CLOSED FORM EXPRESSIONS FOR INTEGRAL RAY GEOMETRIC PARAMETERS FOR WAVE PROPAGATION ON GENERAL QUADRIC CYLINDERS

RM Jha, SA Bokhari, V Sudhakar and PR Mahapatra
Department of Aerospace Engineering
Indian Institute of Science
Bangalore, 560 012 India

Summary

A major obstacle in the application of the powerful ray-theoretic techniques like the UTD [1] to antenna analysis in presence of a general convex surfaces is the difficulties in obtaining the surface ray geometric parameters. While the differential ray geometric parameters (e.g. Frenet-frame field vectors, the various radii of curvature, torsion) can be easily obtained once the parametric form of the surface is known, the integral ray parameters are difficult to derive in general.

The integral ray geometric parameters consist of
(i) the relation between the geodesic coordinates,
(ii) the arc length
and (iii) the generalized Fock parameter.

The difficulty in obtaining them analytically, has often led to numerical techniques. In this paper, we present these integral ray geometric parameters for the quadric cylinders (QUACYLs) in the closed form. The QUACYLs consist of the right circular, elliptic, general parabolic and hyperbolic cylinders. The rectangular hyperbolic cylinder, which is a special case of the general hyperbolic cylinder, has also been included due to its frequent use in surface modeling.

The Geodesic Constant Method (GCM) employed by the authors yields these analytical expressions in the one-parameter form, i.e., in terms of the First Geodesic Constant *h*. Since *h* can be obtained in closed form, in terms of the source and observation point coordinates, all the surface ray parameters derived here are in the closed form [2].

The ray geometric parameters derived here can be applied to the ray-theoretic determination of mutual coupling, radiation pattern of the antennas in presence of large scatterers, and the monostatic and bistatic radar cross section of scatterers.

REFERENCES

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[2] R.M. Jha, **Surface Ray Tracing on Convex Quadrics with Applications to Mutual Coupling between Antennas on Aerospace Bodies**, Ph D Dissertation. Submitted to Department of Aerospace Engineering, Indian Institute of Science, Bangalore, India, Nov. 1988.

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TABLE 1

PARAMETRIC EQUATION FOR QUADRIC CYLINDERS IN GEODESIC COORDINATE SYSTEM

Coordinate surface	Parabolic Cylinder	Rectangular Hyperbolic Cylinder	General Hyperbolic Cylinder	Elliptic Cylinder	Circular Cylinder
Of coordinate system	Parabolic Cylindrical (u,w,z)	Elliptic Cylindrical (n,ϕ,z)	Elliptic Cylindrical (n,ϕ,z)	Elliptic Cylindrical (n,ϕ,z)	Circular Cylindrical (r,ϕ,z)
For parameter = const.	w	ϕ	ϕ	n	r
Ray Parameters					
1. Parametric equation $r = (r_x, r_y, r_z)$					
r_x	au	a cosh n	a cosh n	a cos ϕ	a cos ϕ
r_y	u^2	a sinh n	c sinh n	c sin ϕ	a sin ϕ
r_z	z	z	z	z	z
2. Shaping parameter	a	a	a,c	a,c	a
3. Geodesic coordinates (u,v)	u = u v = z	u = n v = z	u = n v = z	u = ϕ v = z	u = ϕ v = z

TABLE 2

INTEGRAL SURFACE RAY GEOMETRIC PARAMETERS FOR QUADRIC CYLINDERS AND

1. Relation between the geodesic coordinates on the geodesic $z = z(u,h,h')$

General Parabolic Cylinder	$\frac{h}{(1-h^2)^{1/2}}\left[\frac{u(a^2+4u^2)^{1/2}}{2} + \frac{a^2}{4}\ln[2u+(a^2+4u^2)^{1/2}]\right] + h'$	
Rectangular Hyperbolic Cylinder	$\frac{h\,a}{(1-h^2)^{1/2}}\left[\frac{1}{2^{1/2}}[F(r,k)-2E(r,k)] + \frac{\sinh 2u}{(\cosh^2 u)^{1/2}}\right] + h'$	$k = \frac{1}{2^{1/2}}$, $r = \sin^{-1}\left[\frac{(\cosh 2u) - 1}{\cosh 2u}\right]$
General Hyperbolic Cylinder	$\frac{h}{(1-h^2)^{1/2}}\left[c[F(r,k)-E(r,k)] + (\tanh u)(a^2\sinh^2 u+c^2\cosh^2)^{1/2}\right] + h'$	$k = \frac{a}{c}$, $r = \sin^{-1}(\tanh u)$
Elliptic Cylinder	$\frac{h}{(1-h^2)^{1/2}}\left[c\ E(u,k)\right] + h'$	$k = \frac{(c^2-a^2)^{1/2}}{c}$
Circular Cylinder	$\frac{h}{(1-h^2)^{1/2}}\left[au\right] + h'$	

2. Arc length $s(u)$

For all QUACYLs $$s(u) = \frac{1}{h}\left[z - h'\right]$$

3. Generalized Fock parameter $\xi(u)$

General Parabolic Cylinder	$(0.5\varkappa a^2)^{1/3}(1-h^2)^{1/6}\Big[\ln[2u+(a^2+4u^2)^{1/2}]\Big]_{u=u_s}^{u=u_f}$	
Rectangular Hyperbolic Cylinder	$\frac{(\varkappa a)^{1/3}(1-h^2)^{1/6}}{2^{1/2}}\Big[F(r,k)\Big]_{u=u_s}^{u=u_f}$	$k=\frac{1}{2^{1/2}}$, $r=\sin^{-1}\left[\frac{(\cosh 2u)-1}{\cosh 2u}\right]$
General Hyperbolic Cylinder	$\frac{(\varkappa a^2c^2)^{1/3}(1-h^2)^{1/6}}{(a^2+c^2)^{1/2}}\Big[F(r,k)\Big]_{u=u_s}^{u=u_f}$	$k=\frac{a}{(a^2+c^2)^{1/2}}$, $r=\sin^{-1}\left[\frac{1}{\cosh u}\right]$
Elliptic Cylinder	$\frac{\varkappa^{1/3}a^{2/3}(1-h^2)^{1/6}}{c^{1/3}}\Big[F(u,k)\Big]_{u=u_s}^{u=u_f}$	$k=\frac{(c^2-a^2)^{1/2}}{c}$
Circular Cylinder	$\varkappa^{1/3}a^{1/3}(1-h^2)^{1/6}\Big[u\Big]_{u=u_s}^{u=u_f}$	