Performance Analysis of Mobile Cellular Communication Network

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Abstract

In this paper we study a cellular mobile communication system and address the problem of the number of frequency channels required at each cell in fixed channel assignment scheme in order to keep the probability of call failures below acceptable limits. The system considered here is more realistic than those dealt with in the literature since we consider more general motion of the mobiles and also instead of considering each cell in isolation we develop a queueing network model for this system. We prove the stability of this network and show the continuity of the stationary distributions. We show that the network is not product form and develop insensitive upper bounds on the blocking probability at each cell.

1 Introduction

A major problem facing the radio communication industry is the limitation of the available radio frequency spectrum. In setting allocation policy, we seek systems which need low bandwidth but provide high usage. In order to achieve spectrally efficient systems the 'Cellular' approach is used. In a cellular radio system the geographical area in which service is provided is divided into smaller regions referred to as 'cells'. Each cell site has a limited number of frequency channels. The spectral efficiency of this system stems from the fact that frequency channels used within a site are reused in another site located sufficiently far from the first one, so that the co-channel interference level is small enough. Mobile units which initiate a call will be allocated a frequency channel by the cell site in which the mobile unit is located, if there is atleast one channel free. When a mobile moves out of its cell to another cell, the call is transferred to the base station of the new cell. This is referred to as CALL HANDOFF . The handoff is said to be successful if there is atleast one channel free in the new cell, when the handoff is attempted ; otherwise the call dies before its service is complete. The period for which the call uses the frequency channel in a cell is referred to as the 'channel holding time' of the call. In general, the channel holding time is not the same as the service time of the call; it is a random variable which also depends on the motion of the mobile initiating the call.

The main performance issue is the probability of call failure. For this, various channel assignment schemes are used to allocate frequencies to the base stations to be used by the mobiles in its area. The important assignment schemes are the fixed channel assignment schemes ,the dynamic channel assignment schemes and hybrid channel assignment schemes ([1], [3],[4], [5]).

In literature we find that cellular systems have been studied widely. Some simulation studies are presented in ([3]) to assess the performance of the assignment schemes. Guerin [7] shows by simulation that the channel occupancy time distribution is best approximated by exponential distribution. An approximate analytical model and performance characteristics for the cellular communication system is developed by Hong et.al [2]. In all the above studies simplifying assumptions are made on the motion of mobiles. In [2] the mobiles' velocity is assumed to remain the same throughout and each cell is studied in isolation. The mobiles' velocity in [7] is not only assumed to remain constant but they are assumed to move only in one of the four axes directions and the direction cannot change. But such assumptions are totally unrealistic; infact the complexity of analysing such systems arise mainly because of the feedback nature. Also because of the inter-connectivity of the different cells it is not proper, as we show, to study each cell in isolation. In contrast a natural model for studying such systems would be a queueing network.

We develop an analytical model for a cellular communication system in which the mobiles change direction at i.i.d time intervals and for this model we obtain upper bounds on the blocking probability in terms of the call arrival rate, mean of the channel holding time and the number of frequency channels in the cell, under the assumption that the channel holding time has a general i.i.d distribution. In section 2 we give the description of the communication system and in section 3.1 present the analytical model. We show the existence of and convergence to the stationary distributions for our queueing network starting from any initial conditions in section 3.2 In section 3.3 we show that the network is not product form even if the external arrivals are Poisson and the channel holding times are assumed exponential which implies that each cell cannot be studied in isolation. When the network is not product form, it is difficult to obtain the stationary distributions for such a network. Therefore we obtain three upper bounds on the time stationary distributions. We clarify the relationship between customer stationary and time stationary distributions for our network in section 3.4 and also obtain some explicit bounds on the blocking probabilities in terms of the mean channel holding time and mean total arrival rate. In section 3.5 we obtain upper bounds on the mean channel holding time and the total arrival rate to a call in terms of the service requirement of a call, external arrival rates and the motion of the mobiles.

