A LOGICAL MODEL FOR DECISION-MAKING

V. SRIDHAR, M. NARASIMHA MURTY, G. KRISHNA

Department of Computer Science and Automation
Indian Institute of Science
Bangalore 560 012, INDIA

ABSTRACT

The general form of a decision making model raises hypotheses about the dynamics of stimulus, conception, and response and we might suppose that the decision process begins with the perception of some sort of stimulus. The amount of information available for decision making is often incomplete and hence any logic that attempts to model decision making must be nonmonotonic in nature. The existing logics can handle defeasible nonmonotonic inferences. We propose a modified First-Order Logic so that defeasible beliefs can also be handled. The modification is in the form of a set of proper axioms to handle belief revision, and a modified modus ponens to capture nonmonotonic reasoning. The proposed logic is extended to model decision making activity.

1. Introduction

Decision making is a root process. It is intertwined with all human activity. Every one makes all kinds of decisions throughout his life. What school to attend, what profession to choose, what job to take, whom to marry, how to plan for retirement are some of many crucial decisions which each person must make. The decision problem is simply the determination of how people must proceed in order to reach the best decisions. A rational approach to the decision problem must reflect the decision maker's objectives. The reason for making a decision is embodied in the desire of the decision maker to achieve some future state of affairs - his objective. He must choose one strategy in preference to all other alternatives and this choice can only be made rationally in terms of the objective. If there is only one course of action available to him, we do not usually speak of decision problem because the word decision implies choice.

We might suppose that the decision process begins with a perception of some sort of stimulus which suggests to the decision maker that a decision situation is at hand [9]. This initial perception is supplemented from the contents of the decision maker's memory to form an initial conceptualization of the situation. What one perceives is controlled by his past experience and thus by what he recalls. One of the things which characterizes the trained or experienced observer is the presence of conceptual structures which control or sensitize his perception. So the general form of a model is a familiar one in psychology, for it raises hypotheses about the dynamics of stimulus, conception, and response. The initial conceptualization may be characterized by confusion, doubt, and missing information.

Belief is our perception of the fact. Our belief is normally biased and may change with time. We use beliefs in reasoning as though they are facts. Default is an abstraction or generalization of beliefs. These generalizations help us in dealing with incomplete information [13]. In order to allow belief revision, we must be able to remember past perceptions [1]. Material remembered usually has to be set in relation with other material and in most complete cases must be dated, placed, and must be given some kind of personal mark. We represent the perceived information that takes into account the above requirements.

Decision making involves reasoning with vast amount of knowledge. In order to achieve parsimony in knowledge representation, normally inheritance hierarchies are employed. Even though simple inheritance representation can be dealt with easily, multiple inheritance provides tremendous representation flexibility.

In this paper, we propose a logical model for decision making activity. The rest of the paper is organized as follows. Section 2 analyses the decision making process. In section 3, we describe how knowledge used in decision making activity is represented. Sections 4 and 5 contain the proposed logic and some related results. Sections 6 and 7 contain the extension to the modified logic to model decision making activity and some related results.

2. The Analysis of Decision making

A specific decision depends on the analysis, intepretation, and evaluation of information available to the decision maker [7]. A decision that has been made may be countermanded (nonmonotonicity) or supplemented by subsequent decisions. The decision maker wants to achieve something, call it his goal, purpose, or objective. The decision maker will choose an action which he believes will help most to obtain his objectives. There are three main reasons for the notorious fact that we do not always

CH2809-2/89/0000-1142 $1.00 © 1989 IEEE

achieve objectives, despite our best efforts in that direction. The first is the frequent intransigence of society and nature. The second reason is the competition of rational opponents. The third reason is that the available information may not be sufficient to take the correct decision.

