CLOSED FORM EVALUATION OF ELEMENT COUPLING COEFFICIENTS IN CONFORMAL ARRAYS ON GENERAL QUADRIC CYLINDERS

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ABSTRACT

Quadric cylinders (QUACYLs) form an important class of canonical shapes in aerospace engineering from the surface modeling point of view. In designing and analyzing conformal antenna arrays over aerospace bodies, a knowledge of the mutual coupling between individual elements is necessary. In this paper, a closed form Geodesic Constant Method (GCM) is developed to model the ray-geometric aspects of conformal arrays on QUACYLs. The formulation incorporates a shaping parameter, permitting modeling of surfaces of different sharpness. Mutual coupling results for the general parabolic cylinder are presented to illustrate the application of the formulation.

Introduction

It has been suggested that the use of distributed antennas on aircraft wing for radar applications has a significant advantage over the conventional single antenna concept [1]. Distributed antennas on wing surfaces have also been suggested for navigational applications using the Global Positioning Systems (GPS). Such conformal arrays on convex surfaces directly use the theory of planar arrays. In this paper, we derive the surface ray geometric parameters over general quadric cylinders (QUACYLs). Those parameters can be used to determine radiation characteristics of the arrays. As an example, specific results on mutual coupling between element pairs are provided in the paper. QUACYLs, which include circular, elliptic, and general parabolic and hyperbolic cylinders, form convenient canonical surfaces for modeling many major components of aerospace bodies, such as fuselage and wings.

Formulation

A general quadric cylinder (QUACYL) is a regular quadric surface and can be expressed in the geodesic coordinates (u,v) in the parametric form

$$x = f(u) \qquad y = g(u) \qquad z = v \tag{1}$$

which in the particular case of the general parabolic cylinder (GPCYL), can be expressed as

$$x = au \qquad y = u^2 \qquad z = v \tag{2}$$

where a is shaping parameter and determines the "sharpness" of the GPCYL. The effect of a is illustrated in Fig. 1. All the quadric cylinders can be generalized to incorporate the shaping parameter in their parametric equations

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in the geodesic coordinate system. The equation for the right mth-order geodesic on a QUACYL in the geodesic coordinate is expressed as

$$\frac{dz(u)}{du} = \frac{\pm h_m (f_u^2 + g_u^2)^{1/2}}{(1-h_m^2)^{1/2}} \tag{3}$$

The subscripts u and v refer to the partial derivatives w.r.t. u and v respectively. The ray geometric parameters required in the high frequency mutual coupling analysis can be conveniently classified as [2]

1. Surface-dependent ray geometric parameters which depend on the nature of the surface alone and are expressed as a function of the coordinates of the point at which they are evaluated. The unit surface normal vector and the principal curvatures are examples of these.

2. Geodesic-dependent ray geometric parameters depend on the individual space curves traced on these quadric surfaces in addition to the nature of the surface, for example the Frenet-frame field vectors (**t**,**n**,**b**) and the radius of curvature along the geodesic.

3. Interaction-dependent ray geometric parameters which can be described only if the geodesic coordinates of the source and observation points are given. The arc length, the generalized torsion factor and Fock parameter, and the blending functions are examples of interaction-dependent ray geometric parameters.

All these ray geometric parameters can be derived in the closed form using the definition given in [3]. One of the highlights of our method is that all these expressions are now a function of the First Geodesic Constant h and their accuracy depends on the accuracy of h alone. For this reason, we call this method the Geodesic Constant Method (GCM).

However, serious difficulties exist in deriving many of the interaction-dependent parameters; these pertain to the integration of the arc length and generalized Fock parameter. We have been able to derive derived these parameters analytically for the case of the QUACYLs. For the specific case of GPCYLs, these are expressed in the closed form as

$$s(u) = \frac{1}{4(1-h^2)^{1/2}} \left| 2u(a^2+4u^2)^{1/2} + a^2 \ln[2u+(a^2+4u^2)^{1/2}] \right|_{u=u_s}^{u=u_f} \tag{4}$$

$$\xi(u) = \frac{(4a^2)^{1/3}(1-h^2)^{1/6}}{2} \left| \ln[2u+(a^2+4u^2)^{1/2}] \right|_{u=u_s}^{u=u_f} \tag{5}$$

Mutual coupling between finite-dimensional $(0.5\,\lambda \times 0.2\,\lambda)$ slots over GPCYLs of varying shaping parameter has been computed and two specific results are presented here as an illustration of the mathematical model presented in this paper. Figures 2 and 3 show the mutual admittance magnitude and phase plots for increasing separation between the two slots along u-axis. The z-axis separation between the centroids of the slots in the Figures 2 and 3, is taken as zero and $0.5\,\lambda$, respectively.

