ECG SYNTHESIS VIA DISCRETE COSINE TRANSFORM

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ABSTRACT
Results on modelling and synthesis of ECG
from its component waves are presented
after a suitable transformation. The
transformed components add up to the
transformation of the complete beat (CB)
and thus enable synthesis of ECG. Pole-
zero modelling of the transformed signals
by Shanks method proved very successful.

I. INTRODUCTION
The electrocardiogram (ECG) is invariably
used for monitoring the heart function in
routine morphological studies as well as
in critical situations for rhythm analysis
in ICCUs. Inspite of the fact that QRS
complex is the most prominent feature in
the ECG noise sources such as base line
wander, power line interference, muscle
tremor peaky tented T waves and small
premature ventricular contractions make
QRS detection inaccurate and ambiguous.
Time and frequency domain methods or a
combination of these two based on single
lead and/or multilead analysis have been
developed. More sophisticated detectors
are based on syntactic methods, heuristic
approaches   involving   nonlinear
transformations, matched filters, etc.
[1]. Because of the intricate problems,
detection of P and T waves remained as a
challenge   till to-day.

Attempts have been made to model the time
domain ECG signal directly, which required
very high orders [2]. However no method
has been suggested to delineate the QRS
and other component waves by modelling.
It is shown here that modelling with a
lower order system function and
delineation of component waves is feasible
when the ECG cycle is subjected to a
suitable transformation such as discrete
cosine transform (DCT) or discrete sine
transform (DST). The method has been
extended to model several transformed ECG
cycles in a single operation as well as
delineation of component waves in a given
ECG cycle [3]. Because of its simplicity
Shanks method is used here.

II. MODELLING

A. Discrete Cosine Transform (DCT):
The DCT of a discrete time signal $X[n]$, n
= $0,1,2,...,N-1$ is by definition

$$X[k] = 1/\sqrt{N} \sum_{n=0}^{N-1} X[n] \quad k = 0$$

$$= \sqrt{2/N} \sum_{n=0}^{N-1} X[n] \cos [(2n+1)k /2N]$$

$$K = 1,2,...,N-1.$$

The sequence $X[k]$ thus obtained is a minimum
or near minimum phase signal. The DCT of
a complete ECG cycle can be obtained as
the sum of the transforms of the
individual components and vice-versa.
These transformed components are then
modelled by the Shanks algorithm [4].

B. Shanks Algorithm [6]
Aim is to design a system function $F(z)$ to
approximate $X(z)$, the z-transform of the
given sequence $X(k)$. That is

$$F(z) \simeq X(z)$$
$$\text{where } X(z) = \sum_{k=-\infty}^{\infty} X(k) z^k$$

Let $F(z) = [A(z)/B(z)] = f_0+f_1 z+f_2 z^2+...$

and $A(z) = a_0+a_1 z+a_2 z^2+...+a_N z^N$

$$B(z) = 1 + b_1 z+b_2 z^2+...+b_M z^M.$$

N and M are arbitrary numbers and fix the
coefficients a_i and b_j, $i=1,...,N$,
$j=1,...,M$. For the impulse response $f[k]$
of the model to approximate $x[k]$ in the
minimum mean square sense, b_i has to
satisfy the simultaneous equations:

$$\sum_{i=1}^{M} b_i \phi_{ij} = \phi_i \qquad j=1,...,M$$
where

$$\phi_{ij} = \sum_{k=N+1}^{L} x(k-j) . x(k-i)$$

$$\phi_i = \sum_{k=N+1}^{L} x(k) . x(k-i)$$

L being the signal length. Similarly, a_i has to satisfy the following equations:

$$\sum_{i=0}^{N} a_i \ \psi_{ij} = \psi_i \qquad j = 0,1,\ldots,N$$

$$\psi_{ij} = \sum_{k=0}^{L} C(k-i) \ C(k-j)$$

$$\psi_i = \sum_{k=0}^{L} x(k) \ C(k-i)$$

where $C(k)$, $k = 0,1,\ldots,L$ are obtained from $1/B(z)$ by long division. That is

$$1/B(z) = C(0) \ C(1)z + \ldots + C(L) \ z^L$$

III. RESULTS AND CONCLUSIONS

A low pass filtered and digitized at 100 sps ECG signal and the component waves P, QRS and T are shown in Figs. 1a, b, c, d respectively. Their DCTs shown in Figs. 2a, b, c, d, are modelled using orders (6,6) for CB and (2,2) for each of the component waves. The time domain outputs are obtained by computing the IDCT of the impulse response of the model. The pole-zero plot, the model impulse response and its IDCT are shown in Figs. 3a,b and c. The fit of the component waves as well as the CB obtained from the models are in close agreement with the original signals.

It is shown that the ECG signal when transformed can be pole-zero modelled using any algorithm such as Shanks method. The transforms of the component waves add up to the transform of the complete beat and thus enable synthesis from the components. The sum of model orders of individual components add up to that of the entire cycle.

REFERENCES

1. O. Pahlm and L. Sornmo: Software QRS detection in ambulatory monitoring - A review, Med. Biol. Engg. & Comp., 22, pp 289-297, July 1984.
2. G.S.S.D. Prasad: A comparative study of suboptimal ARMA algorithms for biomedical signals, Ph.D. Thesis, Dept. of E.E., I.I.Sc., Bangalore,1988.
3. I.S.N. Murthy, et al: Modelling of ECG signals Part I, II, III, TR3, Dept. of E.E., I.I.Sc., Bangalore, 1988.
4. J.L. Shanks: Recursion filters for digital processing; Geophysics, 32, pp 33-51, Feb. 1967.

Fig.1   a) Complete Beat   b) P-Wave   c) QRS-Complex   d) T-Wave

Fig.2   a) CB DCT   b) P-Wave DCT   c) QRS Complex DCT   d) T-Wave DCT

Fig.3   2.1   a) Pole-Zero Plot   b) Model Impulse Response   c) IDCT of Imp. Resp.