According to Herbert Simon [15], the decisions can be classified into programmed and nonprogrammed decisions. The distinction he proposed was based on the idea that great many business decisions are straightforward, repetitive, and routine; with the result the organization is able to develop a defined and formalized procedures for handling them. The nonprogrammed decisions are those which are novel, unstructured, and consequential. There is no cut-and-dried method of handling them. The simplification of decision problem is usually required to make it intellectually tractable. We may usefully raise some hypotheses about the ways in which this simplification takes place. For example,

1. Resort to rules of thumb (Defaults).
2. Appeal to System of Category. Often policy categories are used to place decisions in broad classes and these policies can change in order to make decisions more effective (Defeasible Premises).
3. Suppress Uncertainty.
   People play many roles (multiple contexts). Each role can be associated with its own objectives. Most people, for example, will establish some kind of objectives for themselves in the area of their professional activities. They will also have other objectives relating to their interpesonal relationships and so on. It appears that most people handle their decision problems in a particular field of activity ignoring other fields of activity.

We discuss briefly in the following the various components of the decision cycle [2]:

1. Setting objectives is the first stage in the decision cycle. These objectives not only provide an end towards which the decision maker wants to proceed but also act as choices between alternative courses of action. No single objective can cover all of the requirements of a decision maker. He is always dealing with a competing set of objectives. An issue is whether all of these possible objectives are in fact consistent. If, on the other hand, it is believed that there is a fundamental inconsistency between these multiple objectives, then it becomes necessary to think in terms of the trade offs between the objectives and this makes the decision making process much more complex. One possible response, however involves attempting to produce an agreed hierarchy of objectives with the most important objectives having to be achieved first.
2. The criteria, derived from the general objective or objectives must be unambiguous and must be stated in a specific form to allow for choices to be made between the alternative courses of action open to the decision maker.
3. The next stage of the decision cycle involves looking for information that will enable it to develop courses of action that will contribute towards achieving the objectives that have been set. The decision maker attempts to identify a limited number of courses of action.
4. Having identified the opportunities open to the decision maker, he is then faced with the task of evaluating their worth in terms of the extent to which each contributes to the achievement of the set objectives. The various alternatives can be distinguished on the basis of outcomes. The way in which the decision maker chooses has to be based on the utility of each of the outcomes. The notion of utility is straightforward and based on the premise that the decision maker will have a preference for certain outcomes. This implies that the decision maker can precisely identify the possible outcomes and their relative likelihoods. In reality, few decisions are taken under conditions of perfect information such as timetabling, machine scheduling where the decision problem is mechanical and in which the decision maker knows all courses of action and their outcomes. In practice, however, decisions are taken along a spectrum of conditions of information ranging from certainty to complete ignorance, where past experience, intuition offers a guide to specifying their alternatives and their outcomes. Quite obviously, if the decision maker should find himself in a position in which desirable consequences of a particular course of action far outweigh any undesirable consequences that are associated with its outcome, the decision choice is relatively straightforward.

For complex problems and particularly those of strategic nature different approaches involving a broader perspective need to be used if full account is to be taken of the social, political and philosophical issues that come into play in decisions involving substantial element of judgement. The behavioral factors influence the decision maker in the process of arriving at a decision, and can act as a severe constraint in the implementation of the decision. The behavior of an individual is governed by a variety of psychological factors which operate at both the conscious level and subconscious level. These can be grouped under: (i) his personality (ii) his perception (iii) his relative willingness to accept risk. An individual's perception influences the way in which he views a decision problem and its environment, and hence affects his approach to the decision. Perception is, therefore, first and foremost a selective process in which the ability of an individual to comprehend the enviornment and the stimuli arising from it is limited by his conscious and subconsious awareness and understanding of what is going on around him.

If we regard a decision maker's conceptualization of a decision problem as hypothesis, then his activities which are aimed at testing the validity of his hypothesis may well be regarded as experiments. The explicit use of Hypothetico-Deductive method permits one to explore the deductive consequences of a hypothesis, the relations among hypotheses and to work towards a unified theory in which every thing of interest could be deduced from a small number of basic laws.

