ADAPTIVE CONTROL WITH OPTIMAL MODEL FOLLOWING

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Abstract

In this paper, a new method is proposed for adaptive optimal model following control of a finite dimensional linear system. Initially, a method is developed to design an optimal model matching control law for a known plant. This is achieved in a novel way by transforming the model matching problem into an output regulator problem. Based on the above approach, a method is proposed for adaptive optimal model following control of an unknown plant. It is a combination of recursive least squares algorithm for parameter estimation from input-output data and the proposed model matching control law. Conditions have been derived for the boundedness of all the signals in the closed loop system.

Introduction

The main objective of control system design is to make the system input-output relation to behave in a certain prespecified manner. In adaptive control problems, the desired performance specifications are given in terms of the model system and the controller is designed at each adaptive step to force the output of the plant to follow the output of the model. Under this situation, the control system is said to be in 'model following' configuration.

Different methods have been proposed in the literature for the above model following control [6,7]. Most of the methods developed so far assume the exact model matching condition, i.e. either the plant is minimum phase or unstable plant zeros are contained in the zeros of the model. These conditions are quite restrictive in practice, and such information is not available a priori in the adaptive control of uncertain plants. Hence it becomes necessary to employ optimal control schemes especially in adaptive control.

In this work, a state space description is chosen for the model system. Initially, a method is proposed for the design of optimal model matching control law for a given plant. This is achieved in a novel way by transforming the model matching problem into an output regulator problem. The resulting control law is such that the impulse response of the compensated plant is as close as possible to the impulse response of the model and is given in terms of the states of the plant and the model.

Based on this approach, a method is developed for adaptive optimal model following control of an unknown plant. It is a combination of the 'recursive least squares' algorithm for parameter estimation from input-output data and the proposed optimal model matching algorithm. This scheme is shown to guarantee overall closed loop stability if the estimated parameter values converge to within an acceptable range of the actual plant parameters. This stability property holds good irrespective of whether the plant is minimum phase or not. The paper ends with the numerical example to show the effectiveness of the method.

Optimal Model Matching For a Known Plant

In this section, we develop a method for optimal model matching control of a known plant

The plant is defined in state space as,

$$X_{k+1} = AX_k + BU_k \qquad A \in R^{n\times n}$$
$$Y_k = CX_k \qquad B \in R^{n\times 1}, \quad C \in R^{1\times n} \tag{1}$$

(A,B) stabilizable
(C,A) detectable
and the model as

$$X^m_{k+1} = A^m X^m_k + B^m U^m_k; \qquad A^m \in R^{m\times m}$$
$$Y^m_k = C^m X^m_k \qquad B^m \in R^{m\times 1}, \quad C^m \in R^{1\times m} \tag{2}$$

We assume here that 'U_m' is an unit impulse

i.e. $U_m = \delta(k)$

where $\delta(k)$ is defined as $\delta(k) = 1$ when $k = 0$
$= 0$ when $k > 1$.

The objective is to find the control law U_{mk} such that y_k is as close as possible to y^m_k when $U^m_k = \delta(k)$, i.e. the output from the compensated plant should follow the output of the model when unit impulse input is applied to both. This is equivalent to minimizing the performance criterion J, given that

$$J = \sum_{k=0}^{\infty} [(y_k - y^m_k)^2 + \rho U^2_k] \quad (3)$$

where > 0 is the penalty imposed on the control energy.

Thus the impulse response of the model system is given by

$$y^m_k = C^m (A^m)^{k-1} B^m \quad (4)$$

[Here $X^m(o)$ is taken to be zero].

This response y^m_k for impulse input $\delta(k)$ is seen to be the response of the following autonomous system, defined by

$$\hat{X}^m_{k+1} = \hat{A}^m \hat{X}^m_k \qquad X^m(o) = \hat{B}^m$$

$$\hat{Y}^m_k = \hat{C}^m \hat{X}^m_k \quad (5)$$

where (C^m, A^m, B^m) is the state-space realization of $Z^{-1} T_m(Z)$. One of such realization is

$$\hat{A}^m = \begin{bmatrix} A^m & 0 \\ C^m & 0 \end{bmatrix}, \hat{B}^m = \begin{bmatrix} B^m \\ 0 \end{bmatrix}, \hat{C}^m = [0 \ldots 0 \ 1]$$

$$\text{Here, } X^m_k = (\hat{A}^m)^k \hat{X}^m(o)$$

$$= (\hat{A}^m)^k \hat{B}^m$$

$$\text{Hence, } Y^k_m = \hat{C}^m (\hat{A}^m)^k \hat{B}^m = C^m (A^m)^{k-1} B^m \quad (6)$$

