VECTOR EXTRAPOLATED FAST ML ALGORITHMS FOR EMISSION TOMOGRAPHY

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ABSTRACT

A new class of fast cyclic iterative algorithms for maximum likelihood estimation of emission densities in ECT is developed. The Approach is based on acceleration of convergence by vector extrapolation. The minimal polynomial and the reduced rank extrapolation techniques are integrated with the iterations in EM and EM search algorithms. The resulting algorithms, tested with simulated phantoms, have shown improved convergence and have given very encouraging results.

METHOD

In this paper we develop a new class of fast cyclic iterative algorithms for emission density estimation based on Convergence acceleration by vector extrapolation. In this approach a finite number of consecutive estimates produced by gradient based algorithms, such as EM or EM search, is considered as a sequence of vectors generated by an operator. These vectors are then extrapolated in the vector space to obtain a new estimate of the emission density. This density vector is then used as the next starting estimate for the gradient based algorithm from which a finite set of iterates are again generated and vector extrapolated to obtain the next emission density estimate. This process of cyclic iteration is continued till acceptable convergence is achieved.

Two methods of vector extrapolation, the Minimal Polynomial Extrapolation (MPE) and the Reduced Rank Extrapolation (RRE), are implemented and tested using simulated phantoms. In MPE, the extrapolated estimate is obtained as a weighted sum of the sequence of estimates generated by the gradient based algorithm, where the weights are related to the coefficients of the minimal polynomial of the operator. In RRE a weighted sum of the correction vectors is used to obtain the extrapolated estimate. These algorithms are briefly outlined here and detailed in [1].

These two approaches, the MPEML and the RREML algorithms, are implemented and tested using simulated phantoms. Fig. 1 shows the loglikelihood functions estimated using the proposed algorithms and compares them with those estimated using the EM and EMS algorithms. Fig. 2 gives the reconstructed images using [MPEML and RREML algorithms with both EM and EMS as base algorithms.

REFERENCE

[1] Rajeevan N., et.al., "Vector extrapolated fast ML algorithms for Emission Tomography," submitted to IEEE Trans. Medical Imaging.

MPEML Algorithm

Initialisation ;
 λ^0 = strictly positive initial estimate
m = degree of minimal polynomial (taken as a small positive integer)
n = 0, cycle index
MPEML
do
{ generate λ^k , k = 1,...,m using gradient based algorithm
compute $\Delta(\lambda^k) = \lambda^{k+1} - \lambda^k$, k = 0,...,m-1
find the coefficient vector, $\underline{c}$, of the minimal polynomial from
 $\underline{c} = -[\Delta]^+ \Delta(\lambda^m)$, $c_m = 1.0$
where $A = [\Delta(\lambda^0) \Delta(\lambda^1) \dots \Delta(\lambda^{m-1})]$
and $[\Delta]^+$, the generalised inverse,
is computed as $[\Delta]^+ = [\Delta^t \Delta]^{-1} \Delta^t$
find weights $w = \underline{c} / (\sum c_i)$
find extrapolated estimate $\lambda^{n,m}$ using
 $\lambda^{n,m} = \sum w_k \lambda^k$
 $= \lambda^{n,m}$; n++ ;
} until convergence.
 λ^0 is the MPEML estimate of the emission density.

RREML Algorithm

Initialisation :
 λ^0 = strictly positive initial estimate.
m = a small positive integer
n = 0, cycle index
RREML :
do
{ generate λ^k , k = 1,...,m using gradient based algorithm
for k = 0,...,m-1 compute
 $\Delta(\lambda^k) = \lambda^{k+1} - \lambda^k$
and $\Delta^2(\lambda^k) = \lambda^{k+2} - \lambda^{k+1} - \Delta(\lambda^k)$
find weights w_k , k = 0,...,m using
 $w = -[\Delta]^+ \lambda^0$; $w_m = 1.0$
where $\Delta = [\Delta^2(\lambda^0) \Delta^2(\lambda^1) \dots \Delta^2(\lambda^{m-1})]$
and $[\Delta]^+$, the generalised inverse,
is computed as $[\Delta]^+ = [\Delta^t \Delta]^{-1} \Delta^t$
find extrapolated estimate $\lambda^{n,m}$ using
 $\lambda^{n,m} = \lambda^0 + \sum w_k \Delta(\lambda^k)$
 $\lambda^0 = \lambda^{n,m}$, n++ ;
} until convergence.
 λ^0 is the RREML estimate of the emission density.

Fig. 1 Plot of loglikelihood vs base iterations.

- a) EM - EM algorithm
 EMMPE 2 - MPE algorithm with two EM iterations per cycle
 EMRRE 2 - RRE algorithm with two EM iterations per cycle

- b) EMS - EM search algorithm
 EMSMPE 2 - MPE algorithm with two EMS iterations per cycle
 EMSRRE 2 - RRE algorithm with two EMS iterations per cycle

Fig. 2 Reconstructed images using MPEML and RREML algorithms.

EM10 : 10 iterations of EM algorithm; EMS10 : 10 iterations of EM Search algorithm; EMMPE 2-3 : 3 cycles of EMMPE with 2 iterations per cycle; EMSRRE 2-2 : 2 cycles of EMSRRE with 2 iterations per cycle; EMMPE 2-3 : 3 cycles of EMMPE with 2 iterations per cycle; EMSMPE 2-2 : 2 cycles of EMSMPE with 2 iterations per cycle.