BEARING ESTIMATION OF A SOUND SOURCE IN A SHALLOW WATER CHANNEL

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ABSTRACT

A solution to the problem of bearing estimation of multiple sound sources in a shallow ocean waveguide is developed. A planar array of N vertical columns of Q hydrophones each is used. Each vertical column is used as a mode-filter to extract the same normal mode from among the many propagating in the waveguide. It is shown that a two-column planar array used in conjunction with a delay-and-sum array processor would suffice to estimate the source bearing. When multiple sources are present, the use of a high resolution spectral estimation technique in place of the simple delay-and-sum array processor provides unbiased estimates. Computer simulation results using the MUSIC method with a planar array in a waveguide with a rigid bottom are presented.

Introduction

Many high resolution spectral estimation techniques exist for the estimation of the direction of a source using hydrophone arrays [1]. While such techniques can provide good bearing estimates for sources in an infinite homogeneous medium, bearing estimation of sound sources in a shallow ocean waveguide is a more difficult problem. This is due to the fact that a source in a waveguide gives rise to a coherent set of normal modes. Due to the interference amongst the different normal modes, the acoustic signal varies from one hydrophone to another.

Buckingham [2] has studied the response of a steered horizontal linear array to the sound field of a source in a waveguide. Unlike a single principal maximum that one observes in the array response due to a plane-wave source, the array response to a single source in a waveguide exhibits many maxima. This is due to the multimodal structure of the sound field. Hence the field appears to be generated by many plane wave sources. If the angular separation between adjacent modal-wave arrival directions is small, one observes beam-broadening, and if the angular separation is large, one observes beam-splitting. Thus, interpretation of any one of the array response maxima as the true source bearing would be erroneous [2,3].

In this paper, a method of obtaining unbiased bearing estimates of one or more sound sources in a shallow ocean waveguide using a planar array is presented.

Problem Formulation

Consider a planar array of NxQ identical omnidirectional hydrophones in the far-field of a monochromatic point source in an isovelocity water layer overlying a rigid bottom. The above assumptions regarding the ocean environment have been made to simplify the ensuing analysis and are not essential for the validity of the proposed method. The geometry of the problem is shown in Fig. 1. The planar array can be thought of as a horizontal array (H-array) of N columns, each column being a vertical array (V-array) of Q hydrophones. The acoustic pressure at the qth hydrophone of the nth column is given by [4]

$$p_{qn} = \sum_{m=1}^{M} A_m \exp[jk_m r_n], \tag{1}$$

where

$$A_m = (2/H)\ (2\pi/k_m r_n)^{1/2} \sin(\gamma_m Z_s) \sin(\gamma_m Z_q), \tag{2}$$

$$\gamma_m = (\pi/H)\ (m - 1/2), \quad m = 1,2,\dots,M, \tag{3}$$

$$M = \mathrm{Int}[(H/\pi)\ (k + 1/2)], \tag{4}$$

$$k = \omega/c = (k_m^2 + \gamma_m^2)^{1/2}, \tag{5}$$

γ_m is the vertical component of the wave number k, k_m is the horizontal component of the wave number k, c is the sound speed in the water layer, H is the depth of the water layer, r_n is the horizontal separation between the source and the nth V-array, Z_s is the source depth, and Z_q is the depth of the qth hydrophone in any V-array. Let h be the horizontal separation between adjacent columns of the H-array, and d be the separation between adjacent elements of a V-array.

The horizontal separation between the sound source and the nth column of the H-array (see Fig. 1) is

$$r_n = r_1\ [1 + 2(n-1)(h/r) \sin\beta + (n-1)^2\ (h/r)^2]^{1/2}, \tag{6}$$

where r_1 is the horizontal separation between the source and the 1st column of the H-array and β is the source bearing in the horizontal plane measured with respect to the H-array normal. For $(N-1)\ h/r \ll 1$, r_n is approximately given by

$$r_n \simeq r_1 + (n-1)h \sin\beta\ . \tag{7}$$

Using this approximation in the phase term and the approximation $r_n = r_1$ in the amplitude term in Eq. (1), we have

$$p_{qn} = \sum_{m=1}^{M} A_m \exp\{jk_m[r_1+(n-1)\ h \sin\beta\ ]\}, \tag{8}$$

where

$$A_m = (2/H)(2\pi/k_m r_1)^{1/2} \sin(\gamma_m Z_s) \sin(\gamma_m Z_q). \tag{9}$$