From the above discussion it follows that any logical model for decision making must have, at least, the following characteristics.

1. It must be able to withdraw/revise the derived theorems [10]. This is required since decisions taken may have to be withdrawn or revised in the light of new information.
2. It must be able to withdraw/revise the premises. This is required since policies, guidelines etc., employed in decision making may change from time to time indicating the dynamic nature of the whole affair and some of the rules of thumb may continuously misfire hinting the decision maker to give a new look to them.
3. It must be able to model multiple contexts. This is required since people play multiple roles and each role can be associated with its own objectives and objectives of different roles may contradict with each other.
4. It must be able to deal with multiple, may be conflicting, objectives.
5. It must be able to deal with conflicting courses of action.

In short, we need a nonmonotonic logic that can withdraw premises. The various nonmonotonic logics proposed to-date [3,4,5,12] can withdraw derived theorems but are not capable of withdrawing premises.

3. Knowledge Representation

The knowledge employed in decision making is represented in the form of an inheritance network. Artificial Intelligence (AI) research has often emphasised the need for multiple inheritance where a more specific object may inherit information from several more general concepts [14,16]. A further requirement is that they should allow exceptions. In systems that permit multiple inheritances, an inheritance tree is replaced by an inheritance directed acyclic graph. In order to provide a formalism for property inheritance with exceptions from multiple more general concepts, we propose an inheritance network as follows. Each node in the network is connected to its neighbors via the following implications:

ISA-Implication, $\supset$, allows the inheritance of properties from the nodes representing more general concepts (ISA properties).

Property-Implication, $\Rightarrow$, allows the inheritance of natural properties of the node.

Default-Implication, $>$, allows the inheritance of the default properties of the node.

The semantics of these operators is exactly identical to the semantics of the usual logical implication. Multiple operators are necessary to overcome some of the problems associated with the multiple inheritance. The property inheritance requires that some properties of some of the ancestors must be preferred over the others. In other words, default properties must be inherited only after inheriting ISA and natural properties.

Any object or individual can be assumed to have a unique name and a set of attributes or properties. The individuals are uniquely identified by the most specific category of the individual perceived by the pereceiver. In other words, the most recent perception about an individual forms the most specific belief. The properties of an individual cannot stand independently. Their existence is always in relation to the category of the individual. Thus we can assume that properties are perceived as follows:

Category(Individual) $\supset$ Property(individual)

4. Modified First-Order Logic

We start with the First-Order Theory (FOT) [6]. The problem in employing FOT for reasoning is that it is monotonic in behavior. In order to achieve complete nonmonotonic behavior, we introduce proper axioms that allow us to treat premises, which represent beliefs, nonmonotonically, and modify the monotonic modus ponens into nonmonotonic modus ponens. We borrow from FOT everything except the inference rule modus ponens. To this, we add a set of proper axioms and a nonmonotonic modus ponens inference rule. a keeps track of belief revisions, b suggests that we do "forget" certain things and recast them again, and g indicates the context. The proposed proper axioms are essentially second order as they contain predicates that describe the properties of properties of individuals. These are divided into three levels. The Level-0 proper axioms deal with input beliefs and take care of change in beliefs. These also handle contexts.The Level-1 proper axiom allows reasoning with the most recent beliefs. The Level-2 deals with the descriptions of individuals.

Level-0 proper axiom (La-0):

$$(\forall a,b,g,T,T1,T2,T3)\ (B(P,a,b,g,T) \wedge -L'(B(P,a+1,b,g,T1)) \wedge -L'(B(P,1,b+1,g,T2)) \wedge -L'(B(P,0,b,g,T3)) \wedge odd(a) \supset (C_g \supset True(P,T)))$$

$$(\forall a,b,g,T,T1,T2,T3)\ (B(P,a,b,g,T) \wedge -L'(B(P,a+1,b,g,T1)) \wedge -L'(B(P,1,b+1,g,T2)) \wedge -L'(B(P,0,b,g,T3)) \wedge even(a) \supset (C_g \supset True(-P,T)))$$

$$(\forall g,d)\ (B(C,g,d) \wedge (\forall t)\ (-L'(B(C,t,d+1))) \supset C_g)$$

where $a,b,g,d \in w$ and w is $\{0,1,2,\ldots\}$

$t \in N$, a finite subset of w.

