EFFICIENT ATMOSPHERIC AND EXTRA-ATMOSPHERIC INTERCEPTION THROUGH OPTIMALLY BIASED PROPORTIONAL NAVIGATION

Uday S. Shukla and Pravas R. Mahapatra

Department of Aerospace Engineering
Indian Institute of Science
Bangalore-560012, India.

ABSTRACT

Starting from a definition of the biased proportional navigation (PN) law and expression of the trajectory parameters, especially the lateral acceleration, in terms of a nondimensional bias parameter, an analytical minimization is made of the integrated (total) control effort with respect to the bias parameter. It is shown that optimum biasing may lead to significantlly more control efficient PN guidance in a wide variety of engagement situations, especially those involving higher target maneuvers. The performance of the optimally biased PN is compared with the standard (unbiased) PN law for the general case of a maneuvering target. The optimum bias is expressed through a simple algebraic equation readily solvable in real time even on small on-board processors. For the special (and very useful) case of the effective navigation being equal to 3, the equation reduces to a quadratic leading to an explicit expression for the optimum bias. Specific examples are provided for interception both inside and outside the atmosphere. It is shown that control effort savings upto a factor of 3 can be achieved through optimum biasing under realistic engagement conditions.

INTRODUCTION

Pure Proportional Navigation is a commonly used pursuit strategy for guided weapons as well as for rendezvous/docking/intercept in space. In this strategy, the turning rate of the guided body is controlled to be proportional to the turn rate of the line of sight (LOS)from the guided body to the target.

Although PN guidance results in intercept under a wide variety of engagement conditions, its control-effort-efficiency is not optimum in many situations especially in the case of maneuvering targets. Scope remains for improving the efficiency. Variants have been suggested over the basic PN scheme to improve its efficiency. The biased PN [1,2] is one such scheme, in which a fixed angular rate is superimposed on the measured LOS rate before computing the commanded projectile turn rate (or lateral acceleration).

Because of the introduction of an extra control parameter (i.e. the bias value), such a biased PN (BPN) may be made to achieve a given intercept with reduced total control effort. This is an important advantage for operations outside the atmosphere where lateral control forces are generated by the operation of control rockets, and the total control effort (integrated lateral force) determines the fuel requirement of the control engine(s). This fuel forms a part of the orbital payload which is at a high premium. For atmospheric flights, a reduction in control effort results in smaller pressure bottles in the case of pneumatic actuators and smaller batteries in the case of the modern all-electric actuators. The resulting space and weight saving could be very important in tactical applications.

To be able to take the best advantage of the BPN scheme, it is necessary to optimize the BPN performance with respect to the bias parameter. The performance of the BPN is maximized to obtain the optimum bias value. The development here may be considered as an extension of the earlier work by Brainin and McGhee[2]. The efficiency of BPN is explicitly compared relative to the PN which is more realistic as compared to normalization (as in Ref. 2) with respect to control effort required by a single-impulse guidance scheme which is not practical. Further, while a numerical approach was taken for the optimization in the earlier work, the optimization process in this chapter has been carried out analytically to the extent of obtaining a simple algebraic equation for the optimum bias parameter, which can be solved in real time even on small airborne computers. For the special and important case of the effective navigation constant being equal to 3, the equation is quadratic and the optimum bias parameter is obtained in closed form. To be able to appreciate the advantages of BPN in terms of physical parameters, trajectories of BPN and PPN are plotted and examples are provided which clearly illustrate the savings in total control effort achieved by using a properly optimized BPN.

DEFINITION OF BIASED PROPORTIONAL NAVIGATION

Consider a target T and a pursuer M as points in a plane moving with constant speeds V_T and V_M respectively as shown in Fig. 1. The line MT from the pursuer to the target is the line of sight (LOS) which is inclined at an angle θ with respect to a reference line. If the pursuer velocity vector V_M makes an angle ϕ with the reference line, then the standard PN law is defined as

$$\dot{\phi}=N\dot{\theta} \tag{1}$$

where N is the navigation constant. , we use a modified form of (1) as follows [2] :

$$\dot{\phi}=N(\dot{\theta}-\dot{\theta}_B) \tag{2}$$

where $\dot{\theta}_B$ is a rate bias on the LOS turn rate. Equation (2) defines the biased PN (BPN) law shown graphically in Fig. 2. The BPN law (2) reduces to the standard PN law (1) when $\dot{\theta}_B$ equals zero.

SOLUTION OF BIASED PROPORTIONAL NAVIGATION

We consider the case of pursuit against a target maneuvering with a constant lateral acceleration A_T. The governing differential equations of motion, considering the geometry only, are obtained by resolving velocity components of the target and the pursuer along and normal to the LOS.

$$\dot{r}=V_T\cos(\theta-\beta)-V_M\cos(\theta-\phi) \tag{3}$$

and

$$r\dot{\theta}=-V_T\sin(\theta-\beta)+V_M\sin(\theta-\phi) \tag{4}$$

where $\kappa=A_T/V_T$ represents the turn rate of the target, and $\beta=\kappa t$.