The arrival directions of each of the modal waves, as measured with respect to the H-array normal, is given by

$$\theta_m = \sin^{-1}[(k_m/k) \sin\beta\ ]. \tag{10}$$

It is thus clear that a single source gives rise to M modal-wave arrivals at the array. Now, if the k_m's are assumed to be known, then the source bearing β can be obtained by using estimates of θ_m in Eq. (10). It is well known that approaches based upon eigen-decomposition of the array covariance matrix for estimating the arrival directions fail when the different arrivals are correlated as in the present case. One way of overcoming this difficulty is to decorrelate the modal-wave arrivals by range-averaging using a towed horizontal array [5]. But, when there are more than one sources, say L, with distinct bearings β_1, $\beta_2, \ldots, \beta_L$ then there will be LxM arrivals at the array, and the eigendecomposition approach in conjunction with a horizontal line array becomes unattractive. In order to overcome this difficulty, we shall use vertical arrays as elements of a horizontal array. The V-arrays, when used as mode-filters, eliminate all but one of the modes propagating in the waveguide and thus reduce the number of arrivals seen by the array to one per source

Mode Filtering

Each V-array is required to function as a mode filter to select the same (say, mth) normal mode. The null-steering technique of mode-filtering [6] is employed for this purpose. A brief outline of this method is given below. In this method, each normal mode is viewed as a pair of quasi-plane waves, and nulls of a vertical array are steered in the directions of arrival of the quasi-plane waves associated with the modes to be rejected. To reject the ith mode, a 3-element vertical array with the weight-sequence

$$W_i = \left\{ a,\ -2a\cos(\gamma_m d),\ a \right\} \tag{11}$$

is required, where a is a normalizing constant. To reject all modes except the mth, a vertical array with Q = 2M-1 elements with the weight sequence

$$W = W_1 * W_2 * \ldots * W_{m-1} * W_{m+1} \cdots W_M \tag{12}$$

is required, where * denotes convolution. The array weighting coefficients are real and symmetric, and, more importantly, they are independent of the source bearing β and the array depth. If the elements of each V-array have the weighting coefficients given by Eq. (12), the response of each V-array is given by

$$s_n = b_m(r_1, Z_s)\exp[jk_m(n-1)h\sin\beta], \tag{13}$$

$$b_m(r_1, Z_s) = (2/H)\,(2\pi/k_m r_1)\,\sin(\gamma_m Z_s)\exp(jk_m r_1). \tag{14}$$

It is seen from Eqs. (13) and (14) that the magnitude of the array response is a function of the depth Z_s and range r_1 of the source. In particular, if the source is situated at the node (zero) of the selected mode the response of each V-array is zero. Hence, it is important that the mode with the largest response be chosen to maximize the SNR.

Bearing Estimation

Now consider the N-element H-array whose columns (V-arrays) are used as mode-filters. If each of the N V-arrays is used for filtering the same mth normal mode, then using Eq.(13), the H-array response can be expressed as

$$s = b_m(r_1, Z_s)\sum_{n=1}^{N} f_n \exp[jk_m(n-1)\,h\sin\beta], \tag{15}$$

where f_n are the weights applied to the outputs of the columns of the H-array. More specifically, consider a two element H-array whose response can be expressed as

$$s = f_1 + f_2\exp[-jk_m h\sin\beta]. \tag{16}$$

Further, if we set $f_1 = 1$ and f_2 is set equal to a phase delay $k_m h\sin\beta$, the H-array response is given by

$$s = 1 + \exp[-jk_m h(\sin\beta - \sin\alpha)], \tag{17}$$

where α is the steering direction. It is clear from Eq. (17) that the H-array response is maximum when $\alpha = \beta$. Hence, a planar array with 2 vertical columns for mode filtering, and a conventional delay-and-sum processor can be used to estimate the bearing of a single source.

But the ability of this conventional processor to resolve two or more signals is inherently limited by the beamwidth of the array. This processor can resolve different sources only if they have an angular separation of more than one beamwidth. Hence, high resolution spectral estimation techniques such as the MUSIC method have to be used for resolving multiple sources.

Let there be L sources present in the waveguide located at $(Z_{si}, r_{1i}, \beta_i)$, $(i = 1,\ldots,L)$. The total acoustic pressure at a hydrophone in the array can be expressed as

$$p_{qn} = \sum_{i=1}^{L}\sum_{m=1}^{M} A_m(Z_{si})\exp\{jk_m[r_{1i}+(n-1)h\sin\beta_i]\}. \tag{18}$$

If each V-array is used as a mode filter to select the mth mode, the H-array signal vector is given by

$$s = [s_1, s_2, \ldots, s_N], \tag{19}$$

where

$$s_n = \sum_{i=1}^{L} b_m(r_{1i}, Z_{si})\exp[jk_m(n-1)h\sin\beta_i]. \tag{20}$$

