsilon).\,p(\varepsilon) / p(U)$.

As U is dependent on the bits a(i),

$$p(U/\varepsilon) = \int_{\text{over all }\alpha} p(U/\varepsilon, \alpha).\,p(\alpha)\,d\alpha$$

Here, $p(\alpha)$ is the joint PDF of A and α is one of $2^{(N+1)}$ possible values assumed by A. As the bits at successive intervals are assumed to be independent of each other, we get the following expression for their joint PDF:

$p(\alpha) = \prod_{k=0}^{N} p\{a(k)\}$, where, p{a(k) is the PDF of a(k)

Using this in the above expression for $p(\varepsilon / U)$, we get:

$p(\varepsilon / U) = p(\varepsilon) / p(U).\,I$

where, $I = \int_{\text{over all } \alpha} \prod_{k=0}^{N} p\{U/(\varepsilon,a(k))\}.p\{a(k)\}\, d\alpha$ (10)

Let us assume that the PDF p (U) is independent of ε and the PDF of ε is uniformly distributed from - 0.5T to 0.5T and 0 elsewhere. Hence, maximizing the integral I in (10) maximizes p (ε / U). For this we proceed as follows: Interchanging the order of integration and multiplication in (10), we get:

$$I = \prod_{k=0}^{N} \int_{\text{over all } \alpha} p\,(U/\{\varepsilon,a(k)\}).\ p\,\{a(k)\}.\ d\alpha$$

Assuming a(k)'s to be equiprobable, p {a(k)} = 0.5 for a(k) = +/- 1 for all k. Using this in the above expression and noting that the integration in this is nothing but averaging p(U/{ε a(k)}) over all values of p {a(k)}'s, we get the following result:

$$I = 0.5 \prod_{k=0}^{N} (p\,(U/\varepsilon,+1) + p(U/\varepsilon,-1)) \quad (11)$$

Consider now the conditional PDFs in (11). Here, components of U are vectors with M components as noted earlier. Consider any of these coefficients v_{ik}. v_{ik} is a Gaussian variable with a mean value of $a(k).h_{ik}$ and a variance of σ_z^2. But z_{ik} has a PDF given by:

$$p\,(z_{ik}) = \frac{1}{\sqrt{2\pi\sigma_z^2}} \exp\,(-\, z_{ik}^2 / 2\sigma_z^2)$$

Hence, $p\,(v_{ik}/\,\varepsilon,\, a(k)\,) = \frac{1}{\sqrt{2\pi\sigma_z^2}} \exp\,\{-\,(v_{ik} - a(k)\,h_{ik})^2 / 2\sigma_z^2\}$

Therefore, PDF of U is given by,

$$p(U/\,\varepsilon,a(k)) = \frac{1}{\sqrt{(2\pi\sigma_z^2)^M}} \exp\{-\,(U - a(k)\,H).\,(U - a(k)\,H)^T / 2\sigma_z^2\}$$

Substituting this into (11), and simplifying [1, 2], we get,

$$I = C. \prod_{k=0}^{N} \text{Cosh}\,\left(\frac{1}{\sigma_z^2} U\,H^T\right) \quad (12)$$

Here, C is a constant independent of ε. Hence, maximizing the conditional PDF means, choosing ε such that I in (12) is maximized. It can be shown that maximizing λ (U, ε) in the following expression is equivalent to maximizing I in (12) [2].

$$\lambda\,(U,\varepsilon) = \sum_{k=0}^{N} \ln \text{Cosh}\,\left(\frac{1}{\sigma_z^2} \int_{(k-1)T+\varepsilon}^{kT+\varepsilon} v(t).h(t-(k-1)T-\varepsilon).\,dt\right) \quad (13)$$

This leads to the following algorithm for estimating ε:

Algorithm: For different values of ε, compute the quantity given by (13). Select that ε, which gives the highest λ (U, ε).

3. A closed loop MAP synchronizer:

An "open loop" approach is followed in the above algorithm for the MAP synchronizer. In a closed loop MAP synchronizer, $\frac{\partial(\lambda(U,\varepsilon))}{\partial\varepsilon}$ is forced to zero and $\frac{\partial^2(\lambda(U,\varepsilon))}{\partial^2\varepsilon}$ is forced to be negative for maximizing λ(U, ε).

Let $X(\varepsilon) = \frac{1}{\sigma_z^2} \int_{(k-1)T+\varepsilon}^{kT+\varepsilon} v(t).h(t-(k-1)T-\varepsilon).\,dt)$,

then, $$\frac{\partial(\lambda(U,\varepsilon))}{\partial\varepsilon} = \{ \sum_{k=0}^{N} \tanh\,\{x(\varepsilon)\}\,\}.\frac{\partial(x(\varepsilon))}{\partial\varepsilon} = 0 \quad (14)$$

But, $$\frac{\partial(x(\varepsilon))}{\partial\varepsilon} = \frac{\partial}{\partial\varepsilon}\left(\frac{1}{\sigma_z^2} \int_{(k-1)T+\varepsilon}^{kT+\varepsilon} v(t).h(t-(k-1)T-\varepsilon).\,dt\right). \quad (15)$$

