

**SPLIT PHASE INDUCTION MOTOR OPERATION FROM PWM  
VOLTAGE SOURCE INVERTER**

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**Abstract:** Split-phase AC motors, with 30 deg. separation between the two groups of stator windings are used in conjunction with current source inverters, in order to eliminate sixth harmonic torque pulsations. The present paper examines the operation of split-phase induction motors from pulse width modulated (PWM) voltage source inverters. Splitting the phase windings leads to reduced voltage ratings for the inverter switches. The inverters are operated with space phasor based PWM. It is well known that with this technique, a three phase inverter can give a maximum peak fundamental of  $0.577 V_{DC}$  for the motor phase voltage (with a circular trajectory for the voltage space phasor), as against  $0.5 V_{DC}$  with sine triangle modulation. In the case of split-phase machines operated from two inverters, it can be shown that the equivalent 3 phase maximum peak fundamental can be  $0.622 V_{DC}$  with the DC bus voltage being half the value  $V_{DC}$  for the conventional 3 phase configuration. If the DC bus voltage is given a slight boost (in order to keep the resultant voltage space phasor magnitude, the same for the split-phase configuration as compared to the 3 phase configuration), and made  $(0.5 V_{DC}) / \cos 15^\circ$ , the maximum peak fundamental motor voltage obtainable is  $0.643 V_{DC}$ , which is more than what is obtainable by resorting to six-step operation in the three phase motor. Further, as compared to seven positions for the space phasor of voltage in three phase machines, forty eight different space phasor locations are possible in the split-phase machine. The boundary of the space phasors is now a 12 sided polygon. By switching between locations within the polygon, it is possible to generate reduced space phasor amplitudes with lower harmonics than in the three phase machines.

**1. Introduction**

Current source inverter drives using split-phase induction motors are used in industrial applications in order to eliminate sixth harmonic torque pulsations [1,2,3]. The sixth harmonic torque pulsations are produced mainly due to the interaction between the fundamental flux and the 5th and 7th harmonic rotor currents. In the split-phase Induction motor configuration, the sixth harmonic torque pulsations produced by the two split-phase groups of coils are in opposition [1], they get cancelled and the resultant torque pulsation frequency is twelve times that of the fundamental, when fed from a CSI [1]. In this paper a split-phase motor drive (fig.1) with voltage source inverters, using voltage space phasor based PWM technique is examined. The split-phase motor

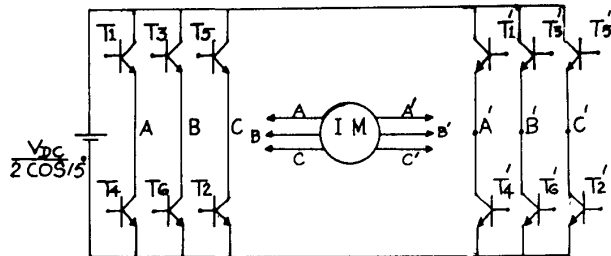


Fig.1. Split-phase induction motor drive configuration is achieved by splitting the phase belt of a conventional three phase motor into two equal halves with a phase separation of 30 deg. between the two (fig.2). The split-phase groups namely ABC and A'B'C' (fig.2) are controlled by two inverters with a DC link voltage of  $V_{DC} / (2 \cos 15^\circ)$  each (fig.1). Since the split-phase configuration is achieved by splitting the phase-belts into two, the equivalent number of turns per phase for the new configuration is  $N_s / 2 \cos 15^\circ$  ( $N_s$  - equivalent number of turns per phase for 3 phase motor configuration), and correspondingly a link voltage of  $V_{DC} / 2 \cos 15^\circ$  gives the same magnitude for the air-gap flux, in the new configuration as compared to the three phase inverter drive with a DC link voltage  $V_{DC}$ .