The equations (3) and (4) for the pursuer motion under PN are

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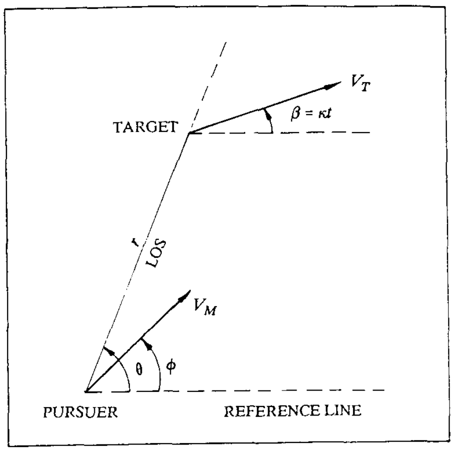

Figure 1. Geometry of Proportional Navigation

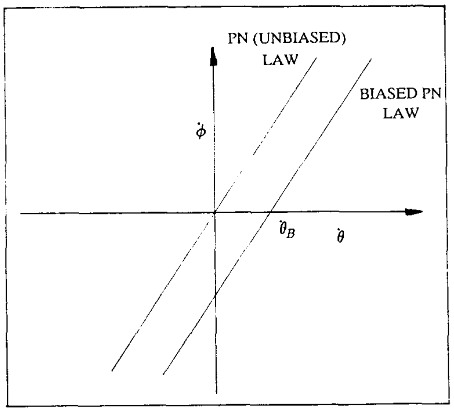

Figure 2. Biased PN scheme

not solvable in closed form. Here PN equations are linearized to make analytical treatment possible. Considering the homing trajectory to be a perturbation over a collision course, we can write (Fig. 1)

$$\phi=\phi_C+\Delta\phi \tag{5}$$

Assuming $\Delta\phi$ and θ to be small, (3) and (4) may be readily combined to yield an equation in $\dot\theta$ only

$$\ddot\theta(t_f-t)-2\dot\theta=\frac{V_M}{V_{rL}}\cos\phi_C\dot\phi+\frac{V_T}{V_{rL}}\cos\beta_i\dot\beta \tag{6}$$

where t is the time from launch, V_{rL} is the initial target-pursuer relative velocity along LOS and $t_f=r_i/V_{rL}$ is the final intercept time. Using the biased PN law (2), (6) reduces to

$$\ddot\theta(t_f-t)-(2-N')\dot\theta-N'\dot\theta_B=\frac{V_T}{V_{rL}}\cos\beta_i\dot\beta \tag{7}$$

where $N'=\dfrac{NV_M\cos\phi_C}{V_{rL}}$: Effective Navigation Constant

Equation (7) describes the behavior of the rate of change of LOS angle ($\dot\theta$) and can be integrated to give

$$\dot\theta=\left[\dot\theta_i-\frac{1}{N'-2}\left(\frac{V_T\cos\beta_i\dot\beta}{V_{rL}}+N'\dot\theta_B\right)\right]\left(\frac{t_f-t}{t_f}\right)^{N'-2}+\frac{1}{N'-2}\left[\frac{V_T\cos\beta_i\dot\beta}{V_{rL}}+N'\dot\theta_B\right] \tag{8}$$

where $\dot\theta_i$: Initial value of LOS angular rate.

Assuming $\beta_i=0$ (i.e. initial target velocity direction coincides with the reference line) without loss of generality, and using $\dot\beta=A_T/V_T$, (by definition of β) eq (8) reduces to

$$\dot\theta=\left[\dot\theta_i-\frac{1}{N'-2}\left(\frac{A_T}{V_{rL}}+N'\dot\theta_B\right)\right]\left(\frac{t_f-t}{t_f}\right)^{N'-2}+\frac{1}{N'-2}\left[\frac{A_T}{V_{rL}}+N'\dot\theta_B\right] \tag{9}$$

The expression (9) represents the LOS turn rate for pursuit against a maneuvering target under the BPN law.

PURSUER LATERAL ACCELERATION

The lateral acceleration A_M of the pursuer under the BPN is obtained as

$$A_{MB}=V_M\dot\phi=NV_M(\dot\theta-\dot\theta_B) \tag{10}$$

Substituting eq (9) in eq (10) and rearranging, we get

$$A_{MB}=\frac{N'}{(N'-2)\cos\phi_C}\left[a-b\left(\frac{T}{T_i}\right)^{N'-2}\right]=\frac{bN'}{(N'-2)\cos\phi_C}\left[p-\left(\frac{T}{T_i}\right)^{N'-2}\right] \tag{11}$$

where

T = *time to go* $=t_f-t$

T_i = *initial value of* $T=t_f$

$$a=A_T+2V_{rL}\dot\theta_B \tag{12}$$

$$b=A_T-(N'-2)V_{rL}\dot\theta_i+N'V_{rL}\dot\theta_B \tag{13}$$

$p=a/b$ = *bias parameter*

From (12)-(13) and the definition of p, the rate bias $\dot\theta_B$ is expressed as

$$\dot\theta_B=\frac{1-p}{(pN'-2)V_{rL}}A_T+\frac{p(N'-2)}{pN'-2}\dot\theta_i \tag{14}$$

$$=K\frac{A_T}{V_{rL}}+L\dot\theta_i \tag{15}$$

where

K = *Acceleration bias coefficient* $=\dfrac{1-p}{(pN'-2)}$

L = *Rate bias coefficient* $=\dfrac{p(N'-2)}{pN'-2}$

It can be seen from eq (15) that $\dot\theta_B$ is singular at $p=2/N'$. On

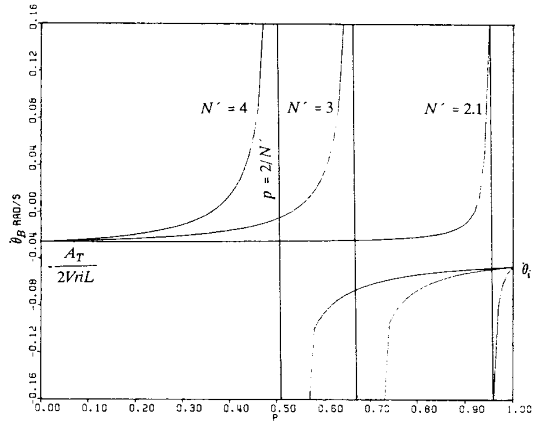

Figure 3. Behavior of the rate bias $\dot\theta_B$ as a function of bias parameter p

