3-D GEODESICS ON CONVEX QUADRICS FOR SURFACE RAY PROPAGATION : A TURBO BASIC PACKAGE FOR COMPUTER-AIDED INSTRUCTION

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ABSTRACT

A computer package has been developed to demonstrate the various types of geodesics on developable as well as nondevelopable quadrics, with particular application to high frequency EM analysis. This user-friendly package, developed in TURBO BASIC, allows the user to simulate the various possible geodesics between any two points located on a convex surface.

Introduction

The ray-theoretic formulations such as the GTD and UTD have the common feature that they require surface ray geometric parameters along the ray paths which are the geodesics of the convex surface. However, the determination of the geodesic ray paths is not quite easy. This is further hindered by the fact that quite often the undergraduate students (and sometimes even researchers!) have preconceived notions about the geodesics which are often incomplete and in several cases even wrong. For example, one is often led to believe that "the geodesic is the shortest path (which is partially true) and consequently between two points on a surface there is only one geodesic (which is wrong)."

The authors have developed a user-friendly package to remove some of these misconceptions of the students, as well as to convincingly demonstrate the various types of geodesics on the various convex surfaces. We have chosen the family of quadrics for the demonstration of geodesics, since they form the class where most of the non-trivial properties of the geodesics can be demonstrated. Besides, the class of quadrics constitute the Eisenhart Coordinate Surfaces [1], which is extremely important from the EM analysis point of view.

Description of the Geodesics

A set of computer codes has been developed in the TURBO BASIC which permits the user to visualize the geodesics in an interactive manner.

The most commonly held notion of a geodesic (as being of the shortest path) corresponds to a primary geodesic. Figure 1 shows a primary geodesic on a right circular cylinder. However, "there is only one geodesic" becomes a questionable proposition, the moment another geodesic - the left primary geodesic appears in the clockwise direction (Fig. 2).

Realizing the geodesics on the developables like the right circular cylinder and cone is relatively easy, since the development of these quadrics maps the geodesics onto the plane as (different) straight lines.

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On the other hand, the geodesics on the nondevelopable surfaces are quite complex. This is demonstrated in Fig. 3, which shows the geodesics on a general paraboloid of revolution (GPOR). In such cases the geodesic differential equation expression in the Geodesic Coordinate System [2], rather than the standard canonical geodesic equation, is used to describe the geodesics. On a surface which is described in two geodesic coordinates u and v, the differential equation of the geodesic takes the form

$$v = \frac{\pm h_{rm} E^{1/2}}{G^{1/2}[G-h_{rm}^2]^{1/2}} \tag{1}$$

where E and G, two of the First Fundamental Coefficients of the set (E,F,G), are functions of u. Implicit in this characterization is the property that the unit principal normal n of the geodesic at any point on the curve is equal to the unit surface normal **N** at that point on the surface on which the geodesic is described. The subscript "rm" refers to the m-th order right geodesic. Similarly, several orders of left geodesics may also exist in general. The "+/-" sign of h_{rm} depends on whether the arc length is a monotonically increasing or decreasing function of the geodesic parameter u as shown in Figs. 4 and 5. In more complex cases, a geodesic may be in parts monotonically increasing as well as decreasing function of the geodesic parameter u of which the right second-order geodesic in Fig. 3 is an example [3]. In such cases the sign is also taken "+/-" depending on the descent/ascent of the geodesic.

Application of the Geodesic Constant Method

The authors have developed a Geodesic Constant Method (GCM) where all the surface ray geometric parameters required in the high frequency calculations [4] are expressed explicitly in terms of the First Geodesic Constant h. This analysis uses a general Geodesic Coordinate System and is applicable to a wide class of quadrics and non-quadric surfaces.

The power of the GCM becomes apparent with the ease of its application to the ogival surfaces, where the geodesics have been hitherto described only by computationally intractable two-parameter numerical methods [5].

Further analysis by the authors [6] has shown that contrary to the popular belief, the number (i.e., order) of geodesics on a circular cone is finite. Finally, the authors have observed a splitting of the geodesics on the GPOR (Fig. 6). The GPOR, being a quadric, is the simplest surface on which this phenomenon is observed [7]. In the ray-theoretic approaches, this implies a doubling of the effort in terms of the ray tracing of all the surface ray parameters, required in the EM field computation.

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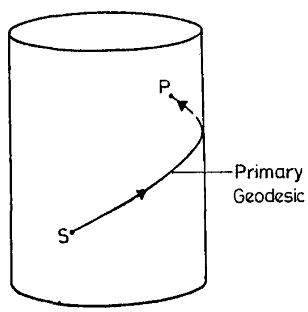
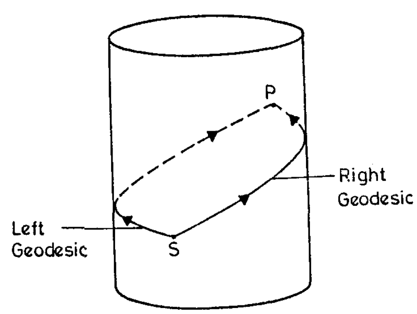


Fig. 1 Primary (or 1st-order) geodesic on a circular cylinder.



. 2 Right and left primary geodesics on a circular cylinder.

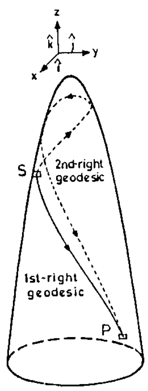


Fig. 3 Right 1st and 2nd-order geodesics on a general paraboloid of revolution (GPOR).

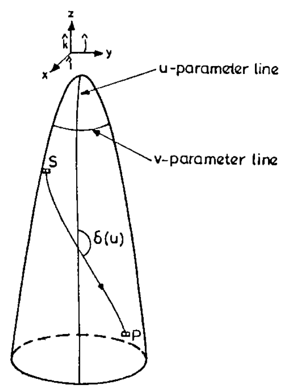


Fig. 4 Right 1st-order geodesic on a GPOR where arc length monotonically increases with u-parameter. Hence h_{rm} is positive.

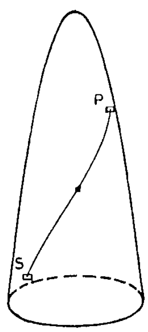


Fig. 5 Right 1st-order geodesic on a GPOR where arc length monotonically decreases with the increase in u-parameter. Hence h_{rm} is negative.

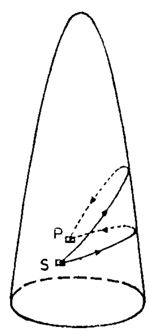


Fig. 6 Splitting of the right primary geodesic on a GPOR, among the simplest surface to show this phenomenon.