For the proposed scheme (Fig.1), the voltage space phasor locations for the split-phase group of coils namely ABC and A'B'C' are shown in fig. 3. The voltage space

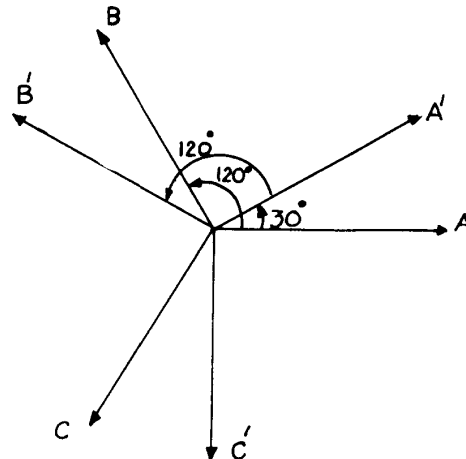


Fig.2. Phasor diagram - Split phase motor

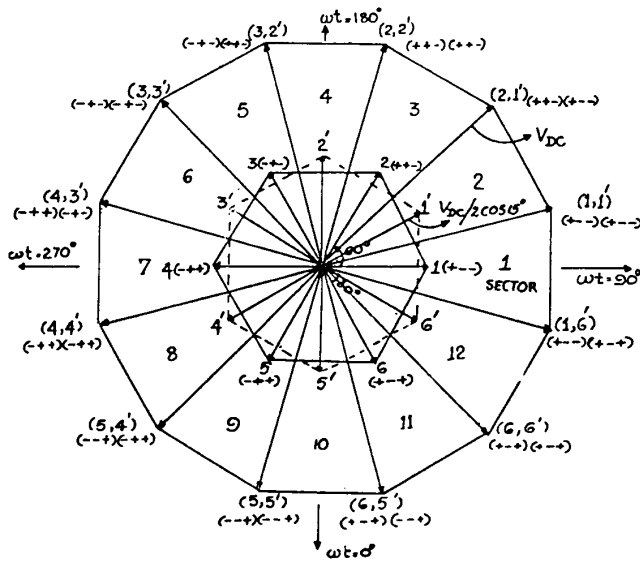


Fig.3(a) Voltage Space Phasor Locations for split phase induction motor drive

phasor locations for the A'B'C' phase group are phase shifted by 30 deg. from those of the ABC phase group. For the same switching state, the sequence 1',2',3',4',5',6' is that of the A'B'C' phases and the sequence 1,2,3,4,5,6 is that of the ABC phases (fig.3) [4,5]. A switching state of (+--) for the ABC phases implies that the top switch of A-phase and bottom switches of B and C phases are on [4,5]. The switching states for the six sequences of inverter for a quasi-square wave operation are shown in Fig.3.

With a three phase inverter, the boundary of the voltage space phasors is a hexagon (fig.3)[4,5]. With a split-phase configuration, by using appropriate switching states for the two inverters a 12-sided polygon forms the boundary as shown in fig.3. Therefore, a given voltage space phasor may be in one of the 12 possible sectors as shown in fig.3.