either side of this singularity $\dot\theta_B$ decreases and is dominated by the A_T term for $p<2/N'$ and by the $\dot\theta_i$ term for $p>2/N'$. In fact at $p=0$, $\dot\theta_B=-A_T/2V_{\dot r L}$ and at $p=1$, $\dot\theta_B=\dot\theta_i$. For the value of $p=1/\left[1+(N'-2)\dot\theta_i V_{\dot r L}/A_T\right]$, $\dot\theta_B=0$. This behavior of $\dot\theta_B$ with respect to the bias parameter p is depicted in Fig. 3. In addition, for $p=\infty$, $\dot\theta_B=\frac{1}{N'}\left[-A_T/V_{\dot r L}+(N'-2)\dot\theta_i\right]$.

CONTROL EFFORT

The cumulative velocity increment ΔV (which determines the total control effort) necessary for interception is defined for any pursuer trajectory as

$$\Delta V=\int_0^{T_i}\left|A_M\right|dT \tag{16}$$

Two cases must be considered for computing the cumulative velocity increment ΔV:

Case I : $(0\leq p\leq 1)$

In this case exactly one change of sign of A_M occurs in the interval $[0,Ti]$. This is apparent from (11), since p is a fraction between 0 and 1, and $\left(T/T_i\right)^{N'-2}$ decreases monotonically from 1 to 0 for $N'>2$. The acceleration reversal occurs at

$$T_r=T_i\left(\frac{a}{b}\right)^{\frac{1}{(N'-2)}} \tag{17}$$

$$=T_i p^{\frac{1}{(N'-2)}} \tag{17a}$$

Then ΔV_B is given by

$$\Delta V_B=\frac{M_i}{T_i}\frac{2N'}{N'-1}\left|\frac{(N'-2)p^{\frac{N'-1}{N'-2}}+\left|1+(N'-2)p^{\frac{N'-1}{N'-2}}-(N'-1)p\right|}{N'p-2}\right| \tag{18}$$

where

$$M_i=\frac{1}{\cos\phi_C}\left|V_{\dot r L}T_i^2\dot\theta_i+\frac{A_T}{2}T_i^2\right| \tag{19}$$

Case II : $p\leq 0$ *or* $p\geq 1$

Since T/T_i has a minimum value of zero and maximum value of unity, the RHS of (11) will remain unipolar during the entire pursuit if $N'>2$. Thus, the lateral acceleration A_M never changes sign during the pursuit, and ΔV_B is given by

$$\Delta V_B=\frac{M_i}{T_i}\frac{2N'(N'p-p-1)}{(N'-1)(N'p-2)} \tag{20}$$

By making use of (18) or (20), the cumulative velocity increment ΔV_B for BPN for any value of the bias parameter p and effective navigation constant N' can be readily obtained as long as $N'>2$, which includes most useful values of N'.

OPTIMUM BIASING OF PROPORTIONAL NAVIGATION

The foregoing treatment provides a mechanism (through the introduction of a rate bias) of controlling the total control effort necessary for achieving a given mission. To make the best use of this freedom, it is necessary to optimize the rate bias to achieve a minimum control effort. Such an optimization is carried out below for the two cases considered in the last section.

Case I : $0\leq p\leq 1$

To minimize ΔV_B with respect to p, we first examine the quantity within the inner modulus in (18) i.e.,

$$F\equiv 1+(N'-2)p^{\frac{N'-1}{N'-2}}-(N'-1)p \tag{21}$$

It can be seen that $F=1$ for $p=0$ and $F=0$ for $p=1$, and

$$\frac{\partial F}{\partial p}=(N'-1)\left(p^{\frac{1}{(N'-2)}}-1\right) \tag{22}$$

For $N'>2$, the factors $(N'-1)$ and $(N'-2)$ are always positive. Also, since $0\leq p\leq 1$, the quantity $p^{1/(N'-2)}$ is always a fraction and hence $\left(p^{1/(N'-2)}-1\right)$ is always negative, implying that $(\partial F/\partial p)$ is always negative. We can thus remove the inner modulus from eq (18) and the problem reduces to the minimization of

$$\Delta V_B=\frac{M_i}{T_i}\frac{2N'}{N'-1}\left|\frac{(N'-2)p^{\frac{N'-1}{N'-2}}+1+(N'-2)p^{\frac{N'-1}{N'-2}}-(N'-1)p}{N'p-2}\right| \tag{23}$$

Since the quantity inside the modulus in eq (23) is not necessarily unipolar, the modulus operation is likely to disturb its extremal behavior. As such, to minimize $\Delta V_B(p,N')$ from eq (23), we would first consider the quantity inside the modulus and find its extremal points in the interval $(0\leq p\leq 1)$, and then choose the one with the least magnitude of $\Delta V_B(N',p)$. Differentiating the quantity within the modulus in eq (23) and equating to zero,

$$p_o^{\frac{N'-1}{N'-2}}-2\frac{N'-1}{N'}p_o^{\frac{1}{N'-2}}+\frac{N'-2}{2N'}=0 \tag{24}$$