2 Description of the Communication System

We consider the following system. The mobiles move randomly in a specified area. The entire area is divided into identical cells. The mobiles move randomly with change of direction occuring at $\{t_i\}_{i=1}^{\infty}$ intervals. At each $T_k = \Sigma_{i=1}^{k} t_i$ instant the direction and the length of displacement is selected according to the $\{V_i\}_{i=1}^{\infty}$ sequence.

The arrival of new calls to the system is assumed to be a renewal process. The service time of a call is assumed to have a general distribution independent of all other random variables. The channel holding time has a general i.i.d distribution. Each cell has a limited number of (say

k_i at cell i) frequency channels for the calls lying in its area. We assume that fixed channel assignment scheme is used for channel allocation. A call on its generation is allotted a frequency channel by the base station in its cell, if a channel is free; otherwise the call is lost. On the other hand the call, on its generation, may be allotted a channel but when the mobile moves from this cell to another cell, the call has to be transfered from the base station of the cell in which it was generated to the base station of the new cell. The call transfer is successful if there is atleast one channel free in the new cell when the call transfer is attempted. If all the channels in the new cell are occupied then the call can no longer be continued and it is lost before its sevice is completed.

We use the following notation :

M = number of cells in the communication system.

$M < \infty$

$\lambda(s)$ = arrival rate of new calls to the cell 's'.

$1/\mu$ = mean channel holding time.

k_i = number of frequency channels at the ith cell ; $1 \leq i \leq M$.

p_i = probability that an external arrival (new arrival) occurs at cell i.

p_{ij} = probability of the call (mobile) moving from the ith cell immediately to or through the jth cell; $1 \leq i,j \leq M$

p_{i0} = probability that the call terminates at the ith cell; $1 \leq i \leq M$.

$p_{ii} = 0$; $1 \leq i \leq M$.

$\Sigma_{k=0}^{M} p_{ik} = 1$; $1 \leq i \leq M$.

$\lambda_T(i)$ = total arrival rate (the arrival rate of new calls along with the rate of calls handed off from other cells) of calls at the ith cell. Z_t represents the system state at arbitrary instant 't'.

Z_n represents the system state at arrival epochs.

We define the state of the system in section 3.1 .

3 Analysis of the System

In this section we model the above system by a queueing network and analyze it. We prove the existence of a stationary distribution for the queue length vector (the number of customers in each cell) of the queueing network in 3.2. In the next subsection we obtain insensitive upper bounds on the time stationary distributions of the network. In section 3.4 we consider bounds on the customer stationary distributions (in particular on the blocking probability). In section 3.5 we relate these parameters to the mobile communication system and get upper bounds for that system.

3.1 Queueing Model for the Mobile Communication System

The cellular communication system described above is modelled as an open network of queues. Each cell in the system corresponds to a node in the network ; thus the network has M nodes. Customers which enter the network from outside correspond to the new calls. We assume that the total external arrival process is a renewal process. Random variable 'b' denotes the inter arrival times for this process. Probability that an external arrival arrives at the ith queue first is denoted by p_i. The services offered by node i to the customers is a sequence of i.i.d. random variables. We denote by s_i a random variable denoting service at node i. The ith node in the network has k_i, $1 \leq i \leq M$, servers. Each node in the network has no waiting places and hence acts as a loss system. Customers who finish service at one node go over to another node according to the transition probabilities in the matrix P , ie; a customer goes from node i to node j with probability p_{ij} (p_{ij} may be equal to 0) and leaves the system with probability p_{i0} (here again we assume that the probability of going from a node to itself is zero ie. $p_{ii} = 0$). Thus the arrivals at any node are the superposition of the external arrivals and those routed from other nodes. A customer who arrives at a node and finds all the servers at that node busy leaves the system immediately. Thus each cell is modelled as $./GI/k_i/0$ queue. The system state at time t, denoted by $Z_t = (Z_t(1),\ldots,Z_t(M))$ where $Z_t(i)$ is a vector representing the service times already undergone by the customers at node i. Similarly we define the Z_n at the nth arrival epoch to the system.