The total array output (including noise) vector can be written as

$$U = D\,b + n \tag{21}$$

where **D** is an NxL direction matrix whose ith column is

$$d_i = [1,\ e^{jk_m h\sin\beta_i}, \ldots, e^{jk_m(N-1)h\sin\beta_i}]^T \tag{22}$$

$$b = [b_m(r_{11}, Z_{s1}),\ b_m(r_{12}, Z_{s2}), \ldots, b_m(r_{1L}, Z_{sL})\ ]^T \tag{23}$$

is the source amplitude vector, and

$$n = [n_1, n_2, \ldots, n_N]^T \tag{24}$$

is the noise vector whose elements are the noise signals at the individual hydrophones. The noise is assumed to be gaussian with variance σ^2 and to be spatially white. The array output covariance matrix is then given by

$$R = E[U\,U^H] = D\,B\,D^H + \sigma^2 I, \tag{25}$$

with

$$B = E[b\,b^H], \tag{26}$$

where H denotes the Hermitian transpose and I is the identity matrix. If the multiple arrivals are uncorrelated, the eigenvalues λ_n and eigenvectors $\mathbf{V}_n$ of R are defined by the relationship

$$R\,V_n = \lambda_n V_n, \quad n = 1,2, \ldots, N, \tag{27}$$

where

$$\lambda_1 > \lambda_2 > \cdots \lambda_L > \lambda_{L+1} = \lambda_{L+2} = \cdots = \lambda_N. \tag{28}$$

The first L eigenvectors corresponding to the largest L eigenvalues span the signal subspace and the remaining (N-L) eigenvectors span the orthogonal noise subspace. Denoting the array steering vector **E** by

$$E = [1, e^{-jk_m h\sin\alpha}, \ldots, e^{-jk_m(N-1)h\sin\alpha}\ ], \tag{38}$$

where α is the steering direction as measured with respect to the H-array normal, the MUSIC spectrum can be expressed as [7]

$$R_{MU} = 1/E^{+}(\alpha)\ N\ N^{+}\ E(\alpha), \tag{39}$$

where N is a Nx(N-L) matrix whose columns are the eigenvectors associated with the (N-L) repeated eigenvalues.

Computer Simulation Results

Simulations have been carried out for a planar array in an isovelocity water layer overlying a rigid bottom.

Numerical values assigned to the parameters in the simulation are as follows: Channel depth H = 50m, sound speed c = 1500 m/sec, source frequency f = 50 Hz, dimensions of the array NxQ = 5x5, distance between the H-array columns h = 15m and interelement distance of each V-array d = 12.5m. For the values assigned to the source and channel parameters, there are three propagating modes in the channel. The modal wave arrival angles with respect to the normal to the V-arrays, and the weighting coefficients for filtering the different normal modes are given in Table 1. This Table indicates that the nulls of the mode filters are in the directions of arrival of the unwanted modal plane waves.

Two sources located at (Z_{s1} = 15m, r_{11} = 2 km, β_1 = 40°) and (Z_{s2} = 33.33m, r_{12} = 2 km, β_2 = 60°) are considered. The MUSIC spectra obtained using the 1st mode and 2nd mode filters are shown in Figs. 2 ,3 respectively. While the spectral peaks in Fig. 2 are seen to correctly point to the source bearings, the spectrum in Fig. 3 has only one peak corresponding to the 1st source. This is due to the fact that the 2nd source is at the node of the second mode. Bearing estimates for two sources with different bearing separations obtained using the ROOT-MUSIC algorithm [8] are given in Table 2. The results in Table 2 are obtained for a noise-free environment. In a noisy environment, the SNR is different for different nodes. Hence the mode with the maximum signal power should be used for the bearing estimation. For a given mode, the mode signal power is a function of the source depth, range and the source strength. For a given mode SNR, the performance of the algorithm in a noisy environment is similar to that in an unbounded medium.

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Table 1. Mode filter weights for filtering the different normal modes ($W_1 = W_5 = 1$)

Mode Number	Null directions $\pm\theta_1$	$\pm\theta_2$	Weighting coefficients $W_2 = W_4$	W_5
1	26.7436	48.5904	0.0000	1.4142
2	8.6269	48.5904	-1.0824	0.5858
3	8.6269	26.7436	-2.6131	3.4142

Table 2. Bearing estimates for two sources with different bearing separations ($Z_{s1} = Z_{s2}$ = 15m, $r_{11} = r_{12}$ = 2km)

Actual bearings $\beta_1°$	$\beta_2°$	Estimated bearings Mode 1 $\beta_1°$	Mode 1 $\beta_2°$	Mode 2 $\beta_1°$	Mode 2 $\beta_2°$
42.0	45	41.9990	45.001	42.005	44.9993
44.0	45	43.9972	45.002	44.0015	44.9983
44.5	45	44.4943	45.0058	44.5033	44.9965
44.8	45	44.786	45.023	44.8081	44.9917
44.9	45	44.876	45.023	44.9137	44.9860