Equation (24) now expresses the optimum bias parameter p_o as a simple algebraic equation with coefficients dependent on N'. In general, eq (24) will involve fractional powers of p_o, but will still be simple enough to be solved in real time even on a small airborne computer. For off-line design computation, a hand-held calculator would suffice. Once p_o is obtained, it can be substituted for p in (14) to determine the optimum rate bias $\dot\theta_{Bo}$. The values of p_o corresponding to $N'=2.1, 2.5, 3, 3.5, 4$ and 4.5 have been obtained by a numerical solution of eq (24) and are given in Table 1, along with the

TABLE 1. Optimum bias parameter for biased proportional navigation. K and L are the acceleration and rate bias coefficients

N'	p_o	K	L
2.1	0.787	-0.614	-0.227
2.5	0.341	-0.574	-0.149
3.0	0.140	-0.544	-0.088
3.5	0.062	-0.526	-0.052
4.0	0.029	-0.515	-0.031
5.0	0.007	-0.505	-0.010

coefficients K and L in eq (15). It is apparent that since L is consistently much smaller than K, the contribution of the initial LOS rate $\dot{\theta}_i$ to optimum value of rate bias $\dot{\theta}_{Bo}$ is minor relative to that of target maneuver A_T. Further, with increasing N', the contribution due to $\dot{\theta}_i$ becomes relatively insignificant as L asymptotically approaches zero, with K tending to 0.5.

For the important special case of $N'=3$, eq (24) reduces to the quadratic form

$$6p_o^2-8p_o+1=0 \tag{25}$$

having the roots

$$p_o=1.19 \text{ and } p_o=0.14 \tag{26}$$

Since only one of the roots in (26) lies within the domain of interest ($0\leq p\leq 1$), the only admissible optimum corresponds to $p_o=0.14$.

For other integer values of N', although a closed form solution is not guaranteed, considerable simplification of eq (24) can be achieved. For example, when $N'=4$, eq (24) reduces to

$$p_o^{\frac{3}{2}}-1.5p_o^{\frac{1}{2}}+0.25=0 \tag{27}$$

which is a simple cubic in $(p_o)^{1/2}$.

Case II : $p\leq 0$ *or* $p\geq 1$

The velocity increment ΔV_B here is given by (20). The gradient of ΔV_B is given by

$$\frac{\partial \Delta V_B}{\partial p}=\frac{M_i}{T_i}\frac{2N'(2-N')}{(N'-1)(N'p-2)^2} \tag{28}$$

For $N'>2$, this gradient is negative for all values of p, and hence ΔV_B is a monotonic function and has no distinct optimum. However, (28) indicates the existence of asymptotic stationary points since $\partial\Delta V_B/\partial p=0$ for $p\to\pm\infty$, at which $\Delta V_B\to 2M_i/T_i$, from (20).

Global Optimum Biasing

To facilitate visualization of the function behavior, the dimensionless quantity $\Delta V_B/(Mi/Ti)$ is plotted in Fig. 4 for $N'=2.1$, 3 and 4 with p varying from -2 to +3. $N'=2$ is not plotted since eq (18) is singular when N' equals 2. For $0\leq p\leq 1$, the formula (18) is used, and outside this domain (20) is used. It is seen that for $p\leq 0$, the asymptotic stationary point represents a maximum and that for $p\geq 1$, it represents a minimum.

Also an analysis of the behavior of ΔV_B at $p=0$ and $p=1$ shows that

- ΔV_B at $p=0$ has a value $(M_i/T_i)[N'/(N'-1)]$ which is less than the asymptotic value $2M_i/T_i$ for all $N'>2$. Also the gradient of ΔV_B at $p=0$ is $\left[M_i/T_i\right]\left[N'(2-N')/2(N'-1)\right]$ (from (28)), which is negative for all $N'>2$.
- ΔV_B at $p=1$ has a value $(M_i/T_i)[2N'/(N'-1)]$ which is greater than the asymptotic value $2M_i/T_i$ for all $N'>2$. Also, the gradient of ΔV_B at $p=1$ is $\left[M_i/T_i\right]\left[2N'/(N'-1)(2-N')\right]$ (from (28)) which is negative for all $N'>2$.

From the above argument and an examination of Fig. 4 it is clear that the global minimum of ΔV_B will be within the domain $0\leq p\leq 1$, and will occur at the optimum rate bias parameter p_o given by (24).

The singularity in ΔV_B in the region $0\leq p\leq 1$ seen in Fig. 4 is predictable from eq (18) to occur at $p=2/N'$ and is in fact a direct result of relation (15) between the rate bias $\dot{\theta}_B$ and bias parameter p which is the variable of optimization. However, the singularity imposes no practical problems since it corresponds to infinite rate bias, as seen by letting $p=2/N'$ in eq (14) and also seen in Fig. 3.

RESULTS AND DISCUSSIONS

For reasons of generality, a non-dimensional rate bias parameter p has been used in the formulation. It has also resulted in improving the tractability of the problem. However, the transformation used in the non-dimensionalization has resulted in a certain blurring of the physical insight into the behavior of the biased PN system. To be able to visualize the potential benefits of the biasing in clearer focus, two specific examples are provided below.