Remark 1 : The p_{ij}s mentioned here would depend on the motion of the mobiles. In section 3.5 we get some bounds on p_{ij}s . The mean service time at each node $1/\mu$, mentioned here corresponds to the mean channel holding time. In section 3.5 we obtain an upper bound on this.

We now analyze this network to get expressions or bounds for the stationary distributions and probability of lost calls, the performance measure we are interested in. But before we do this we prove the existence of a stationary distribution for the queuelength at each node.

3.2 Stability and Continuity of the Queueing Network

In this section we prove the stability of the queueing network described above and show the continuity of the stationary distributions.

Theorem 3.1 : *Assume $p_j > 0$ for all j. Let there be constants $A > 0$ and $\epsilon > 0$ s.t.*

$$P(s_j > t+A) \leq (1-\epsilon).P(s_j > t) \qquad (*)$$

$$P(b > A) > 0$$

for all $t \geq 0$ and for all $j=1,\ldots,M$.

Also, let $E(b) < \infty$ and $E(s_j) < \infty$ and either all b and s_j have lattice distribution with same span or atleast one of them have nonlattice distribution. Then there are stationary distributions s.t. starting from any initial distributions, Z_n and Z_t converge to those distributions (these may be different for Z_n and Z_t).

Proof : See ([8], [9]).

The assumptions made in the theorem are similar to that of Borovkov [10]. In [9] we have also shown these results without the assumptions in (*). For this, the proof is somewhat similar to that of Sigman [11].

For the non lattice case, if the distributions of b is spread out, then the convergence in theorem 3.1 is in total variation. Also, for the lattice case, if the distribution of b is aperiodic, then the limit is in total variation.

We shall also need the stationarity of the system state process at the arrival epochs of customers (external as well as from other cells) to a particular cell. The arrival process to a particular cell is determined by certain jumps in the time stationary process $\{Z_t\}$ (We denote by $\{Z_t\}$ here the state process after it has reached time stationarity). We can consider this process at times $Z_{a_n^-}$ where a_n is the arrival epoch to a cell (which is fixed in this paragraph). Although this process is not stationary at the times a_n^-, from this we can form a process which is stationary at these epochs and describes the behaviour of the system (see Franken et al. [12]).

In ([8],[9]) we have also shown that under appropriate conditions the prestationary and stationary distributions of the queueing network are continuous functions of the distributions of p_{ij}, p_i, b and s_j.

3.3 Upper Bounds on the Time Stationary Distributions

We have established, in the previous section, the stationarity of the queue length process. Here we would be interested in obtaining some information about the stationary probabilities of the system. Let us first notice that our network does not have a product form solution. For this we can take the example of two nodes, each with a single server. The external arrival process is Poisson and the service times at each node are exponential. Then it is easy to calculate its stationary distributions (see[8],[9] for explicit expressions) which turn out to be non product form unless $p_{12}=p_{21}=0$. Since our system does not have a product form solution, in general it will be difficult to get the stationary probabilities. Also, since the queue length distributions at different nodes are dependent, it is not correct to consider each cell in isolation as has been done in previous studies.

We now develop several insensitive upper bounds on the time stationary probabilities for our system when the external arrival process is Poisson. The insensitive bounds are particularly useful since the channel holding times may not have simple distributions. To develop the upper bounds we construct auxillary systems, which under the same external arrival rates λ and service rate μ form an upper bound on the stationary distributions of our network. We develop three such 'upper bound systems' and compare their tightness under different conditions.

Jackson Network : The system is an open Jackson network with M nodes, each node having infinite servers. Let the external arrivals to each node be Poisson with the same arrival rate as our original system, λ and let the Poisson streams to the different nodes be independent of each other. The service distributions at each node are general with mean $1/\mu$ and are independent of all other r.v.'s. Let the routing matrix P, be the same as in our original system

It is well known that this has a stationary queue length distribution if the mean service times at each node are finite and further the stationary distribution has a product form insensitive solution. In the following we write X $\leq_{st}$ Y, for random variables X and Y, to mean that for all non negative non decreasing functions f for which E[f(X)] and E[f(Y)] exist, E[f[X]] $\leq$ E[f[Y]]

Proposition 3.2 :

$q=(q(1),\ldots,q(M)) \leq_{st} (q^J(1),\ldots,q^J(M))=q^J$

where q(i) represents the stationary number of customers at node i in our system and q^J(i) represents the stationary number of customers at node i in the Jackson's network.