Letting $I1 = \frac{1}{\sigma_z^2} \int_{(k-1)T+\varepsilon}^{kT+\varepsilon} v(t).\frac{\partial h(t-(k-1)T-\varepsilon)}{\partial t}.\,dt.$, applying the well known Leibnitz theorem to (15), and noting that h(0) = h(T) = 0, (14) can be reduced to:

$$\sum_{k=0}^{N} \{I1\}.\{\tanh\left(\frac{1}{\sigma_z^2} \int_{(k-1)T+\varepsilon}^{kT+\varepsilon} v(t).h(t-(k-1)T-\varepsilon).dt\right)\} = 0 \quad (16)$$

A realization of (16) is shown in fig. 4. Here, the receiver symbol clock is derived from a voltage controlled oscillator (VCO) whose phase is varied by evaluating and using quantity represented by the left hand side of (16) as the feedback signal. The VCO adjusts ε progressively in such that $\frac{\partial(\lambda(U,\varepsilon))}{\partial\varepsilon}$ goes to zero. Also, it is important to ensure that $\frac{\partial^2(\lambda(U,\varepsilon))}{\partial^2\varepsilon}$ is negative (< 0) when the above condition is reached. This is achieved as follows: A correction of VCO in the proper direction is forced by making the sign of the feedback signal depend on the sign of $\frac{\partial(\lambda(U,\varepsilon))}{\partial\varepsilon}$.

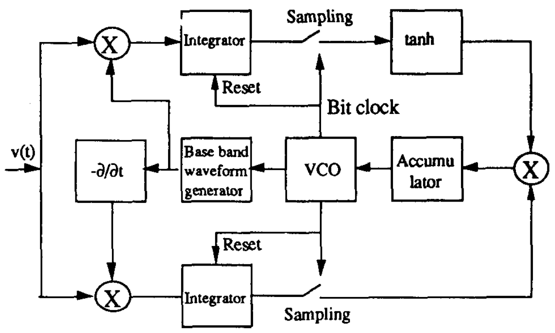

Fig. 4: A closed loop MAP synchronizer

4.: A Simplified MAP synchronizer:

A simplified MAP synchronizer is shown in fig. 5, in which the following simplifications have been introduced:

a) Digitizing the input signal and considering only its polarity for further processing,

b) Replacing the base-band waveform generator by a polarity-coincidence DPSK demodulator, with a shift register delay line, EXOR gate and a "continuous integrator". A proof that simplification (a) and (b) do not unduly affect the synchronizer performance is given in [2].

c) Replacing the differentiator by a subtractor and

d) Replacing the tanh function by its sign .

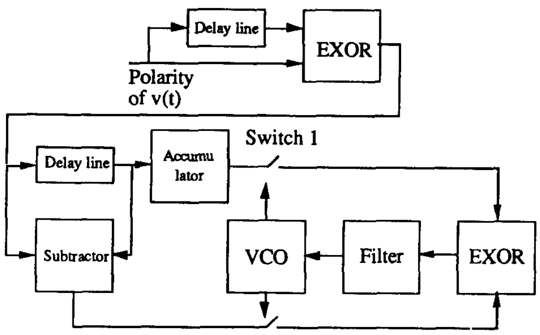

Fig. 5: Simplified MAP synchronizer for the DPSK scheme

5. The Data Transition Tracking Loop (DTTL):

Another form of MAP synchronizer is the DTTL described in [3]. In this scheme, two timing generators are used to integrate the input signal "along" and "across" the data transitions. The outputs of these integrators are then used to drive a VCO to achieve the required synchronization. In the original form the DTTL also requires a lot of hardware for its realization. Simplifications similar to the ones given above lead to reasonably simple all-digital synchronizer for the DPSK scheme. This is shown in fig. 6. Here output of the polarity coincidence DPSK demodulator is fed to two accumulators (integrators) which integrate "along" and "across" the data transitions. These integrators are reset at the appropriate instants by the timing generators. The sampled outputs of these integrators are multiplied (in an EXOR gate) after suitable processing shown in fig. 6. The multiplied output is filtered and used to drive the VCO, the output of which generates the required symbol clock and the timing waveforms to operate the sampling switches as well as the integrators.

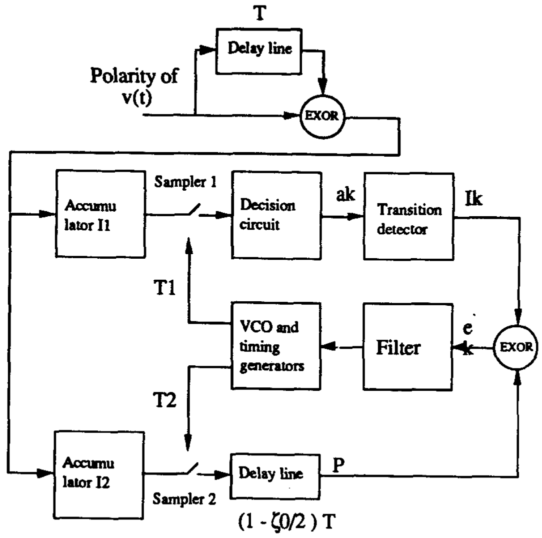

Fig. 6: Simplified DTTL synchronizer

6. Conclusions:

MAP estimator topologies may be used to derive synchronizers for the DPSK scheme. Several simplifications may then be used to realize easy to realize synchronizers.

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