Level-1 proper axiom (La-1):

$$(\forall P,T,x)\ (True(P(x),T) \wedge (\forall Q)\ (-F(True(Q(x),T))) \supset P(x))$$

Level-2 proper axiom (La-2):

$$(\forall P,Q,T)\ (True(P \supset Q,T) \wedge L'(P) \wedge -L(Q)\ (\forall R,Tr)\ (True(P \supset R,Tr) \wedge Tr > T \wedge -L(-R) \supset L'(R)) \supset Q)$$

ISA Modus Ponens (ISA-MP): This replaces monotonic modus ponens of FOT.

From P, infer every Q such that $P \supset Q \wedge -L(-Q)$,
infer every $Q \in$ NMTC($\Rightarrow$,P),
infer every $Q \in$ NMTC($>$,P).

Notes:

(1) P is a First-Order predicate representing a belief or a default.

(2) B(P,a,b,g,T) is a second order sentence and for a given P and g, a is incremented from 0 onwards till some maximum is reached when b (indicating relearning) is incremented. B(P,a,b,0,T) indicates a context independent default or context independent belief. B(P,0,b,g,T) indicates our voluntary or involuntary

intension of withdrawing belief in P. We observe that this attempts to model "forgetfulness". T is the time stamp. Note that P in B(P,a,b,g,T) can be either P or $P \supset Q$.

(3) B(C,g,d) is a second order sentence. C_g indicates a particular context and d indicates the number of times we have switched from one context to another. C_0 , which is always true, takes care of context independent defaults and beliefs. At any point in time, only one C_g , $g > 0$, is true.

(4) True(P,T) is a second-order sentence.

(5) The context of belief assertions is explicitly input in the form B(C,g,d).

(6) Finiteness of t indicates that we allow only finite number of contexts.

(7) L is a modal (belief) operator [8] defined as:

$$L(P) \in TH \text{ if } P \in TH$$

and $-L(P) \in TH$ if $P \notin TH$ where TH is any theory.

(8) L' is L with TH being theory generated so far.

(9) F is defined as follows:

$F(True(P(x),T)) \in$ Theory if
$(\forall T0)\ (True(P(x),T0) \wedge T0 \leqslant T \wedge$
$-(\exists Q)\ (True(Q(x),T))$
else $-F(True(P(x),T)) \in$ Theory.

(10) NMTC($\supset$,P) is a nonmonotonic transitive closure with respect to the operator $\supset$ and is defined as follows:

$$NMTC(\supset,P) \equiv R^1_{\supset} \cup R^2_{\supset} \cup R^3_{\supset} \cup \ldots$$

where $R^1_{\supset} = \{Q \mid P \supset Q \wedge -L(-Q)\}$
and $R^i_{\supset} = \{Q \mid P \in R^{i-1}_{\supset} \wedge P \supset Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{\supset} \wedge -L(-Q)\}$

(11) NMTC($\Rightarrow$,P) is a nonmonotonic transitive closure with respect to the operator $\Rightarrow$ and is defined as follows:

$$NMTC(\Rightarrow,P) \equiv R^1_{\Rightarrow} \cup R^2_{\Rightarrow} \cup R^3_{\Rightarrow} \cup \ldots$$

where $R^1_{\Rightarrow} = \{Q \mid P \Rightarrow Q \wedge -L(-Q)\}$
and $R^i_{\Rightarrow} = \{Q \mid P \in R^{i-1}_{\Rightarrow} \wedge P \Rightarrow Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{\Rightarrow} \wedge -L(-Q)\}$