Proof: Trivial.

Remark 2: This upper bound is applicable when the service at a node (ie. the channel holding time) has a general i.i.d distribution. We can further strengthen the upper bound if the service distribution at a node is assumed to have an exponential distribution. In the following we develop upper bounds when the service distribution can be approximated by an exponential distribution.

We develop another auxillary system which has an insensitive product form solution and which stochastically upper bounds our system in terms of the queue length vector. This system would be refered to as the 'JUMP OVER SYSTEM '. The system is described as follows. The number of nodes, the number of servers at each node, external arrival processes, the routing matrix etc. are all the same as in our system. The only difference is that when a call arrives to a node which does not have any free server, the call jumps over instantaneously to the next node on its route. It has been shown in [17] that the jump over system has product form solution. We show in ([8],[9]) that if our system has channel holding time at each node exponential, then the stationary and prestationary distributions (for same initial conditions) of the queue length vector for our system is stochastically smaller than the jump over system with same parameters.

We have developed another upper bound for the time stationary distributions at each node in our network using the technique given in [13].

3.4 Bounds on the Blocking Probabilities

The bounds obtained in the section 3.3 are on time stationary distributions of our network while for blocking probabilities we need the customer stationary distributions. Thus we first clarify the relationship between the time stationary and customer stationary distributions for our system. These provide us bounds, in association with the results in the last section on the stationary distributions of queue lengths seen by arriving customers to a cell. Our results in this regard are not complete. Therefore since blocking probability is a major performance index for the practical system, we also obtain some bounds in this section on blocking probabilities alone by using the $G/G/k_i/0$ systems (ie. systems with general arrivals, general service distribution, k_i servers and no waiting places).

By the well known PASTA property (see Melamed and Whitt [14]) if we assume that the external arrival process is a Poisson process, then the limiting distributions of Z_n and Z_t in theorem 3.1 are same. Also an external arrival coming to a particular node will see in the steady state, the limiting distribution of Z_t. We present the bounds on the blocking probabilities obtained from these systems in figures (1),(2) and (3). These figures correspond to a network with 2 nodes and 2 servers at each node. The external arrival process is Poisson, the service distributions are exponential with mean 1 and $p_{ij}=p_{ji}=p$, for i,j=1,2 and i$\neq$j. We notice from these figures that for different system parameter values, different upper bounds are tighter and in general atleast one of them gives a reasonably good approximation.

Now let us consider the total arrival process to a particular cell. Since the conditional intensity of this arrival process at time t is obviously dependent upon Z_t, by theorem 4 in Melamed and Whitt [14], at these arrival epochs an arriving customer does not see time stationary distribution of Z_t. In fact, by taking the example of M=2, one server at each node and exponential service we can easily check that the total arrival conditional intensity at a cell is an increasing function of the queue length process q_t in the system but the joint probability of both cells having a customer is strictly less than the product of their marginals. Therefore by corollary 2 in [14], the time stationary distributions of q_t cannot be stochastically compared to the customer stationary distributions at the total arrival epochs to a cell. But if we consider the process of number of customers (queue lengths) in the cell to which the total arrival process is being considered, then we believe that at least for the exponential service distributions and Poisson external arrivals assumed in this paragraph the customer stationary process is stochastically smaller than the time stationary process. This can be easily checked for the above example of two cells. At the moment we do not have a proof for the general case. If this is true then the upper bounds we have obtained in the previous section will provide upper bounds for the customer stationary process at each cell. Now we obtain some bounds on the blocking probabilities only using the results of Disney and Franken [15] on $G/G/k_i/0$ systems. For such a system if a is the expected arrival time and b is expected service time, then denoting $\rho=b/k_ia$ and blocking probability by B, we obtain

$B \geq 1 - 1/\rho_s$
where $\rho_s = \rho$ for $G/GI/k_i/0$ system. Also, for $GI/GI/k_i/0$ system with NBUE inter arrival distributions,
$B \leq \rho/(1+\rho)$.
We will obtain upper bounds on a and b in the next section in terms of the system input parameters : the external input arrival rate, the distribution of the velocity and the service requirement of a mobile.

