GEODESIC SPLITTING ON GENERAL PARABOLOID OF REVOLUTION AND ITS IMPLICATIONS TO THE SURFACE RAY ANALYSIS

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ABSTRACT

The authors have observed an interesting phenomenon of geodesic splitting in the case of surface ray propagation over a general paraboloid of revolution. Since even the primary geodesics are split in both clockwise and anticlockwise directions, this leads to a doubling of the ray paths to be considered in the antenna characteristics computations. This work provides an insight into the ray-splitting phenomenon on simplest (i.e. of lowest order) possible convex surface.

Introduction

It is well known that between any two arbitrarily located points on a cone or a cylinder, there exist primary and higher-order (of multiple encirclements) geodesics in both anticlockwise (right) and clockwise (left) directions. However, the number of geodesics of a given order and sense does not exceed one. In contrast, a detailed study of the surface ray propagation on general paraboloids of revolution (GPORs) by the authors has shown that the geodesics of a given order and sense (e.g. right primary geodesic) could be split into two (Fig. 1).

The implication of such split geodesic is an increase in the number of geodesics (and thereby the surface ray parameters associated with the various geodesic paths) that must be treated for a complete ray-theoretic analysis.

Effects of Geodesic Splitting

Since the geodesic does not split in the case of the right circular cylinder, cone and sphere, the GPOR as a member of the quadric surfaces family represents the simplest example where the phenomenon of geodesic splitting may be observed.

In the case of the high frequency EM antenna computations [1,2], knowledge of several ray geometric parameters associated with each of the existent geodesic is required a priori. This in effect means a doubling of the ray geometric parameters to be determined in the case of the GPOR as compared to those in the case of the right circular cylinder and cone for any given order of geodesics. Thus, for example, even if the primary geodesics alone are considered on a GPOR, an elemental point-to-point treatment would require the determination of four primary geodesics which may be present.

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Since the GPOR is a nondevelopable surface, in general it requires a bivariate search for the surface ray geometric parameters which makes the problem computationally intractable for most of the practical cases. In contrast, the authors have developed a geodesic constant method (GCM) which requires only a simple univariate search [3,4]. This univariate search is for the First Geodesic Constant associated with the geodesics.

The phenomenon of split geodesics serves to emphasize the advantage of the Geodesic Constant Method (GCM) developed by the authors for ray analysis over general convex surfaces. It may however be noted that the ray geometric parameters in the case of the GPOR are extremely sensitive to the value of h which must be determined accurately. It has been observed by the authors that it usually suffices to compute h accurate upto 8 decimal places for a given arbitrary set of source and observation on the GPOR.

In general, there are now two First Geodesic Constants h_{r11} and h_{r12} associated with the two right primary geodesics. Once these geodesics constants (h_{r11} and h_{r12} for the two right primary and h_{l11} and h_{l12} for the two left primary geodesics) are obtained the arc length, the generalized Fock parameter and the Frenet-frame fields vectors **(t,n,b)** required in the ray-theoretic methods like the UTD can be readily determined.

Summary

The phenomenon of geodesic splitting has been observed in the case of the general paraboloid of revolution. The GPOR represents the simplest surface over which geodesic splitting can be studied.

In general the ray tracing over a general paraboloid of revolution would require a bivariate search. The ray splitting in the case of the GPOR tends to further increase the computer time required substantially for the determination of the surface ray geometric parameters.

The authors have developed a Geodesic Constant Method (GCM) involving an accurate simple univariate search which has brought the EM field computations within the ambit of tractability.

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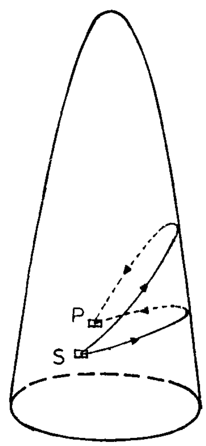

Fig. 1 Splitting of the right primary geodesic on the GPOR.