Thus it is equivalent to stating that for $U^m_k = \delta(k)$, the output of the model system (2) is same as that of the autonomous system (5). Hence, the minimization of the performance criterion 'J' becomes equivalent to solving the output regulator problem for the augmented system with the following system matrices,

$$\bar{X}_{k+1} = \begin{bmatrix} x_{k+1} \\ x^m_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \hat{A}_m \end{bmatrix} \begin{bmatrix} X_k \\ X^m_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U_k$$

$$= \bar{A} X_k + \bar{B} U_k \text{ with } \bar{X}(o) = \begin{bmatrix} X(0) \\ \hat{B}^m \end{bmatrix}$$

$$\bar{y}_k = y_k - y^m_k$$

$$= (C, -\hat{C}^m) \begin{bmatrix} X_k \\ X^m_k \end{bmatrix}$$

$$= \bar{C} \bar{X}_k \quad (7)$$

$$J = \sum_{k=0}^{\infty} (\bar{y}^2_k + U^2_k) \quad (8)$$

It is seen that the model system is not affected by the control U_k. The control law for U_k, is of the form $U_k = Fx_k + Fm\ x^m_k$ where $F^k \varepsilon R^{nx1}$, $F^m \varepsilon R^{mx1}$, obtained by recursively evaluating the expressions (9) - (12). The iteration can be stopped after F, Fm converge to the constant values. These expressions are obtained by solving the above output regulator problem using dynamic programming, followed by some algebraic manipulations to reduce the computational burden in the solution of the algebraic Ricatti equation. The detailed derivation is not given for the sake of brevity. The final form of recursion is given as follows:

$$F(i+1) = -(\rho + B^T P_{11} B)^{-1} B^T P_{11}(i) A \quad (9)$$

$$P_{11}(i+1) = C^T C + A^T P(A - B F(i+1)) \quad (10)$$

$$F_m(i+1) = -(\rho + B^T P_{11} B)^{-1} B^T P_{12}(i) \hat{A}^m \quad (11)$$

$$P_{12}(i+1) = \hat{C}^T C^m - A P_{12}(i) F_m(i+1)$$

$$\text{with } P_{11}(0) = C^T C \text{ and } P_{12}(0) = -C^T \hat{C}^m \quad (12)$$

The implementation is to be done as shown.

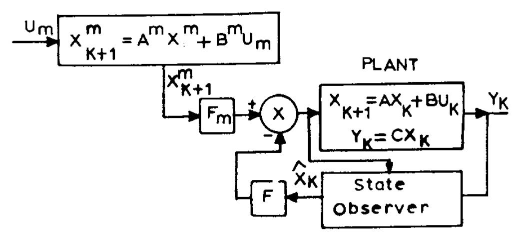

Figure 1
Optimal model matching scheme

Comment: It should be noted that in the implementation, additional state added in the augmented model, is not required for control as the last-entry of F_m is always zero. This is because all the entries of $\hat{A}^m$ in the expression (11) are zero. The other states required for control can be generated from the original model (A^m, B^m, C^m) itself. Also the states of the model must be available in advance by one sampling period because $\hat{X}_m(0) = \hat{B}^m$ is used in the derivation of the control law. Hence the control law is to be written as

$$u_k = Fx_k + F_m x^m_{k+1}$$

The total squared error in the impulse response is given by

$$J_o = (X(0)^T, B^{mT}) \ P(X^T(0), B^{mT})^T$$

Adaptive Model Following Control

In self tuning adaptive control, the plant parameters A, B, C are unknown. Only the plant input U_k and the output Y_k are measurable. Hence parameter estimation is required at each step to design the control law. We assume that the form of the transfer function of the plant is known. The system matrices are taken to be in the form

$$A = \begin{bmatrix} -a_1 & 1 & 0 & \ldots & 0 \\ -a_2 & 0 & 1 & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & 1 \\ -a_n & \cdot & \cdot & \ldots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix}, \quad C=[1 \ 0 \ \ldots \ 0] \quad (13)$$

For this plant the input-output relation can be written as,

$$y_k = -a_1 y_{k-1} - a_2 y_{k-2} \cdots -a_n y_{k-n} + b_1 u_{k-1} + \cdots + b_n u_{k-n} \qquad (14)$$