3.5 Bounds on the System Parameters

In this section we will obtain various bounds for the distributions and the mean values of the channel holding time, total arrival rate to a node and also p_{ij}. For the mobiles in a cellular system, the parameters that can be assumed to be known in the beginning are the distributions of the external arrival process to a cell and of the sequence $\{(V_i,t_i,\tau)\}$ where τ is the time for which a call will last if it is allowed to continue and $\{(V_i,t_i);\ i \geq 0\}$ is a sequence which represents the motion of the vehicle. We will assume that τ is independent of $\{(V_i,t_i);\ i \geq 0\}$. The vehicle moves in a particular direction for a time t_i and then changes direction and speed independently of everything else. The total displacement of the motion during time t_i is represented by V_i.

Now we obtain a simple bound on the channel holding time of a call in a cell. Let us denote by T_b the stopping time which represents the first time, in terms of number of direction changes at epochs t_i that a call which originated in a cell, exits the cell, if the random variable $\tau = \infty$ a.s. . Define $S_n = \Sigma_{i=1}^{n} V_i$ and $T_n = \Sigma_{i=1}^{n} t_i$. Then T_b is a stopping time w.r.t. the sequence S_n. Also if r_c denotes the radius of a cell, then
$P[T_b = n/S_1, \ldots, S_{n-1}] \geq P[||V_n|| > 2.r_c]$
If $P[||V_n|| > 2.r_c] > 0$, then we can easily show that
$E[T_b/S_0] \leq 1/P[||V_n|| > 2.r_c] < \infty \ldots (1)$
where S_0 is the position, of the vehicle, in the cell when the call was initiated. The above bound is uniformly on all S_0 in the cell and hence is also an upper bound on $E[T_1]$. Now, the channel holding time $T_c \leq min[\tau, \Sigma_{i=1}^{T_b} t_i]$. If we can assume that V_i is independent of t_i or that T_b is a stopping time w.r.t. the sequence t_i then
$E[\Sigma_{i=1}^{T_b} t_i] = E[T_b].E[t_1] \ldots (2)$
Thus, by (1) and (2)
$E[T_c] \leq min(E[\tau], E[T_b].E[t_1]) \leq min(E[\tau], E[t_1]/P(||V_n|| > 2.r_c))$
$\ldots (3)$

If we make some more assumptions on the distribution of V_1, then we can get tighter upper bounds on the distribution of T_b and the bound in (1) can be made dependent upon S_0. This will provide us with better bounds than in (3) .

In order to obtain other bounds on T_c and the total arrival rate to the cell, we first study the process S_n, T_n and τ. Let us assume τ to be of the following type. Let X be a r.v. with distribution F. Then define $p = P\{X > t_1\}$. At each epoch t_i, with probability p the call continues till time instant t_{i+1}. If the call terminates before t_{i+1}, then it has a distribution equal to that of r.v. X conditioned on the event $X < t_1$. Thus when the call is initiated at T_0, then with probability p, independently of everything else, the call will continue till time t_1. If τ is an exponentially distributed r.v. then this assumption is satisfied but it is strictly more general than exponential distribution. Then of course
$E[t_1].p/(1-p) \leq E[\tau] \leq E[t_1]/(1-p)$
Under the above assumptions, T_n can be considered a renewal process in which we are interested until the call lasts. The distribution of the renewal interval F' is given in terms of the distribution of t_1 (denoted by F_t) by $F' = p.F_t + (1-p).\delta_\infty$
where δ_∞ denotes a delta dirac measure concentrated at ∞. If $F_t(x) \longrightarrow 1$ as $x \longrightarrow \infty$, then a renewal will not take place with probability (1-p). If U(t) represents the expected number of renewals till time t, then from standard results in renewal theory (see Asmussen [16]) if m represents the time till the last renewal
$P(m \leq t) = (1-p).U(t)$
$E[m] = (\int_0^\infty (1-p-F_t(x))dx)/(1-p)$,
while the total number of renewals N has distribution $P[N = k] = p^{k-1}.(1-p)$
and $U(t) = \Sigma_{n=0}^{\infty} (F')^{*n}(t)$,
where F'^{*n} is n times convolution of F'. Also, if n(t) represents the number of renewals till time t, then
$P[n(t) \leq n] = 1 - F'^{*(n+1)}(t)$..
In the following we use these results to find the probability that a call goes out of the cell in which it originated, before dying.