Summary

For the cases of the circular cylinder and the GPCYL, the expressions obtained for the ray parameters are in the closed form, since the geodesic constant h is also derivable in the closed form. On the other hand, for the coordinate surfaces of the Elliptic-cylinder Coordinate System, such as the elliptic and general hyperbolic cylinders, the explicit nature of the ray parameters as a function of h remains since h can be expressed explicitly in terms of the asymptotic series expansions of the incomplete elliptic integral functions.

Using different shaping parameters one can generate family of QUACYLs of different flatness. The shaping parameter is employed as a control parameter in Surface Modeling for matching the quadrics to the actual surfaces considered.

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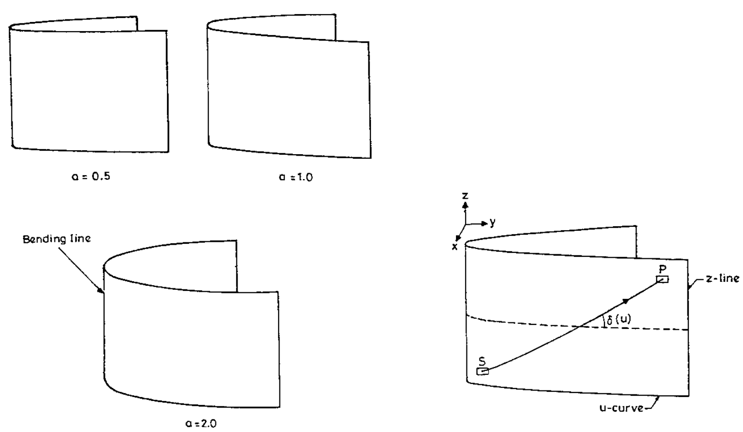

Fig. 1 General parabolic cylinders (GPCYLs) for different value of the shaping parameter a (simulated on an IBM PC-XT compatible).

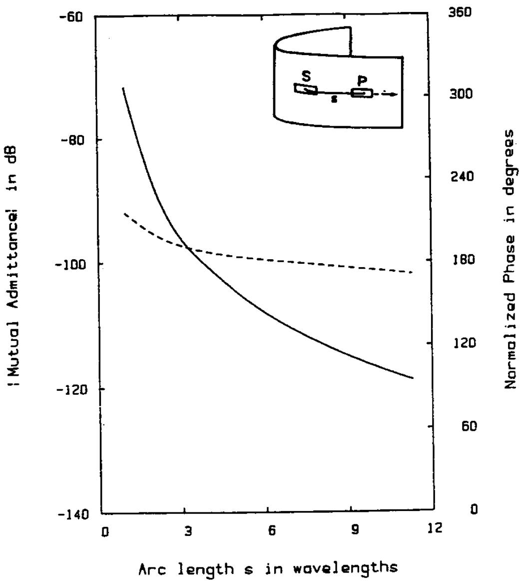

Fig. 2 Mutual admittance vs. the arc length for separation along u-axis between two rect. slots (0.5λ x 0.2λ) on a GPCYL of a = 0.50, z-axis separation =0.0 (constant). Magnitude (——), Phase (----).

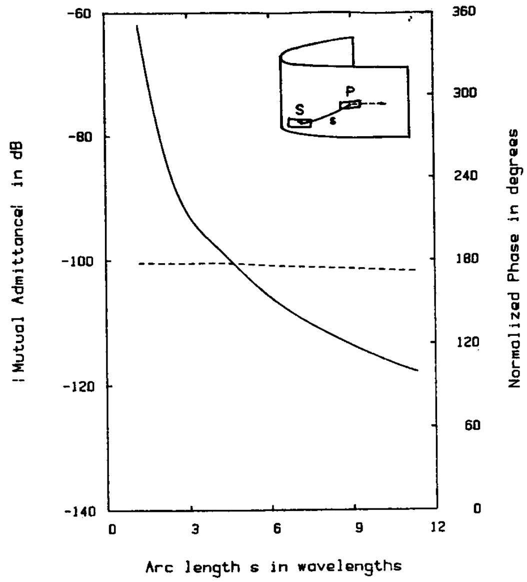

Fig. 3 Mutual admittance vs. the arc length for separation along u-axis between two rect. slots (0.5λ x 0.2λ) on a GPCYL of a = 0.50, z-axis separation =0.5λ (constant). Magnitude (——), Phase (----).