PARAMETRIC DESCRIPTION OF SPECTRAL DENSITY OF ECG

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Abstract - A parametric description of the spectral density of the ECG signal using its Discrete Cosine Transform is proposed. This transformation enables modeling of the ECG efficiently using lower order polynomials. The Steiglitz-McBride, Shanks and the Maxim Likelihood Estimator algorithms are used for modeling. Certain parameters obtained from the spectrum are used to determine an estimate of the autocorrelation function that takes the form of a sum of exponentials and damped sinusoids. Several normal and abnormal ECGs have been analyzed with highly satisfactory results. An abnormal beat required more parameters for its description than a normal.

INTRODUCTION

Of the several ways of representing biosignals, spectral characterization is commonly used. Zetterberg [1,2] employed a rational function in the frequency variable f^2 of the type $Q(f^2)/P(f^2)$, where $Q(x)$ and $P(x)$ are polynomials of low order, to study EEG modeled as an ARMA process with white noise input, under conditions of stationarity. In this paper we study the ECG signal considered as the impulse response of a linear time invariant system. Modelin the direct ECG signal requires orders of (20,207 or higher (30,30) [3]. Instead we model the Discrete Cosine Transform (DCT) coefficients of the time domain signal. A (2,2) order was sufficient to model the transform of the component waves while a (6,6) order was required for the transform of the complete ECG cycle. The auto correlation function (ACF) is characterized by a set of parameters, the resonant frequency, the half power bandwidth and the power content at the resonant frequency. While the first two parameters are obtained from the spectrum, the third one is evaluated from the model coefficients.

METHOD

The transformed signal is modeled as a pole-zero process, whose system function

$H(z) = [B(z^{-1})/A(z^{-1})]$ is to be determined

Define
$$B(z^{-1}) = b_o + b_1 z^{-1} + \dots b_q z^{-q} \qquad (1)$$
$$A(z^{-1}) = a_o + a_1 z^{-1} + \dots + a_p z^{-p} \qquad q \le p.$$

From eqns. (1) and (2) the power spectral density (PSD) is given by

$$\sigma^2 \frac{B(z^{-1})\, B(z)}{A(z^{-1})\, A(z)}$$

The ACF is obtained by taking the inverse fourier transform of the PSD.

Thus
$$r_k = \frac{\sigma^2}{2\pi j} \oint_{|z|=1} \frac{B(z^{-1})\, B(z)}{A(z^{-1})\, A(z)} z^{k-1}\, dz. \qquad (3)$$

With the notation

$$A(z) = a_o + a_1 z + a_2 z^2 + \dots + a_p z^p,$$
$$A_1(z) = z^p A(z^{-1}),\ A_2(z) = [dA_1(z)/dz],$$
$$A_3(z) = z^{p-1} A_2(z^{-1}),\ B_1(z) = z^q B(z^{-1}),$$

(3) can be rewritten as

$$r_k = \frac{\sigma^2}{2\pi j} \oint_{|z|=1} \frac{B_1(z)\, B(z)}{A_1(z)\, A(z)} z^{k+p-q-1} \qquad (4)$$

Using the residue technique we evaluate the integral at the poles of the system function to yield

$$r_k = \sigma^2 \sum_{i=1}^{p} \frac{B_1(z_i)\, B(z_i)}{A_2(z_i)\, A(z_i)} z_i^{k+p-q-1}$$
$$= \sigma^2 \sum_{i=1}^{p} \frac{B(z_i^{-1})\, B(z_i)}{A_3(z_i^{-1})\, A(z_i)} z_i^{k} \qquad (5)$$

For the p1 real poles $z_i = \exp(-\alpha_i)$ we get

$$r_k = \sigma^2 \sum_{i=1}^{p_1} \frac{B(z_i^{-1})\, B(z_i)}{A_3(z_i^{-1})\, A(z_i)} \exp(-k\alpha_i) \qquad (6)$$
$$= \sum_{i=1}^{p_1} F_i \exp(-k\alpha_i)$$

where F_i is the power content at DC given by

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$$F_i = \sigma^2 \frac{B(z_i^{-1})\, B(z_i)}{A_3(z_i^{-1})\, A(z_i)}$$

For the p2 complex poles $z_i = \exp(-\beta_i \pm j\, w_i)$ we evaluate r_k as follows:

Denote

$$B(z_i) = B_x^- \pm jB_y^-, \quad B(z_i^{-1}) = B^+ \pm jB_y^+,$$

$$A(z_i) = A_x^- \pm jA_y^-, \quad A_3(z_i^{-1}) = A_{3x}^+ \pm j\, A_{3y}^+$$

Substituting in (5) yields

$$r_k = \sigma^2 \sum_{i=1}^{p_2} \frac{(B_x^- \pm jB_y^-)(B_x^+ \pm jB_y^+)}{(A_x^- \pm jA_y^-)(A_{3x}^+ \pm jA_{3y}^+)} \exp(-k\beta_i \pm kjw_i)$$

$$= \sum_{i=1}^{p_2} \exp(-k\alpha_i)\, G_i \cos(kw_i) \qquad (8)$$

where $G_i = 2\sigma^2 \dfrac{(B_x^- \pm jB_y^-)\,(B_x^+ \pm jB_y^+)}{(A_x^- \pm jA_y^-)\,(A_{3x}^+ \pm jA_{3y}^+)}$

Finally when $q = p$, we get an additional term

$$E = \frac{b_p\, b_o}{a_p\, a_o}$$

Hence the expression for the ACF takes the form

$$r_k = E\delta_k + \sum_{i=1}^{p_1} F_i \exp(-k\alpha_i) + \sum_{i=1}^{p_2} G_i \exp(-k\beta_i)\cos(kw_i) \qquad (9)$$

where $\delta_k = 1$ if $q = p$
$= 0$ otherwise.

RESULTS

Both normal and abnormal ECGs, sampled at 500 Hz, were used for analysis. Of the three algorithms, the maximum likelihood estimator gave the best spectral fit in terms of the normalized root mean square error (NRMSE), but had the highest time complexity. Steiglitz-McBride and Shanks algorithms gave similar results. For purposes of illustration we have shown the results obtained on analyzing a normal beat and its component waves, after modeling them using the Steiglitz-McBride algorithm.

Figure 1(a) shows the complete ECG cycle. Fig. 1(b) gives the power spectral density (PSD) evaluated from the ACFs, obtained directly from the transformed signal and evaluated using Eq. (9). Figs. 2(a) - 2(b), 3(a) - 3(b) and 4(a) - 4(b) give the corresponding plots for the component waves.

CONCLUSIONS

In [1], a parameter H_i, defined as the skewness of the spectral density about the resonant frequency, is used for characterization. Our analysis shows that this parameter is not necessary. The ACF is thus a function of only four parameters σ_i, w_i, G_i and F_i. Using these parameters we have not only been able to reconstruct our signal but also to simulate a whole class of ECG signals.

REFERENCES

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