Let us calculate the probability that a call before completing moves out of a circle of radius r from S_0. Now, (in the following we take $S_0 = 0$, we can take it at any other point in the cell but then we would have to use $S_n - S_0$ in the following)

P[vehicle does not go out of distance r when the call is on] $\geq$
$P[max_{1,\ldots,N+1}||S_n|| < r] = \Sigma_{k=1}^{\infty} P[max_{1,\ldots,k}||S_n|| < r/N = k-1].P[N = k-1]$.
If $\{V_i\}$ sequence is independent of $\{t_i\}$ then we can get further explicit bounds, as the RHS reduces to
$\Sigma_{k=1}^{\infty} P[max_{1,\ldots,k}||S_n|| < r].p^{k-2}(1-p)$.
Also, if $E[V_1] = 0$ and $var[V_1]$ is finite, we have
$P[max_{1,\ldots,k}||S_n|| > r] \leq k.var[V_1]/r^2$. Therefore,
$P[max_{1,\ldots,k}||S_n|| < r] \geq 1 - k.var[V_1]/r^2$.
Thus we obtain the lower bound as
$\Sigma_{k=1}^{\infty} (1 - k.var[V_1]/r^2).p^{k-2}.(1-p) \ldots (4)$
The upper bound on the probability of call not going out of circle with radius r is
$\Sigma_{k=1}^{\infty} P[max_{1,\ldots,k}||S_n|| < r/N = k].p^{k-1}.(1-p) \ldots (5)$
These bounds can be used to obtain bounds on the probability that a call which originated at some point in a cell will not leave that cell. For example, if the distance of S_0 from the nearest boundary of the cell is r, then the probability that the call will never leave the cell will have lower bound given by expression (4). If we are given a distribution of where in a cell the call originates then we can get an overall lower bound on the probability of a call generated in a cell not to go out of the cell. Instead of using N if we use n(t), then we obtain bounds on the probability that a call generated in a cell will not leave the cell till time t. Then the bounds can also be used to get bounds on the channel holding time distribution.

Once a vehicle goes out of the cell, it will enter one of the neighbouring cells. If the distributions of V_i and the call generation process in the cell is such that a call generated in the cell is equally likely to go out of the cell in all directions, then conditioned on the event that a call goes out of the cell, it will go with equal probability to the cells on its boundaries. For example, if the cells are approximately having a hexagonal shape, then with probability 1/6 the call will enter one of the neighbours. Actually these probabilities can be very easily obtained explicitly in terms of the distribution of V_i and the call generation process directly in the general case also. Ω The upper and lower bounds in expressions (4) and (5) can be used to obtain bounds on the total arrival rate to a cell. The probability that a call which originates at a point (x,y) outside the cell and never reaches the cell is lower bounded by (4) if we take r to be the distance of (x,y) from the cell boundary. We can tighten this bound by adding to it the probability that it does go out of radius r but not in a cone which encloses the cell. This probability can be easily lower bounded if we use the bound in (5) and if V_i has distribution which is direction homogeneous (even more general case can be considered if we know the distribution of V_i). Explicitly, we get the lower bound