**SPACE PHASOR PWM GENERATION FOR SPLIT PHASE INDUCTION MOTOR**

A rotating voltage space phasor with a

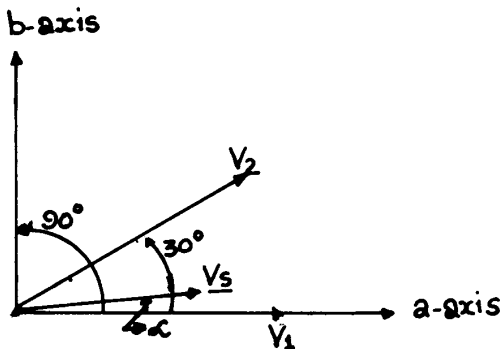


Fig.4. Sampled reference vector in a sector

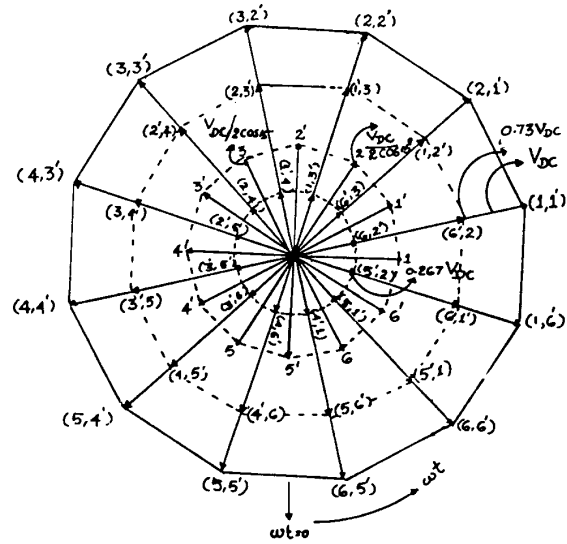


Fig.3(b) Interior space phasor combinations

constant amplitude along a circular trajectory implies a sinusoidal excitation for the motor. A circular trajectory can be approximated by switching between any of the appropriate vectors in a sector (fig.3), for a 3 phase motor [4,5]. For the split-phase motor configuration, as in the three phase inverter, a rotating reference voltage space phasor is sampled at regular intervals[4,5]. For example, let the reference vector  $V_s$  be in sector 1 (fig.4). Let the state (+--)(+--) which defines the start of the sector be represented by  $V_1$  and the state (+--)(+--), which defines the end of the sector be represented by  $V_2$ .

TABLE.1

SECTOR	INVERTER-1			INVERTER-2				
	O	V <sub>1</sub>	V <sub>2</sub>	O	O	V <sub>1</sub>	V <sub>2</sub>	O
1	+++	+--	+--	---	+++	+--	+--	---
2	---	+--	++-	+++	---	+--	+--	+++
3	---	++-	++-	+++	---	+--	++-	+++
4	+++	++-	-+-	---	+++	++-	++-	---
5	+++	-+-	-+-	---	+++	++-	-+-	---
6	---	-+-	++-	+++	---	-+-	-+-	+++
7	---	-+-	++-	+++	---	-+-	++-	+++
8	+++	-+-	---	---	+++	-+-	-+-	---
9	+++	---	---	---	+++	-+-	---	---
10	---	---	+--	+++	---	---	+--	+++
11	---	+--	+--	+++	---	---	+--	+++
12	+++	+--	+--	---	+++	+--	+--	---

O-ZERO VECTOR STATE

During a sampling interval  $T_s$ , let the reference vector  $V_s$  make an angle  $\alpha$  with respect to  $V$  (fig.4). The stationary vector  $V_s$  during a sampling interval  $T_s$  can be generated by switching between  $V_1$  and the zero vector - 0 (table-1), for periods  $T_1$ ,  $T_2$  and  $T_0$ , such that the volt-seconds produced by the  $V_1$  and  $V_2$  states along the axis-a and axis - b (fig.4) will be equal to that of the volt-seconds produced by the  $V_s$  vector along 'a' and 'b' axes [3,4].

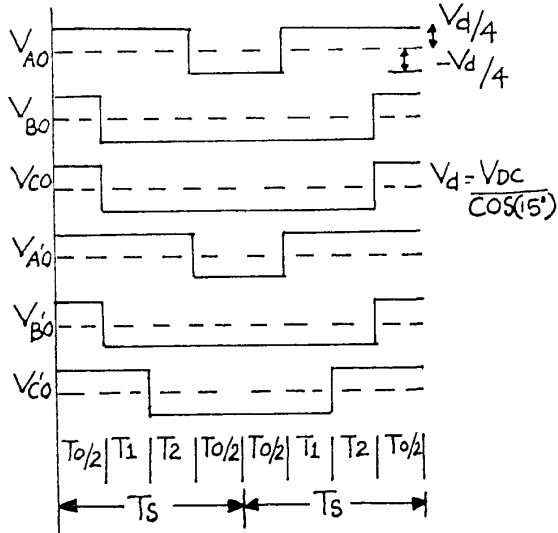


Fig.5. Pole voltage waveform for an inverter switching period in sector -1.