The first example considers an air-to-air tactical situation with a

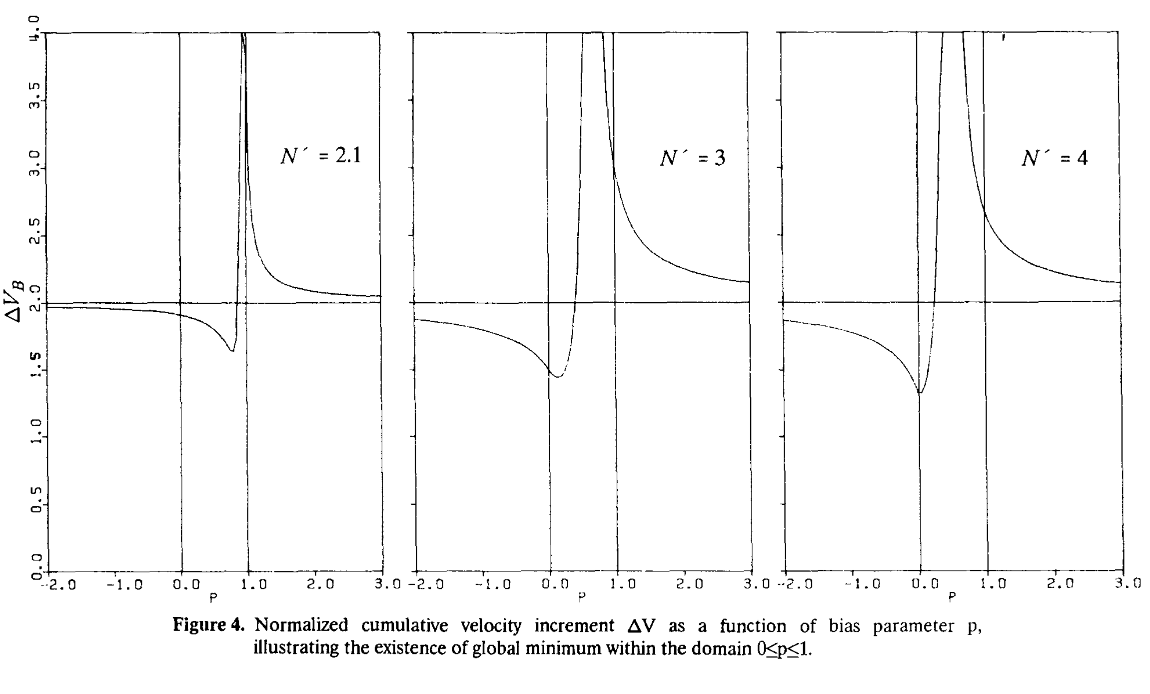

Figure 4. Normalized cumulative velocity increment ΔV as a function of bias parameter p, illustrating the existence of global minimum within the domain $0\leq p\leq 1$.

target speed of 300 m/s, a pursuer speed of 900 m/s, an initial pursuer-target separation of 5000 m, and an initial LOS angle of 60°. Table 2 shows the optimum rate bias $\dot{\theta}_{Bo}$ from (14) for a range of realistic values of the effective navigation constant N'. This computation requires knowledge of the target maneuver A_T and the initial LOS rate $\dot{\theta}_i$ (or, equivalently, initial heading error $\Delta\phi_i$). Here, both the non-maneuvering and maneuvering targets are considered. For the non-maneuvering target case shown in Table 2 (a) two representative values of heading error ($\Delta\phi_i$=15° and 40°) are considered. For the maneuvering target case the target lateral acceleration of $0.5g$, $1g$, $2g$, $3g$ and $4g$ are considered, with heading error $\Delta\phi_i$ taking two values, 0° and 15° in each case. The results are shown in Table 2 (b) through (f). The cumulative velocity increment ΔV_B required for intercept using BPN is computed from (18). For comparison, the cumulative velocity increment ΔV_{PN} for standard PN is also obtained from (18) with $\dot{\theta}_B$=0 and is tabulated alongside.

The optimum rate bias $\dot{\theta}_{Bo}$ exhibits a strong dependence on the effective navigation constant N' for a given level of target maneuver and initial heading error. The most important observation from Table 2, however, concerns the cumulative velocity increments. It is clear from Table 2 that the optimally biased PN always requires lower total control effort than a standard PN for all maneuver levels and effective navigation constants, with or without heading error. Depending on the combination of heading error, target maneuver and N', an optimum BPN could score over a classical PN in terms of required total control effort by a factor of 1:3 as seen from Table 2 (f) for N'=5, A_T=4, $\Delta\phi_i$=15°.

While the results in Table 2 provide the behavior with respect to the total control effort, derived from the pursuer lateral acceleration, it is worthwhile viewing the optimum BPN vis-a-vis the classical PN in a more primary parameter domain such as the trajectory itself during the engagement. Both the non-maneuvering and the maneuvering

TABLE 2. Cumulative velocity increment (ΔV) requirement for biased proportional navigation for air-to-air engagement. The requirement for basic PN is also shown for comparison.