$\Sigma_{k=1}^{\infty}(1-k.var(V_1)/r^2).p^{k-2}.(1-p)+[1-\Sigma_{k=1}^{\infty}P[max_{1,\ldots,k}||S_n|| \leq r/N = k].p^{k-1}.(1-p)).(1-r_c/\rho.r) = \alpha(x,y) \ldots (6)$

Therefore, the upper bound on the probability that a call generated at (x,y) will reach the cell is 1-α(x,y). Then, if the rate of external arrivals in the neighbourhood of point (x,y) is $\lambda(x,y)$, the arrival rate of calls coming to the cell for the first time is upper bounded by

$\int_A[1-\alpha(x,y)]\lambda(x,y)dxdy = \lambda' \ldots (7)$

where A is the over all area of the cellular system. Once a call enters the cell, then at a regeneration point t_n inside the cell, it will behave in the same way as the calls which originated in the cell. Now we have to include in the arrival rate, the calls which have once entered the cell and then reenter after exiting the cell. But because of the memoryless property in the continuation of a call, once a call exits the cell and at the first regeneration time t_i outside the cell it is at point (x,y) then again the upper bound on the probability that it reenters the cell is 1-α(x,y). Combining it with the upper bounds and the probability, that a call inside the cell will exit it at all we obtain the upper bound (let us call it β) that a call entering a cell will exit and enter again. Hence the upper bound on the total arrival rate to the cell now becomes

$\lambda_T = \lambda' + \lambda'.\beta$ where λ' is defined in (7). In calculating these upper bounds we have neglected the blocking probabilities at intermediate cells, but these of course stay as upper bounds. The effect of blocking probabilities can be included by iteratively calculating these bounds.

The bounds shown in this section can be easily tightened by taking into account boundary conditions etc. The purpose of obtaining these bounds is basically to show that it is possible to obtain the bounds that are needed to use the analytical results obtained in earlier sections on the queueing network model. Of course it is theoretically also possible to get exact distributions once we are given the distributions of V_i, t_i, τ.

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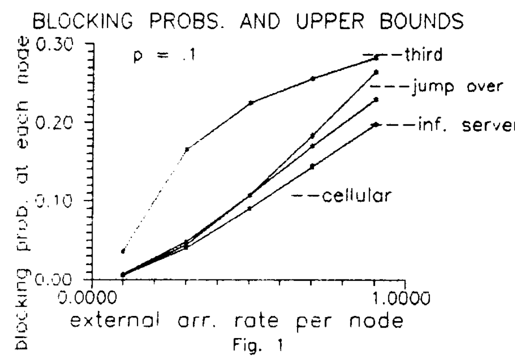

Fig. 1

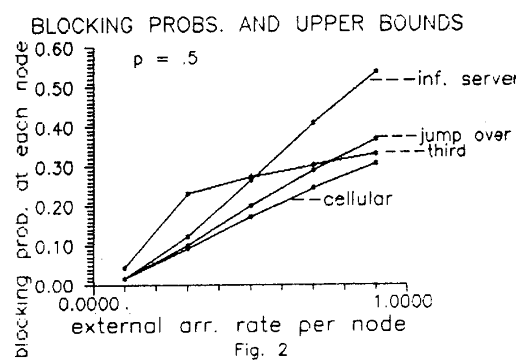

Fig. 2

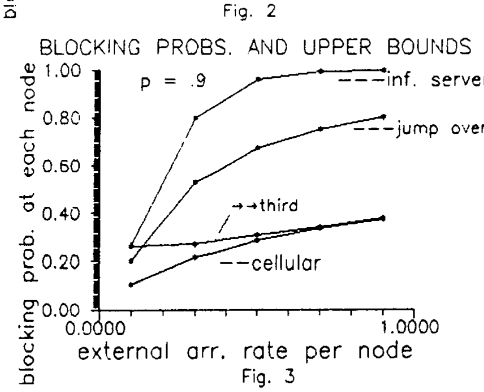

Fig. 3

29.5.5