The vector of parameter estimates is written as

$$\theta = [-a_1 \ \ldots \ -a_n \ \ b_1 \ \ldots \ b_n]^T$$

and a vector of regressors

$$\phi^T(k-1) = [y_{k-1} \cdots y_{k-n} \ u_{k-1} \cdots u_{k-n}]$$

then $y_k = \phi^T(k-1)\ \theta$

The recursive least squares estimation algorithm is given by

$$\theta(k) = \theta(k-1) + K(k-1)\ [y(k) - \phi^T(k-1)\ \theta(k-1)] \qquad (15)$$

$$K(k) = P(k-1)\ \phi(k)\ [1 + \phi^T(k)\ P(k-1)\ \phi(k)]^{-1} \qquad (16)$$

$$P(k) = P(k-1) - \frac{P(k-1)\ \phi(k)\ \phi^T(k)\ P(k-1)}{1 + \phi^T(k)\ P(k-1)\ \phi(k)} \qquad (17)$$

with $P(-2) = NI$, $N \gg 1$ and $\theta(0)$ are given. (18)

Adaptive Control Algorithm

Now the steps involved in adaptive control algorithm are as follows:

1. The plant parameters are estimated using recursive least squares scheme (15) - (18).
2. Replace the estimated parameters $(\hat{A},B,C)$ in place of true parameters (A,B,C) in the observer to estimate the states S.
3. Design the observer parameter matrix 'G' by pole placement or Kalman filter technique using the estimates $\hat{A}$, $\hat{B}$, $\hat{C}$.
4. Compute F, Fm using the equations (9) - (12) and compute $U_k = Fx + Fm\ x_m$.

Repeat the above steps at each sampling interval.

The overall scheme is shown in the following figure.

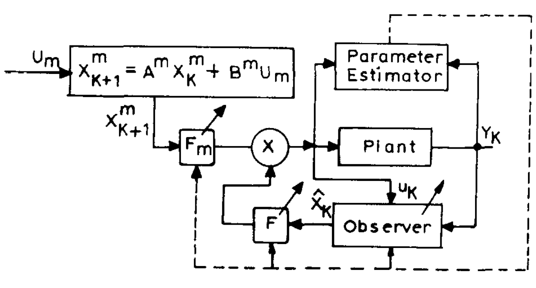

Figure 2
Scheme for adaptive model following

The stability analysis is done in somewhat different way than the proofs given in [2,5], with a view to applying and analysing this method to the control of infinite dimensional systems described in the general semigroup setting. Here a state space equation is derived governing the evolution of u(k), y(k) and conditions are derived for the boundedness of all the signals in the closed loop system.

Now $u_k = Fm(k)\ X^m_{k+1} + F(k)\ X_k$

Thus the evolution for $\phi(k)$ is given by $\phi(k) =$

$$\begin{bmatrix} y_k \\ y_{k-1} \\ \cdot \\ \cdot \\ \cdot \\ y_{k-n} \\ u_k \\ u_{k-1} \\ \cdot \\ \cdot \\ \cdot \\ u_{k-n} \end{bmatrix} = \begin{bmatrix} \begin{matrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ & & & \\ 0 & \cdot & 1 & 0 \end{matrix} & \mathbf{0} \\ \mathbf{0} & \begin{matrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 0 & \cdot & 1 & 0 \end{matrix} \end{bmatrix} \begin{bmatrix} y_{k-1} \\ \cdot \\ \cdot \\ \cdot \\ y_{k-n-1} \\ u_{k-1} \\ \cdot \\ \cdot \\ \cdot \\ u_{k-n-1} \end{bmatrix} + \begin{bmatrix} C(k)X_k \\ \\ F(k)X_k + F_m(k)X^m_k \\ \\ 0 \end{bmatrix}$$

and can be written as $\phi(k) = Q\ \phi(k-1) + w(k)$ (19)

Here Q is always stable. Hence $\phi(k)$ solely depends on w(k). When the estimated parameters become sufficiently close to the actual values, $\phi(k)$ is mainly determined by X_k, $\hat{X}_k$ and X^m_k, if F(k), $F^m(k)$ attain constant bounded values and the model is stable. The conditions for the boundedness of X_k and $\hat{X}_k$ can be got by writing the evolution for $^k(X_k, X^m_k - \hat{X}_k)^T$ as follows. Let $\hat{A},\hat{B},\hat{C}$ denote the estimated parameter matrices.