Non-maneuvering target						
	$\Delta\phi_i$ = 15°			$\Delta\phi_i$ = 40°		
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s	
		BPN	PPN		BPN	PPN
2.1	10.51	378.918	442.654	27.86	1004.459	1173.416
2.5	6.89	355.732	386.444	18.27	942.997	1024.410
3.0	4.15	334.241	347.800	10.99	886.028	921.969
3.5	2.42	318.291	324.613	6.41	843.745	860.505
4.0	1.43	306.121	309.155	3.78	811.484	819.528
5.0	0.50	289.106	289.833	1.31	766.380	768.308

(a)

Maneuvering target						
	A_T = 0.5g $\Delta\phi_i$ = 0°			A_T = 0.5g $\Delta\phi_i$ = 15°		
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s	
		BPN	PPN		BPN	PPN
2.1	-4.23	28.158	65.789	6.50	352.183	380.779
2.5	-3.96	26.435	57.435	3.13	330.633	337.196
3.0	-3.75	24.838	51.691	0.58	310.658	310.859
3.5	-3.63	23.653	48.245	-1.02	295.833	296.654
4.0	-3.55	22.748	45.948	-1.95	284.522	287.929
5.0	-3.48	21.484	43.076	-2.81	268.707	278.012

(b)

Maneuvering target						
	A_T = 1.0g $\Delta\phi_i$ = 0°			A_T = 1.0g $\Delta\phi_i$ = 15°		
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s	
		BPN	PPN		BPN	PPN
2.1	-8.45	56.316	131.578	2.48	325.448	329.761
2.5	-7.92	52.870	114.869	-0.63	305.533	305.775
3.0	-7.51	49.676	103.382	-2.98	287.076	292.852
3.5	-7.25	47.306	96.490	-4.47	273.376	286.630
4.0	-7.10	45.497	91.896	-5.32	262.923	283.269
5.0	-6.97	42.968	86.152	-6.12	248.309	280.242

(c)

Maneuvering target						
	A_T = 2.0g $\Delta\phi_i$ = 0°			A_T = 2.0g $\Delta\phi_i$ = 15°		
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s	
		BPN	PPN		BPN	PPN
2.1	-16.91	112.632	263.155	-5.55	271.978	290.601
2.5	-15.83	105.740	229.739	-8.14	255.335	290.987
3.0	-15.02	99.352	206.765	-10.11	239.910	293.144
3.5	-14.50	94.611	192.981	-11.35	228.461	295.757
4.0	-14.21	90.994	183.791	-12.06	219.726	298.390
5.0	-13.93	85.936	172.304	-12.73	207.513	303.253

(d)

Maneuvering target						
	A_T = 3.0g $\Delta\phi_i$ = 0°			A_T = 3.0g $\Delta\phi_i$ = 15°		
	mrad/s	m/s		mrad/s	m/s	
		BPN	PPN		BPN	PPN
2.1	-25.36	168.948	394.733	-13.57	218.507	313.324
2.5	-23.75	158.610	344.608	-15.66	205.137	317.919
3.0	-22.53	149.028	310.147	-17.24	192.744	323.350
3.5	-21.75	141.917	289.471	-18.24	183.546	328.355
4.0	-21.31	136.490	275.687	-18.81	176.528	332.906
5.0	-20.90	128.904	258.456	-19.35	166.716	340.766

(e)

Maneuvering target						
	A_T = 4.0g $\Delta\phi_i$ = 0°			A_T = 4.0g $\Delta\phi_i$ = 15°		
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s	
		BPN	PPN		BPN	PPN
2.1	-33.82	225.264	526.311	-21.60	165.037	370.642
2.5	-31.66	211.481	459.478	-23.17	154.939	369.265
3.0	-30.03	198.705	413.530	-24.37	145.579	371.573
3.5	-29.01	189.222	385.961	-25.12	138.631	375.320
4.0	-28.42	181.987	367.582	-25.55	133.331	379.420
5.0	-27.86	171.872	344.608	-25.96	125.920	387.365

(f)

target cases are considered. In Fig. 5 the trajectory of a pursuer following biased PN guidance law against a non-maneuvering target is plotted for the initial conditions corresponding to results in Table 2 (a). Two most commonly used N' values of 3 and 4 are used. Next, a laterally maneuvering target is considered and pursuer trajectories are plotted for the moderate and high target maneuvers corresponding to the initial conditions in Table 2 (d) and (f) respectively. Fig. 6 corresponds to $N'=3$ while a value of $N'=4$ is used in Fig. 7. It is seen from these plots that the BPN trajectory for the pursuer is always flatter (i.e. less curved) as compared to the standard PN. As anticipated from the results of Table 2, the cases showing a larger saving in control effort result in a higher level of flattening of the optimal BPN trajectory relative to the standard PN. In the case of non-maneuvering targets, this flattening takes the trajectory close to the ideal case of collision course and in the case of maneuvering targets the flattening takes the trajectory towards the line joining the start of the engagement and the intercept point.

The second example corresponds to an engagement scenario in extra-atmospheric space. The initial target-pursuer separation is 185 km and the relative initial closing speed is 9000 m/s. These values are the same as those used for illustration in [1]. Here also, an initial LOS angle of 60° is assumed, as also an "initial miss distance" of 15 km , corresponding to an initial LOS rate of 3.944 mrad/s. In Table 3, in addition to the cumulative velocity increments ΔV_{Bo} and ΔV_{PN}, the quantity of propellant required for effecting these velocity increments are also presented. The latter quantity is computed assuming, as in [1], an initial interceptor weight of 270 kg and a liquid propellant with a specific impulse of 300 s. Either the cumulative velocity increment or the propellant requirement can be taken as a measure of the required control effort.

TABLE 3. Cumulative velocity increment and propellant required for biased proportional navigation for extra-atmospheric engagement. The requirement for basic PN is also shown for comparison.