(12) NMTC(>,P) is a nonmonotonic transitive closure with respect
to the operator > and is defined as follows:

$$NMTC(>,P) \equiv R^1_{>} \cup R^2_{>} \cup R^3_{>} \cup \ldots$$

where $R^1_{>} = \{Q \mid P > Q \wedge -L(-Q) \wedge -IS\text{-}A(-Q) \wedge -KIND\text{-}OF(P,-Q)\}$
and $R^i_{>} = \{Q \mid P \in R^{i-1}_{>} \wedge P > Q \wedge Q \notin \bigcup_{j=1}^{i-1} R^j_{>} \wedge -IS\text{-}A(-Q) \wedge -KIND\text{-}OF(P,-Q)\}$

(13) IS-A(Q) is defined as follows:

For any $P \in TH$, if $Q \in NMTC(\supset,P) \vee Q \in NMTC(\Rightarrow,P)$ then $IS\text{-}A(Q) \in TH$
For all $P \in TH$, if $Q \notin NMTC(\supset,P) \wedge Q \notin NMTC(\Rightarrow,P)$ then $-IS\text{-}A(Q) \in TH$

(14) KIND-OF(P,Q) is defined as follows:

For any Q' such that $Q' \in INHERITORS(P) \wedge Q' \notin TH$, if $Q \in NMTC(>,Q')$ then $KIND\text{-}OF(P,Q) \in TH$.
For all Q' such that $Q' \in INHERITORS(P) \wedge Q' \notin TH$, if $Q \notin NMTC(>,Q')$ then $-KIND\text{-}OF(P,Q) \in TH$.

(15) INHERITORS(P) is defined to be a set of all Q such that Q is below P in the inheritance network.

5. Some Results

Theorem 1: The proposed logic has no redundant implicative operators.
Theorem 2: ISA-MP subsumes MP.
Theorem 3: ISA-MP permits Simple multiple inheritance.
Theorem 4: Hierarchy of operators ensures correct inheritance.
Theorem 5: ISA-MP permits multiple inheritance.
Theorem 6: The set of proper axioms does not introduce any inconsistency.
Theorem 7: The belief revision is automatic.
Theorem 8: The proposed logic reasons with the most specific information in the corresponding ISA hierarchy.
Proofs of the above theorems are not included due to lack of space.

6. A Model for Decision Making

The decision making activity begins with the perception of some sort of stimulus which suggests to the decision maker that a decision situation is at hand. This stimulus is transformed into concrete objectives. The mental states of the decision maker, which reflect the decision maker's personality, prefer some objectives over the rest. So, we can assume that the objectives of the decision maker are ordered indicating the preferences of the decision maker. The decision maker's knowledge, past experience, guidelines, policies, rules of thumb, his behavioral factors, his social, political, and philosophical outlook, his family relationships - suggest to him the courses of action available in order to achieve the set objectives. These multiple objectives require that decision maker must consider multiple contexts each with multiple courses of action. This suggests that in order to achieve an objective, the decision maker must do some actions in some contexts. In order to resolve conflicts, it can be assumed that we have a hierarchy of objectives (O1,...,Oi,...,On), each (say Oi) with an ordered list of contexts (Ci0,...,Cim). The courses of action ($Aij0,\ldots,Aijn_j$) under each context (Cj) is also assumed to be ordered. Then, the decision generated to meet the objective Oi is as follows:

$[Ai00 \oplus Ai01 \oplus \ldots \oplus Ai0n] \wedge$
$[Ai10 \oplus \ldots \oplus Ailn] \wedge$
$\vdots$
$[Aim0 \oplus \ldots \oplus Aimn]$ where $\oplus$ stands for the XOR operation.

The decision generated is such that these selected courses of action do not conflict not only with each other but also with the courses of actions generated for the objectives O0,O1,...,Oi-1. In the following we provide a set of proper axioms to model decision making process.

