**CLOSED FORM EXPRESSIONS FOR INTEGRAL RAY GEOMETRIC
PARAMETERS FOR WAVE PROPAGATION ON GENERAL
QUADRIC SURFACES OF REVOLUTION**

RM Jha, SA Bokhari, V Sudhakar and PR Mahapatra
Department of Aerospace Engineering
Indian Institute of Science
Bangalore, 560 012 India

Summary

The quadric surfaces of revolution (QUASORs) consisting of right circular cone, sphere, ellipsoid of revolution and general paraboloid and hyperboloid of revolution, which are coordinate surfaces of the Eisenhart coordinate systems, form a versatile class of surfaces from the surface modeling point of view. (The right circular cylinder has been treated elsewhere by the authors as a quadric cylinder). The Eisenhart coordinate system permits a simple separation of the scalar and vector potentials in the field equations.

The ray-theoretic formulation developed by Pathak et al [1] for the determination of the radiation characteristics of the antennas in the vicinity of azimuth-symmetric quadrics is hindered by the fact that the integral surface ray geometric parameters are not readily available. This difficulty mainly arises due to the fact that, unlike the quadric cylinders, the QUASORs in general are non-developable.

We summarily present a table of these ray parameters consisting of
(i) the relation between the geodesic coordinates v and u ,
(ii) the arc length
and (iii) the generalized Fock parameter for the complete class of
the QUASORs.

A Geodesic Constant Method (GCM) developed by the authors permits the derivation of these ray parameters in terms of the geodesic constant h alone. Since h can be expressed in terms of the source and observation point coordinates in the case of sphere and cone, in these cases the ray parameters are in the closed form. On the other hand, in the case of the ellipsoid of revolution and the general paraboloid and hyperboloid of revolution (GPOR and GHOR), h can be obtained using a simple univariate search. Hence in these cases, the ray parameters are in a one-parameter dependent form [2]. Using this approach, it is possible to readily calculate the various radiation characteristics of the antenna in vicinity of a general QUASOR.

REFERENCES

- [1] P.H. Pathak and N. Wang, "Ray analysis of mutual coupling between antennas on a convex surface", IEEE Trans. Antennas & Propagat. (USA), vol. AP-29, no. 6, pp. 911-922, Nov. 1981.
- [2] R.M. Jha, **Surface Ray Tracing on Convex Quadrics with Applications to Mutual Coupling between Antennas on Aerospace Bodies**, Ph D Dissertation. Submitted to Department of Aerospace Engineering, Indian Institute of Science, Bangalore, India, Nov. 1988.

TABLE 1

**INTEGRAL SURFACE RAY GEOMETRIC PARAMETERS OBTAINED
FOR QUADRIC SURFACES OF REVOLUTION**

1. Relation between the geodesic coordinates
u and v on the geodesic

$$v = v(u, h) + h'$$

Geodesic coordinates correspondence
in individual Eisenhart Coordinates

General Paraboloid of Revolution	$v = \frac{h_{rm}}{a^2} \ln \frac{a[4u^2+a^2]^{1/2}+2[a^2u^2-h_{rm}^2]^{1/2}}{a[4u^2+a^2]^{1/2}-2[a^2u^2-h_{rm}^2]^{1/2}} + \sin^{-1} \left[\frac{[a[a^2u^2-h_{rm}^2]^{1/2}]}{u[a^2+4h_{rm}^2]^{1/2}} \right] + h'_{rm}$	$u = u, \quad v = \phi$
Circular Cone	$v = \frac{1}{\sin a} \sec^{-1} \left[\frac{u \sin a}{h_{rm}} \right] + h'_{rm}$	$u = r, \quad v = \phi$
Sphere	$v = -\tan^{-1} \left[\frac{h_{rm} \cos u}{(a^2 \sin^2 u - h_{rm}^2)^{1/2}} \right] + h'_{rm}$	$u = \theta, \quad v = \phi$
Ellipsoid of Revolution	As a function of incomplete elliptic integrals of first and second kind $E(u, k)$ and $F(u, k)$, and h_{rm} and h'_{rm} , where the modulus k is a function of shaping parameters (a, c) .	$u = \theta, \quad v = \phi$
General Hyperboloid of Revolution	As a function of incomplete elliptic integrals of first and second kind $E(u, k)$ and $F(u, k)$, and h_{rm} and h'_{rm} , where the modulus k is a function of shaping parameters (a, c) .	$u = n, \quad v = \phi$

2. Arc length $s = s(u)$

General Paraboloid of Revolution

$$s_{rm} = \left[\frac{[(4u^2 + a^2)(a^2 - u^2 - h_{rm}^2)]^{1/2}}{2a} + \frac{a^4 + 4h_{rm}^2}{8a^2} \ln \frac{a[4u^2 + a^2]^{1/2} + [a^2 - u^2 - h_{rm}^2]^{1/2}}{a[4u^2 + a^2]^{1/2} - [a^2 - u^2 - h_{rm}^2]^{1/2}} \right]_{u=u_s}^{u=u_f}$$

Circular Cone

$$s_{rm} = \left[\frac{(u^2 - h_{rm}^2)^{1/2}}{\sin a} \right]_{u=u_s}^{u=u_f}$$

Sphere

$$s_{rm} = \frac{a}{h_{rm}} \left[\sin^{-1} \left[\frac{a \cos u}{(a^2 - h_{rm}^2)^{1/2}} \right] \right]_{u=u_s}^{u=u_f} \quad u_f > u_s, \text{ i.e., } \theta_f > \theta_s$$

Ellipsoid of Revolution

As a function of incomplete elliptic integrals of first and second kind $E(u, k)$ and $F(u, k)$, and h_{rm} where the modulus k is a function of shaping parameters (a, c) .

General Hyperboloid of Revolution

As a function of incomplete elliptic integrals of first and second kind $E(u, k)$ and $F(u, k)$, and h_{rm} where the modulus k is a function of shaping parameters (a, c) .

3. Generalized Fock parameter $\xi_{rm}(u)$

General Paraboloid of Revolution $\xi_{rm} = \left[\frac{(4\pi)^{1/3} (a^4 + 4h_{rm}^2)^{2/3}}{4a^2} \ln \frac{a[4u^2 + a^2]^{1/2} + 2[a^2u^2 - h_{rm}^2]^{1/2}}{a[4u^2 + a^2]^{1/2} - 2[a^2u^2 - h_{rm}^2]^{1/2}} \right]_{u=u_s}^{u=u_f}$

Circular Cone $\xi_{rm} = \left[\frac{(\pi h_{rm} \cos^2 a)^{1/3}}{\sin a} \sec^{-1} \left[\frac{u \sin a}{h_{rm}} \right] \right]_{u=u_s}^{u=u_f}$

Sphere $\xi_{rm} = \left[\frac{\pi^{1/3} a^{1/3}}{h_{rm}} \sin^{-1} \left[\frac{a \cos u}{(a^2 - h_{rm}^2)^{1/2}} \right] \right]_{u=u_s}^{u=u_f}$

Ellipsoid of Revolution

As a function of incomplete elliptic integrals of first and second kind $E(u,k)$ and $F(u,k)$, and h_{rm} where the modulus k is a function of shaping parameters (a,c) .

General Hyperboloid of Revolution

As a function of incomplete elliptic integrals of first and second kind $E(u,k)$ and $F(u,k)$, and h_{rm} where the modulus k is a function of shaping parameters (a,c) .