The switching states appropriate for all the sectors are shown in Table-1. The zero vector period  $T_0$  is divided equally between the two zero states (+++) and (---), and is located at the start and end of a sampling period  $T_s$  [4]. It can be shown that

$$\begin{aligned} T_1 &= 2KT_s \sin(30-\alpha) \\ T_2 &= 2KT_s \sin\alpha \\ T_0 &= T_s - (T_1+T_2) \end{aligned} \quad (1)$$

where  $K = V_s/V_{DC}$   
 $T_s =$  Sampling period

### 3. INVERTER POLE VOLTAGE WAVEFORM

The inverter pole voltage waveforms for an inverter switching period in sector-1 are

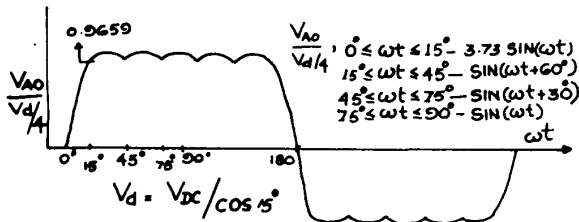


Fig.6. Average pole voltage variation

shown in fig.5. The average value of the pole voltage  $V_{A0}$  in sector-1 can be determined from eqn.2

$$V_{A0} \text{ average} = [T_0/2 + T_1 + T_2 - T_0/2] \times V_d / T_s \quad (2)$$

where  $V_d = V_{DC} / \cos 15$

substituting for  $T_1, T_2$  and  $T_0$  from eqn.(1)

$$V_{A0} \text{ average} = (2KV_d/4) \times 0.5176 \sin(\alpha+75) \quad (3)$$

Similarly the average variation of the pole voltages for other phases can also be determined.

$$V_{B0} \text{ average} = -(2KV_d/4) \times \sin(\alpha+75) \quad (4)$$

$$V_{C0} \text{ average} = -(2KV_d/4) \times \sin(\alpha+75) \quad (5)$$

$$V'_{A0} \text{ average} = (2KV_d/4) \times \sin(\alpha+75) \quad (6)$$

$$V'_{B0} \text{ average} = -(2KV_d/4) \times \sin(\alpha+75) \quad (7)$$

$$V'_{C0} \text{ average} = -(2KV_d/4) \times 1.932 \sin(\alpha-15) \quad (8)$$

By considering the average pole voltage variation in the other sectors also, the average pole voltage variations for a cycle can be computed, and is shown in fig.6, for the maximum reference space phasor, with a circular trajectory. This method of pulse width modulation can therefore be regarded as equivalent to a carrier based modulation technique where the reference waveform has the shape shown in fig.6. For a maximum reference space phasor with a circular trajectory (fig.3), the value of  $K$  is  $V_s/V_{DC} = \cos 15 = 0.9659$ . For lower values of  $K$  the waveform is preserved except for the corresponding reduction in amplitude.

### 4. HARMONIC ANALYSIS OF THE AVERAGE POLE VOLTAGE WAVEFORM

The harmonic analysis of the pole voltage for the maximum reference space phasor with a circular trajectory waveform can be obtained from fig.6. The equation for the fundamental component is

$$\begin{aligned} A_1 = & (V_d/4) (4/\pi) \left[ \int_0^{15} 3.732 \times \sin^2 \omega t \, d(\omega t) \right. \\ & + \int_{15}^{45} \sin(\omega t+60) \times \sin \omega t \, d(\omega t) + \int_{45}^{75} \sin(\omega t+30) \times \\ & \left. \sin \omega t \, d(\omega t) + \int_{75}^{90} \sin \omega t \, d(\omega t) \right] = 0.3109 V_d \end{aligned}$$

$$\text{where } V_d = V_{DC} / \cos 15 \quad (10)$$

Now the fundamental components of the 12 pole voltages are transformed to two orthogonal axes, the resultant voltage space phasor for the fundamental component can be obtained. Assuming one axis, along the A-phase coil, the resultant along that axis is [1]

$$\begin{aligned} V_d * 0.3109 [ & \cos \omega t - \cos(\omega t-120)/2 - \\ & \cos(\omega t+120)/2 + (\sqrt{3}/2) \cos(\omega t-30) - \\ & (\sqrt{3}/2) \cos(\omega t-150) ] = 0.933 V_d \cos \omega t \quad (11) \end{aligned}$$

Now, substituting for  $V_d$ , the fundamental component for the line to neutral voltage for an equivalent 3 phase system can be calculated, and is equal to [4,5]

$$(2/3) 0.933 V_d \cos \omega t = 0.64 V_{DC} \cos \omega t \quad (12)$$

The other higher order harmonics of the average pole voltage waveform can be determined from eqn.(13).