Non-maneuvering target $\dot{\theta}_i = 3.94$ mrad/s					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	.89	1192.383	1392.949	91.611	103.692
2.5	.59	1119.421	1216.067	87.007	93.080
3.0	.35	1051.795	1094.460	82.637	85.406
3.5	.21	1001.601	1021.496	79.328	80.647
4.0	.12	963.305	972.853	76.766	77.408
5.0	.04	909.761	912.050	73.126	73.283

(a)

Maneuvering target $\dot{\theta}_i = 3.94$ mrad/s $A_T = 0.5$g					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	.56	1109.999	1202.128	86.404	92.217
2.5	.27	1042.078	1063.782	82.001	83.419
3.0	.06	979.124	979.945	77.828	77.883
3.5	-.08	932.398	934.640	74.673	74.825
4.0	-.16	896.748	906.771	72.231	72.921
5.0	-.23	846.904	875.028	68.768	70.729

(b)

Maneuvering target $\dot{\theta}_i = 3.94$ mrad/s $A_T = 1.0$g					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	.23	1027.614	1043.073	81.050	82.066
2.5	-.04	964.734	965.160	76.862	76.890
3.0	-.24	906.453	922.973	72.899	74.030
3.5	-.37	863.195	902.514	69.906	72.628
4.0	-.44	830.191	891.353	67.593	71.859
5.0	-.51	784.046	881.078	64.316	71.149

(c)

Maneuvering target $\dot{\theta}_i = 3.94$ mrad/s $A_T = 2.0$g					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	-.44	862.845	914.950	69.882	73.482
2.5	-.67	810.048	914.500	66.169	73.451
3.0	-.83	761.111	920.206	62.667	73.841
3.5	-.94	724.789	927.756	60.031	74.357
4.0	-1.00	697.077	935.562	57.997	74.888
5.0	-1.06	658.331	950.201	55.121	75.881

(d)

Maneuvering target $\dot{\theta}_i = 3.94$ mrad/s $A_T = 3.0$g					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	-1.11	698.076	977.692	58.071	77.732
2.5	-1.29	655.361	992.509	54.899	78.723
3.0	-1.43	615.769	1009.680	51.918	79.865
3.5	-1.51	586.383	1025.361	49.679	80.902
4.0	-1.56	563.963	1039.564	47.956	81.836
5.0	-1.61	532.616	1064.026	45.525	83.435

(e)

Maneuvering target $\dot{\theta}_i = 3.94$ mrad/s $A_T = 4.0$g					
N'	$\dot{\theta}_{Bo}$ mrad/s	ΔV m/s		PROPELLANT Kg	
		BPN	PPN	BPN	PPN
2.1	-1.78	533.307	1148.531	45.579	88.857
2.5	-1.92	500.674	1146.493	43.021	88.729
3.0	-2.02	470.428	1155.072	40.624	89.271
3.5	-2.09	447.978	1167.494	38.830	90.053
4.0	-2.13	430.849	1180.715	37.451	90.882
5.0	-2.16	406.901	1205.947	35.510	92.454

(f)

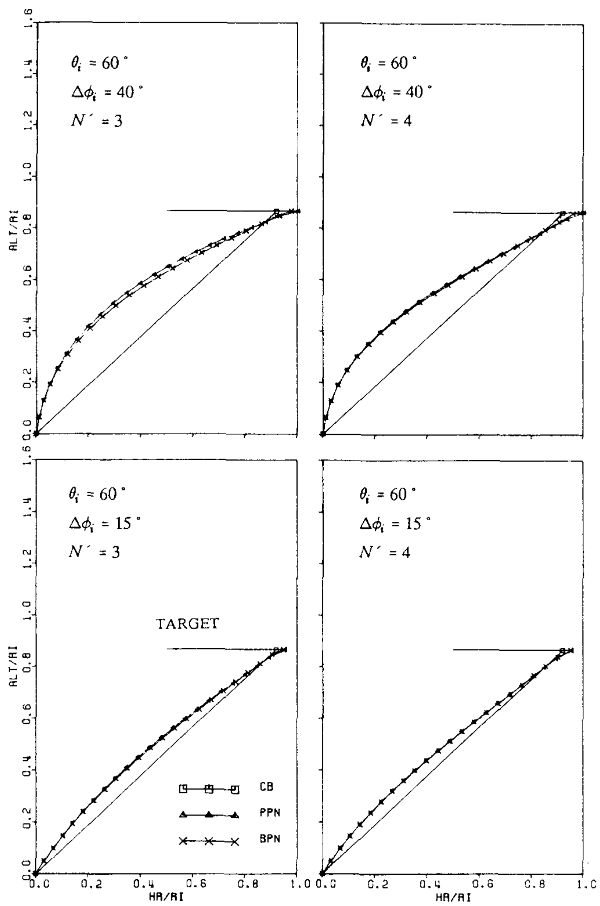

Figure 5. Pursuer trajectory for a biased PN law against a non-maneuvering target. The trajectory under PPN and the Collision course are also shown for comparison.

