ECG COMPONENT DELINEATION BY PRONY'S METHOD

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ABSTRACT

A simple, non iterative method for component wave delineation from the Electrocardiogram (ECG) is derived by modelling its DCT as a sum of few damped cosinusoids. The parameter set $p_k [a_k, \theta_k, f_k, \alpha_k]$ i.e., amplitude, phase, frequency and damping factors of the cosinusoids are estimated by the extended Prony method. Based on the clusters of poles appropriate number of them are combined and the component waves derived thereby are in most cases within a percent rms difference (PRD) of 10. The method performed well even in the presence of different artifacts, with a data compression of 1 in 5.

INTRODUCTION

ECG has been extensively used as a diagnostic tool in hospitals, ICCUs for the detection of many heart ailments. One of the active areas of research in automated ECG signal analysis is the delineation of the P, QRS and T waves. While several existing methods can detect the most prominent QRS complex, there seems to be no way for the unfailing detection of smaller P and T waves [1]. We provide a technique for the delineation of component waves irrespective of their size, shape and location.

METHOD

The Discrete Cosine Transform (DCT) of the ECG signal suits well for modelling it as a sum of damped cosinusoids. p_k the parameter set characterizing these cosinusoids is estimated by the extended Prony method. The component wave delineation then boils down to combining appropriate number of cosinusoids based on the clusters of model poles and computing their Inverse Discrete Cosine Transform (IDCT).

THEORY

Assume $x(n)$ is an ECG signal, made up of three component waves P, QRS and T.

$$x(n) = \sum_i x_i(n) \quad ; \quad i = p, q, t \quad (1)$$

$$n = 0, 1, 2, \dots, N-1$$

where $x_p(n) = x(n)$ for $0 \leq n \leq n_0$, $x_q(n) = x(n)$ for $n_0+1 \leq n \leq n_1$, $x_t(n) = x(n)$ for $n_1+1 \leq n \leq N-1$ and $x_p(n) = x_q(n) = x_t(n) = 0$ elsewhere.

The DCT of $x(n)$ in eqn. (1) is, [2]

$$X(m) = \sum_i X_i(m) = \sum_i \sqrt{(2/N)} C_m \sum_{n=0}^{N-1} x_i(n) \cos \frac{(2n+1)m\pi}{2N}$$

$$i = p, q, t \quad ; \quad m = 0, 1, 2, \dots, N-1 \quad (2)$$

$$C_m = 1 \text{ for } m = 0 \quad ; \quad C_m = 1/\sqrt{2} \text{ for } m \neq 0 \quad ;$$

The IDCT of $X(m)$ is

$$x(n) = \sum_i \sqrt{(2/N)} \sum_{m=0}^{N-1} C_m X_i(m) \cos \frac{(2n+1)m\pi}{2N} \quad (3)$$

$$i = p, q, t \quad ; \quad n = 0, 1, 2, \dots, N-1$$

EXTENDED PRONY METHOD

We use the extended Prony method to approximate the signal $X(m)$ as a sum of r damped cosinusoids, [3] i.e.,

$$\hat{X}(m) = \sum_{k=1}^r a_k \exp(\alpha_k m) \cos(2\pi f_k m + \theta_k) \quad (4)$$

$$m = 0, 1, 2, \dots, N-1$$

where $N > 4r$ and p_k is the parameter set for the k th cosinusoid. The nonlinear problem of estimating p_k is separated into two linear problems of determining $[a_k, f_k]$ and $[a_k, \theta_k]$. Thus $X(m)$ is expressed as

$$\hat{X}(m) = \sum_{k=1}^r h_k z_k^m + \sum_{k=1}^r h_k^* (z_k^*)^m \quad (5)$$

where $h_k = a_k \exp(j\theta_k)$, $z_k = \exp(\alpha_k + j2\pi f_k)$ and $*$ denotes complex conjugate. $X(m)$ can also be generated by the recursive difference equation

$$X(m) + \sum_{j=1}^{2r} b(j)X(m-j) = e(m); \quad 2r \leq m \leq N-1 \quad (6)$$

where $e(m)$ is the prediction error. Parameters $b(j)$ obtained by solving the covariance Linear Prediction (LP) problem are related to the modes z_k in eqn. (5) by

$$B(z) = 1 + \sum_{j=1}^{2r} b(j)z^{-j} = \prod_{k=1}^r (1 - z_k z^{-1})(1 - z_k^* z^{-1}) \quad (7)$$

The remaining unknown parameters h_k , h_k^* , in eqn. (5) are estimated by minimizing the squared error with respect to h_k , h_k^* . The resulting normal equation is

$$[z^H z] h = z^H x \quad (8)$$

where z^H is $2r$ by N Vandermonde matrix obtained using the poles z_k ; h is $2r$ by 1 vector of h_k , h_k^* and x is N by 1 data vector. For the real data case the matrix $z^H z$ is usually invertible.

RESULTS

Figs. a-d show good agreement between the model output (whole beat shown as o) and delineated component waves (solid line) with the originals (dashed lines). Estimated DCTs of whole beat, P, QRS and T comprising of 11, 3, 3 and 5 damped sinusoids respectively are shown in Figs. e-h and their PRDs are 9.08, 24.8, 11.5 and 5.2 respectively. For small and noisy P waves as in this example the PRD may go high though the fit is satisfactory. Fig. i shows the distinct and well separated pole clusters which enable easy component delineation. On an average P and T waves require 2 to 4 and QRS needs 3 to 6 damped sinusoids in their models, only 1 or 2 of them characterize the basic waveshape while

others add finer correction in the form of a small kink or bend to give proper shape to a component wave.

DISCUSSION

A practical problem is the choice of number of sinusoids for a given signal. The order is estimated using the Akaike's Information Criterion (AIC) and Levinson Recursion for covariance LP. For a signal with 300 samples, on an average about 15 damped sinusoids are needed. Considering 4 parameters for each sinusoid in p_k , this resulted in a data compression of 1 in 5. The reconstructed component waves differed from the actual ones by a constant dc shift. The first DCT coefficient is modified such that this shift is corrected.

The peak samples of component waves $x_i(n)$ appear also as distinct peaks in the magnitude spectrum of $X(m)$ which in turn are represented by the well separated clusters of poles in the LP model. The method gave satisfactory results even in presence of EMG noise, mains interference and baseline wander. The DCT of most abnormal ECG signals is of mixed phase and did not pose any problem in their modelling. No instability problem associated with the covariance LP was encountered.

REFERENCE

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