From eqn.(13) the higher order harmonics can be determined and the order of the harmonics present are shown in table-2. From table-2 it can be seen that the harmonics of the order 11,13,23,25,35,37 etc are absent in the average pole voltage

waveform. Therefore the only harmonics that

$$\begin{aligned}
 A_n = & 3.732 \frac{V_d}{\pi} \left[ \frac{-\sin((n+1)\pi/12)}{2(n+1)} + \frac{\sin((n-1)\pi/12)}{2(n-1)} \right] \\
 & + \frac{V_d}{\pi} \left[ \frac{\sin((n+1)\pi/12+60)}{2(n+1)} - \frac{\sin((n-1)\pi/12+60)}{2(n-1)} \right] \\
 & + \frac{1}{2(n-1)} \left[ \frac{\sin((n-1)\pi/4-60)}{2(n-1)} - \frac{\sin((n-1)\pi/12-60)}{2(n-1)} \right] \\
 & + \frac{V_d}{\pi} \left[ \frac{\sin((n+1)\pi/4+30)}{2(n+1)} - \frac{\sin((n+1)5\pi/12+30)}{2(n+1)} \right] \\
 & + \frac{\sin((n-1)9\pi/12-30)}{2(n-1)} - \frac{\sin((n-1)\pi/4-30)}{2(n-1)} \\
 & + \frac{V_d}{\pi} \left[ \frac{\sin(9\pi/12(n+1))}{2(n+1)} - \frac{\sin(\pi/2(n+1))}{2(n+1)} + \frac{\sin((n-1)\pi/2)}{2(n-1)} \right. \\
 & \left. - \frac{\sin((n-1)5\pi/12)}{2(n-1)} \right] \dots (13)
 \end{aligned}$$

will be present in the motor phase voltages will be of the order 5,7,17,19,29,31 etc. The absence of the first group (11,13 etc..) of harmonics means that torque pulsations of order 12,24,36 etc. will be absent. The winding disposition ensures that torque pulsations of order 6,18,30 etc will also be absent [1,3]. Thus with the

Order of harmonics - average Pole voltage - Wave form										
5	7	17	19	29	31	41	43	53	55	65

Table - 2

proposed scheme all low order torque pulsations are eliminated and also the proposed scheme offers an equivalent 3 phase fundamental line to neutral voltage of 0.643 V<sub>dc</sub> (eqn.12), which is greater than that for a quasi-square wave operation (0.637 V<sub>dc</sub>) while still being in modulation.

### 5. THE PWM PATTERN GENERATION

The simple V/f control scheme is used for motor drive. The reference signal for the V<sub>s</sub> and the motor output frequency is derived from the speed reference signal. The amplitude of voltage space phasor V<sub>s</sub> is quantized into 64 states by using the lower 6 bit output of an A/D converter. In each sector the 'α' positions are divided into 16 parts by using the lower 4 bits of a 8 bit counter, where the upper 4 bits are used for the sector information. A 5 bit up down counter working at 64 KHz is used for dividing a sampling interval into 32 parts. These 15 bits are used for addressing a 32 Kbyte EPROM in which the interval for T<sub>1</sub>, T<sub>2</sub> and T<sub>0</sub> are stored. The 15 bit address for the EPROM are as shown in fig.7.

The values of T<sub>0</sub>, T<sub>1</sub> and T<sub>2</sub> are computed for different 'α' positions in a sector for different V<sub>s</sub> amplitudes and a stream of bit pattern '00' (for the first T<sub>0</sub>/2 period) '01' (for the T<sub>1</sub> period), '10' (for the T<sub>2</sub>

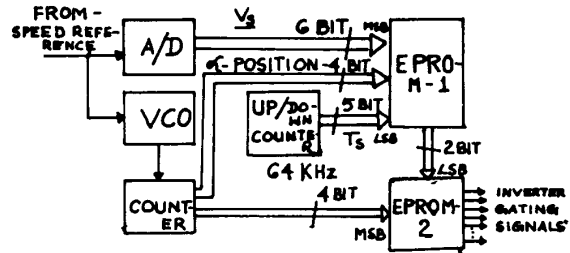


Fig.7. PWM pattern generation

period) and '11' (for the last T<sub>0</sub>/2 period) (table-1) are stored in the first EPROM for different values of V<sub>s</sub> and 'α'. The two bit pattern from the first EPROM and the higher 4-bit of the 8 bit counter for sector information are used for addressing a second EPROM in which switching patterns for various sectors are stored. The output of the second EPROM is used to drive a transistorised inverter, for which the inverter switching frequency is 1KHz.