Let $A - \hat{A} = \tilde{A}$; $B - \hat{B} = \tilde{B}$ and $C - \hat{C} = \tilde{C}$

then it is easy to see that

$$\begin{bmatrix} \hat{X}_{k+1} \\ X_{k+1} - \hat{X}_{k+1} \end{bmatrix} = \left(\begin{bmatrix} \hat{A}_k + \hat{B}_k F_k & -G_k C_k \\ 0 & \hat{A}_k + G_k \hat{C}_k \end{bmatrix} + \begin{bmatrix} G_k \tilde{C}_k & 0 \\ \tilde{A}_k + \tilde{B}_k F_k + G_k \tilde{C}_k & \tilde{A}_k + G_k \tilde{C}_k \end{bmatrix} \right) \begin{bmatrix} \hat{X}_k \\ X_k - \hat{X}_k \end{bmatrix} + \begin{bmatrix} B_k\ F^m_k\ X^m_{k+1} \\ 0 \end{bmatrix} \qquad (20)$$

Hence the condition for stability becomes (A_k, B_k, C_k) stabilizable and detectable for all k, and $\tilde{A}(k)$, $\tilde{B}(k)$, C(k) sufficiently small.

In other words, the estimated transfer function should not have unstable common zeros. When $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{C}_k$ are sufficiently small, $\hat{X}_k$ and X_k are bounded. Since C(k), $\hat{C}(k)$ are uniformly bounded from Eqs. (19) and (20), we infer that w(k) and hence $\phi(k)$ depends purely on x^m_k and becomes independent of x(0), $\hat{x}(0)$ as $k \to \infty$. Thus the system acts only to follow the model and the effect of initial states and disturbances in the output is reduced exponentially in the output.

This type of stability analysis helps to derive the bounds for $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ for the overall closed loop stability when this procedure is applied to the control of infinite dimensional systems described in the general infinite dimensional Hilbert space setting. Roughly speaking if $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$ are the estimated parameters for the reduced order representation, then for the overall stability.

$$\left\| \begin{bmatrix} -G_k \tilde{C}_k & 0 \\ \tilde{A}_k + \tilde{B}_k F_k + G_k \tilde{C}_k & \tilde{A}_k + \tilde{B}_k F_k \end{bmatrix} \right\| < \varepsilon$$

for some $\varepsilon > 0$

where
$$\left\| \begin{bmatrix} \hat{A}_k + \hat{B}_k F_k & -G_k C_k \\ 0 & \hat{A}_k + G_k \hat{C}_k \end{bmatrix}^n \right\| \leq M r^n$$

$M > 0$, $r < 1$ and ε depends on M, r.

When estimated parameters converge to the exact parameters, through persistent excitation, the impulse response error is given by

$$(x(0)^T, (B^m)^T)\; P \begin{bmatrix} x(0) \\ B^m \end{bmatrix}$$

and the controller becomes "Optimal model matching controller".

Example

An initial simulation result is given to show the efficacy of the method. The plant taken is a simple first order plant given by

$$T(z) = \frac{0.7z^{-1}}{1 - 1.5z^{-1}}, \quad \text{here} \quad = [1.5, .7]$$

The above algorithm is applied to the his plant to the model transfer function.

For the square wave input the response of the model and the closed loop plant is shown in Fig.3.

$$T_m(z) = \frac{z^{-1}}{1 - .2z^{-1}}$$

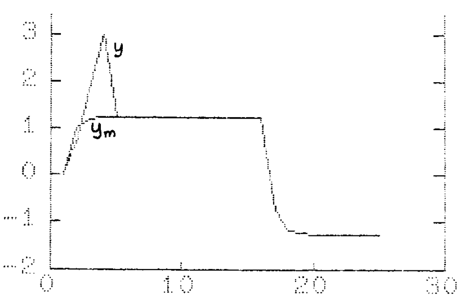

Figure 3

Response of the closed loop plant and the model

The initial values were taken as [0, .7]. Detailed simulation study is under progress.

Conclusion

A new method is proposed for the self tuning adaptive control of linear finite dimensional systems. The model matching problem was approached in a different way and turned out to be design of an optimal regulator followed by the design of precompensator. When estimated parameters become sufficiently close to the actual parameters, the overall system tends to be exponentially stable hence the scheme is robust against small parameter variations, unmodelled dynamics, etc. The design of regulator involves the iterative solution of Ricatti equations, but the procedure can be stopped after one iteration in each sampling period if the same covariance P_{11} used in the previous sampling period as initial covariance. This is possible because the problem turned out to be infinite stage regulator problem [3]. This method is extendable to the plants with integral multiples of sampling period as dead time by enlarging A, B, C with suitable zeros. It is hoped that this method is particularly robust against modelling error and dead time error. Unlike other methods [5], this does not require spectral factorization, solution of disphantive equations, etc. Detailed simulation study is being carried out to evaluate the effectiveness of the method under different conditions. The computational requirement is also moderate. The extension of this method to the self tuning control of infinite dimensional systems is presently under investigation.

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