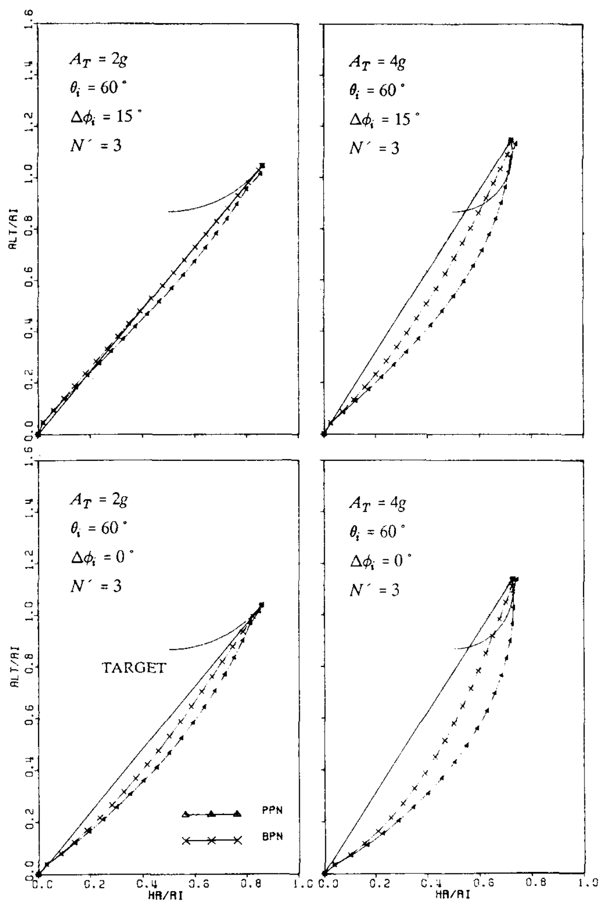

Figure 6. Pursuer trajectory for a biased PN law against a laterally maneuvering target. The trajectory under PPN is also shown for comparison. N'=3.

In section (a) of Table 3 a non-maneuvering space target is considered while in sections (b) and (c), target maneuver A_T value of 0.5g and 1g are assumed. It is apparent that in space pursuit scenarios, control effort can be saved by employing optimum BPN even for relatively low target maneuvers. Thus, for a non-maneuvering target, a 13% propellant saving over the standard PN is possible if N' =2.1 is used while it is 3% for commonly used value of N' =3. A saving of about 3% is possible for A_T=0.5g and it increases to about 10% over the standard PN, for A_T=1.0g and N' =5. Target maneuver A_T is progressively increased in section (d) and (e) of Table 3 and it is seen that the saving is as high as 27% for A_T=2g and 45% for A_T=3g using N' =5. In Table 3(f), a high target maneuver of 4g is deliberately included, keeping in view the possible space-based pursuit-evasion applications of the near future. For such target maneuvers, the propellant saving is by a factor of 1:2 to 1:2.6 for N' varying from 2.1 to 5.0.

CONCLUSIONS

Biased Proportional Navigation (BPN) has been studied from the point of view of control effort requirement. It has been shown that with optimal choice of the rate bias, it is possible to effect large savings in control effort required for intercepting maneuvering targets.

An analytical optimization of the BPN problem has been carried out in terms of a non-dimensional rate bias parameter, resulting in a simple algebraic equation for the optimum value of the parameter from a minimum-control-effort point of view. The equation can be easily solved in real time even in the simple on-board computers of small projectiles. For the special but very useful case of N' =3, the solution for the optimum rate bias parameter is explicit.

Two examples have been provided to concretely illustrate the gains possible by using an optimal BPN over the standard PN both in terms of total control effort and the trajectory behavior. The examples concern both atmospheric and extra-atmospheric pursuits. It has been shown that for highly maneuvering targets, the optimal BPN may require a total control effort as low as one third of the effort necessary for PN without bias. Such savings can be extremely valuable especially in extra-atmospheric engagements where maneuvers are carried out at the direct expense of propellant which forms part of the precious payload.

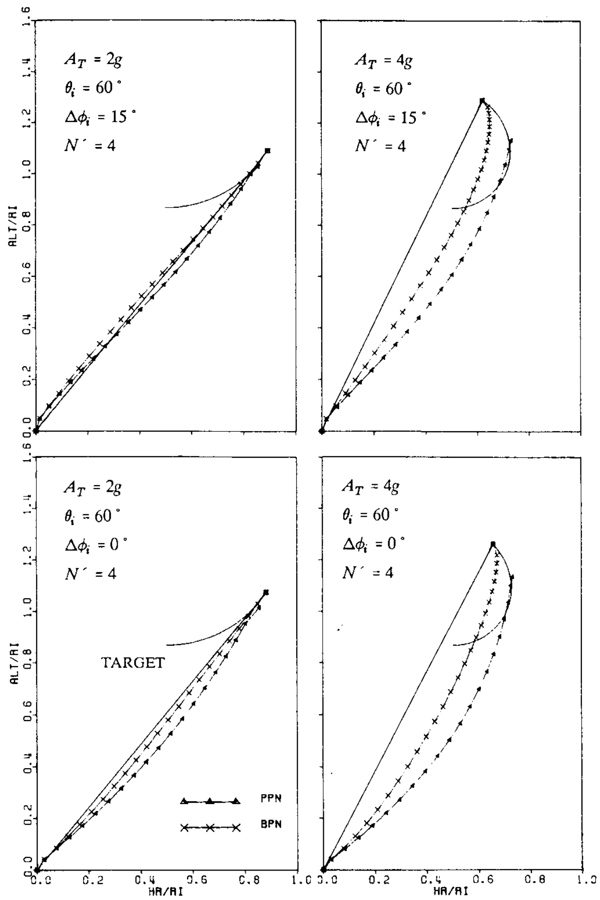

Figure 7. Pursuer trajectory for a biased PN law against a laterally maneuvering target. The trajectory under PPN is also shown for comparison. N'=4.

REFERENCES

[1] Murtaugh, S.A. and Criel, H.E. (1966): "Fundamentals of Proportional Navigation," *IEEE Spectrum*, Vol. 3, No. 12, Dec. 1966, pp. 75-85.

[2] Brainin, S.M. and McGhee, R.B. (1968): "Optimal Biased Proportional Navigation," *IEEE Transactions on Automatic Control*, Vol. AC-13, No. 4, Aug. 1968, pp. 440-442.