### 6. EXPERIMENTAL RESULTS

A 2.5 Kw split-phase induction motor is tested with a dual voltage source inverter (fig.1) using space phasor based PWM technique. The 12 sided polygon with its 12 vectors, derived from the control signals are shown in fig.8 and the resultant circular trajectory for the reference space phasor is shown in fig.9. The motor phase current and its harmonic spectrum are shown in fig.10, for no load operation. The harmonic spectrum shows the fundamental component along with the 5th and 7th harmonic with reduced higher order harmonics. Since the motor is running at no load, the fundamental component is only the magnetising component. Due to the split phase configuration [1][3][4], the 5th and 7th component will not produce any air-gap flux, the impedance for the harmonics are due to the stator impedance only. Hence the higher amplitude for these components as



Fig.8. Twelve sided polygon for the voltage space phasor - split phase motor

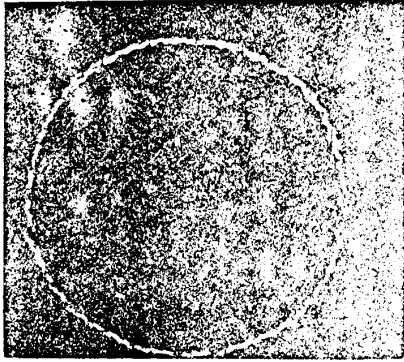


Fig.9. Circular trajectory of the average reference space phasor voltage

compared to the fundamental, at no load operation (fig.10). fig.11 shows the line to neutral waveform and its harmonic spectrum. Fig.12 and fig.13 shows the line to neutral voltage waveform of A phase and A' phase and line to line voltage waveform of AB and A'B' phases respectively. Motor line to neutral waveform and the phase current waveform shows that all the harmonic components of the order 11,13,23,25,35,37 etc. are absent in the current waveform and the harmonics of the order 5,7,17,19 etc. will not contribute anything to the air-gap flux. Hence the torque produced will be due to the fundamental component only. Thus with this proposed scheme an equivalent 3 phase fundamental voltage of  $0.643 V_{DC}$ , which is more than a quasi square wave operation ( $0.637 V_{DC}$ ) can be obtained, while still being in modulation.