[P2-0] $(\forall X,O,P)$ (OBJECTIVES(X) $\wedge$ {O,P} $\in$ X
$\supset$ OBJECTIVE(O,P))

[P2-1] $(\forall O,A)$ $(\exists P,Q)$ (OBJECTIVE(O,P) $\wedge$
GENERATE-CONTEXT-NODE(O,A,Q)
$\supset$ UNORDERED-CONTEXT-NODE(O,A,Q)

[P2-2] $(\forall O,A)$ $(\exists B,U,Q)$
(UNORDERED-CONTEXT-NODE(O,A,Q) $\wedge$ UTILITY(A,U) $\wedge$
SORT(A,U,B) $\supset$ ORDERED-CONTEXT-NODE(O,B,Q)

[P2-3] $(\forall O,A)$ $(\exists B,P,Q)$ (
$((\forall A')$ $(\exists Q')$ (L(ORDERED-CONTEXT-NODE(O,A',Q'))))
$\wedge$ OBJECTIVE(O,P) $\wedge$
$((\forall P')$ $(\exists O')$ (P' < P $\wedge$ OBJECTIVE(O',P') $\supset$
$((\forall A')$ $(\exists Q',C')$ (ORDERED-CONTEXT-NODE(O',A',Q')
$\supset$ COURSE-OF-ACTION(O',Q',C'))))) $\wedge$
ORDERED-CONTEXT-NODE(O,A,Q) $\wedge$
$((\forall Q')$ $(\exists O',A',C')$ (Q' < Q $\wedge$
ORDERED-CONTEXT-NODE(O',A',Q')
$\supset$ COURSE-OF-ACTION(O',Q',C')) $\wedge$

SELECT(MERGE-COURSES-OF-ACTION(O,P), MERGE-CONTEXT-NODES(O,P),A,B) ⊃ COURSE-OF-ACTION(O,Q,B))

Notes:

1. OBJECTIVES(X) denotes a set of (Objective,Priority) as perceived by the decision maker.
2. OBJECTIVE(O,P) denotes an objective O with priority P. Priorities are ordered from 0 onwards, 0 indicating the top priority.
3. GENERATE-CONTEXT-NODE(O,A,Q) generates a context node for an objective O with A as a set of courses of action and with priority Q. Again priorities are oredered from 0 onwards, 0 indicating the top priority.
4. UTILITY(A,U) assigns a utility to each course action $\in$ A. U is a set $\{u \mid u \text{ is the UTILITY of a } A\}$.
5. MERGE-COURSES-OF-ACTION(O,P) is a function that returns the set of courses of actions for all objectives generated prior to the objective O and courses of action for the objective O generated so far.
6. MERGE-CONTEXT-NODES(O,P) is a function that returns the set of all considered context nodes of the objective O.
7. SELECT(M1,M2,A,B) selects the course of action for the next context node that is not present in M2. It can look ahead to make sure that selected course action is an optimal one and does not conflict with any course of action present in M1. If it cannot select such a course of action, then it instantiates B with Null.
8. COURSE-OF-ACTION(O,Q,B) indicates that one of the courses of action for the objective O with priority Q is B.

7. Some results

Theorem 9: The courses of action generated is in accordance with the hierarchy of objectives.
Proof: The proof is based on induction on the number of objectives. Let {O0,O1,...On} be the ordered set of objectives. When there is only one objective, trivially, courses of action generated by SELECT meet the requirement.
Let this be true when we have considered i objectives
O0,...,Oi-1.
It is required to show that it is true when the courses of action required to meet the objective Oi are generated. Consider the context tree indicating the hierarchy of objectives, context nodes, and courses of actions. The priorities of the objectives are properly handled by the third conjunct of (P2-3). The fifth conjunct of (P2-3) ensures that the priorities of the various contexts are properly handled. SELECT can choose an alternative at each context node only if it meets the requirement. If it cannot select any alternative at any context node then that objective cannot be met. Hence the result.
Theorem 10: The decisions taken do not have any conflicting courses of action.
Proof: One important requirement is that the chosen set of courses of action must be implementable. In other words, we require that generated courses of action must not conflict with each other. The SELECT employed in (P2-3) takes care of this requirement.
Theorem 11: The model can deal with conflicting objectives.
Proof: Normally, when multiple objectives are set, they may conflict with each other. One way to resolve the conflict is to have a hierarchy of objectives. Thus by preferring the most important objective, we can handle the conflicts and this is precisely what (P2-3) does (Theorem 9).