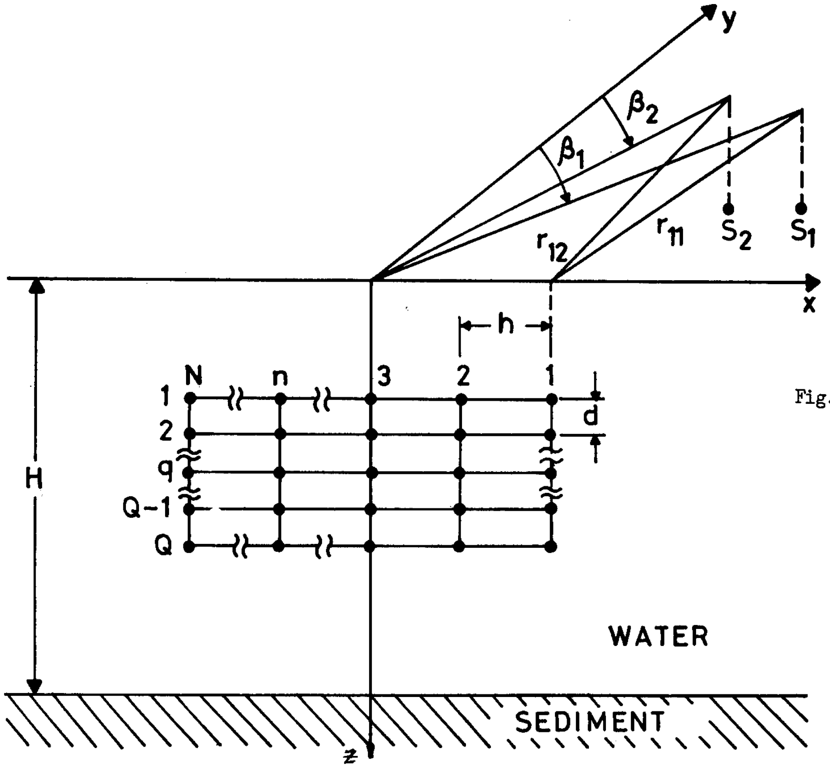

Fig. 1. Geometry of the bearing estimation problem.

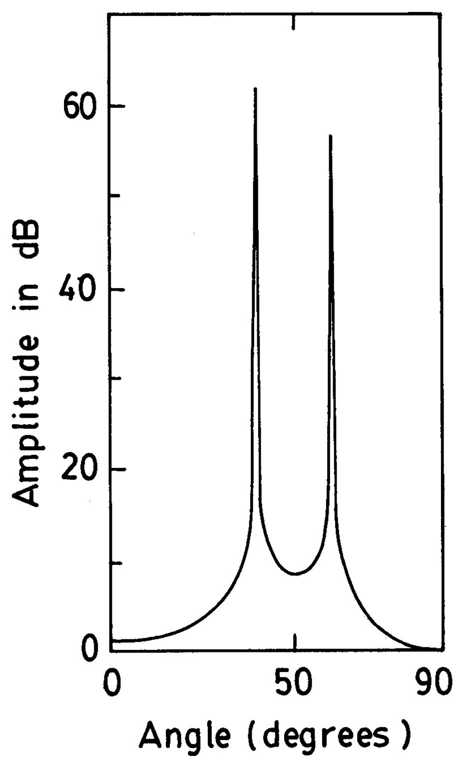

Fig. 2. MUSIC spectrum obtained using V-arrays for filtering the 1st normal mode.

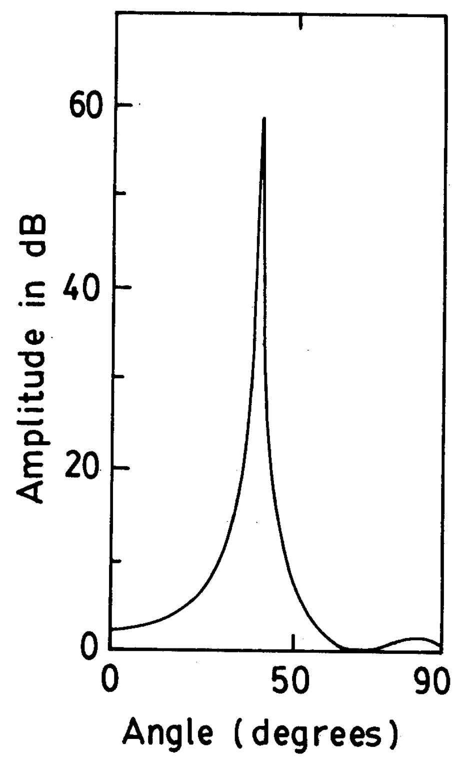

Fig. 3. MUSIC spectrum obtained using V-arrays for filtering the 2nd normal mode.