MOTOR PHASE CURRENT SPECTRUM  
91/04/22 16:56 BAND:2 KHz

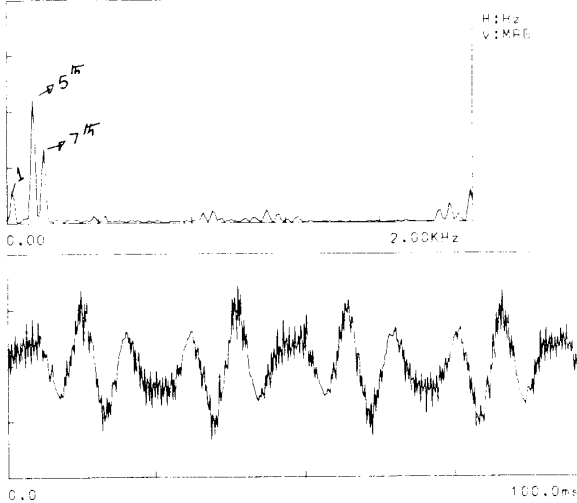


Fig.10. Motor phase current and its harmonic spectrum

LINE-NEUTRAL VOLTAGE SPECTRUM  
86/01/01 00:01 BAND:2 KHz

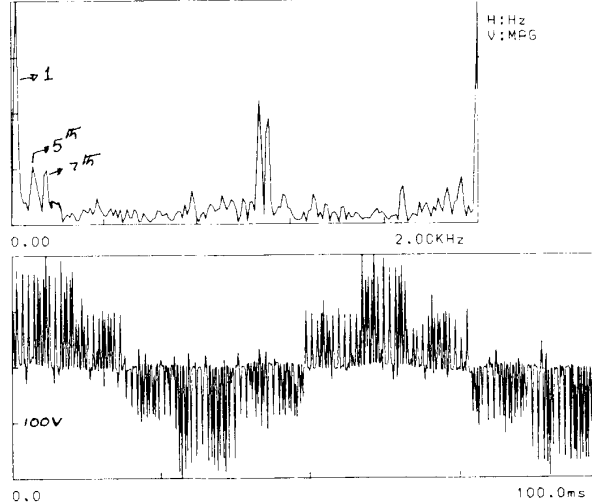


Fig.11. Line to neutral voltage waveform and its harmonic spectrum

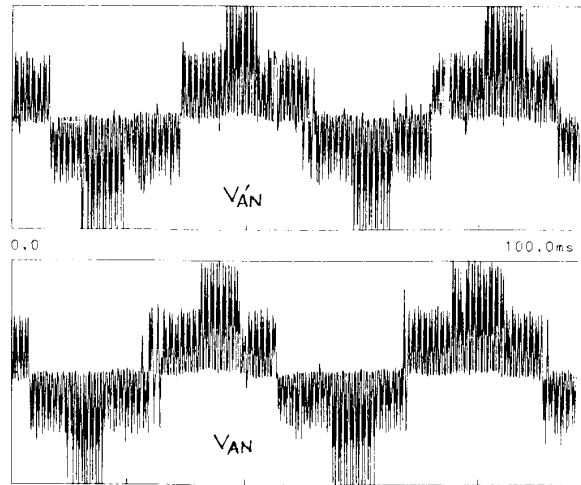


Fig.12. Line to neutral voltage

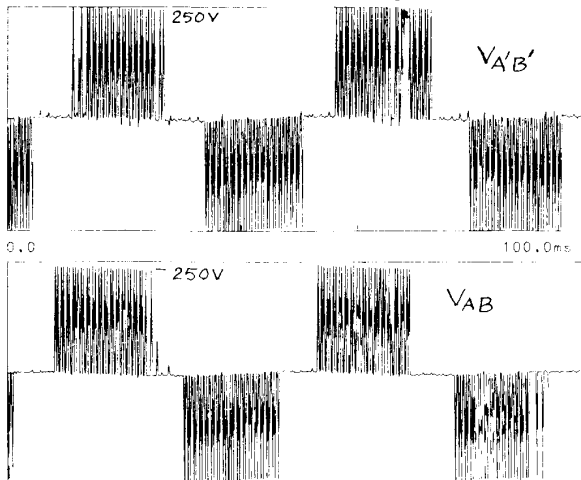


Fig.13. Line to line voltage

## 7. CONCLUSION

Operation of split phase induction motors from pulse width modulated voltage source inverter has been examined. The salient conclusions that emerged from the study are

1. There are 48 possible locations for the resultant voltage space vector as compared to 8 in the conventional 3 phase configuration.
2. The boundary of the voltage space phasor locations is a 12 sided polygon as compared to a hexagon for the 3 phase systems
3. The radius of the largest circular trajectory for the voltage space phasor is  $V_{DC} \times \cos 15 = 0.9659 V_{DC}$  as compared to  $V_{DC} \times \cos 30 = 0.866 V_{DC}$  for the 3 phase system. In terms of the fundamental line to neutral voltage of the equivalent 3 phase machine, this translates to  $0.643 V_{DC}$  for the split phase arrangement as compared to  $0.577 V_{DC}$  for the 3 phase configuration.
4. The average inverter pole voltage waveform contains 5th and 7th harmonic components. These harmonic components do not contribute to the resultant air gap flux, because the corresponding stator MMF's get cancelled due to the winding disposition.
5. Reference vectors of smaller amplitudes (for lower speed operations) can be generated with reduced harmonic contents by switching between the vertices of the inner polygons. The polygon which is closest to the desired trajectory should be selected to give the lowest harmonic content.

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