8. Conclusion

In this paper, we have proposed a logical model for decision making. Decision making involves reasoning with defeasible beliefs. Our approach to the solution is based on proposing modified First-Order Logic and reasoning with human-oriented beliefs and permitting property inheritance with exceptions from multiple more general concepts. Reasoning with multiple inheritance requires reasoning with the most specific information. This is made possible by permitting belief revision so that when more specific information is available, more general information can be withdrawn. The salient features of the proposed approach include:

1. The logical model attempts to model "Contextual Reasoning" and "forgetfulness".
2. It handles defeasible beliefs.
3. The nonmonotonic behavior is characterized by using a modified version of modus ponens as an inference rule.
4. The defaults can be used in the reasoning process without including any abnormal aspects [4].
5. The most specific information [11] in the ISA hierarchy is used for reasoning.
6. The model permits property inheritance with exceptions from multiple more general concepts.
7. The model can deal with multiple, may be conflicting, objectives.
8. The decisions taken do not have any conflicting courses of action.
9. The courses of action generated are in accordance with the hierarchy of objectives.

Acknowledgment

The authors would like to thank the Defence Research Development Organisation, India, for the project grant DRDO/CSA/GK/13 that supported the research reported in this paper.

References

1. F.C. Bartlett, Remembering, (Cambridge University Press, 1932).
2. C. Gilligan, B. Neale, and D. Murray, Business Decision Making (Philip Allan, 1983).
3. J. McCarthy, Circumscription - a form of nonmonotonic reasoning, Artificial Intelligence, 13(1980) 27-39.
4. J. McCarthy, Applications of circumscription to formalising commonsense knowledge, Artificial Intelligence, 28(1986) 89-116.
5. D. McDermott, and J. Doyle, Nonmonotonic Logic I, Artificial Intelligence, 13(1980) 41-72.
6. E. Mendelson, Introduction to Mathematical Logic (Van Nostrand Reinhold Company, 1964).

7. D.W. Miller, and M.K. Starr, The Structure of Human Decisions (Prentice-Hall, Inc., NJ, 1967).
8. R.C. Moore, Autoepistemic Logic. in: Smetts, Mamdani, Dubois, and Prade (Eds.), Non-Standard Logics for Automated Reasoning, (Academic Press, 1988).
9. W.T. Morris, Management Decisions - Art or Science? in: G.A. Yewdall, (Ed.), Management Decision-Making (David & Charles (Publishers) Ltd., 1971).
10. D. Nute, Defeasible Reasoning & Decision Support Systems, Decision Support Systems 4, No. 1(1988).
11. D.L. Poole, On the comparision of theories: Preferring the most specific explanation. in: Proceedings IJCAI-85, Los Angeles, CA(1985) 144-147.
12. R. Reiter, A logic for default reasoning, Artificial Intelligence, 13(1980) 81-132.
13. R. Reiter, Nonmonotonic Reasoning. in: Traub, Grosz, Lampson, and Nilsson (Eds.), Annual Review of Computer Science, (Annual Reviews Inc., California, 1987).
14. E. Sandewall, Nonmonotonic Inference Rules for Multiple Inheritance with Exceptions. in: Proceedings of the IEEE Vol 74, No. 10 (October 86).
15 H.A. Simon, Models of Man (John Wiley, 1975).
16. D.S. Touretzky, The Mathematics of Inheritance Systems (Morgan Kaufmann Publishers, Inc., California,1986).