ANALYTICAL EVALUATION OF ELEMENT COUPLING COEFFICIENTS ON GENERAL PARABOLOIDS OF REVOLUTION

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ABSTRACT

Mutual coupling results have been presented between individual elements in a conformal phased array on a general paraboloid of revolution (GPOR). The formulation presented yields all the surface ray geometric parameters required in the ray analysis in explicit one-parameter form. The parameter involved is the First Geodesic Constant h, whose determination involves a simple univariate search.

Introduction

A general paraboloid of revolution (GPOR) is a structure often encountered in aerospace engineering. Aircraft radomes and and satellite launch vehicle nose cones may be modeled by GPORs. Although conformal phased arrays on the right cylinder and cone have been treated extensively in literature, no results are available for the case of GPORs [1,2]. This is due to the fact that treating the surface geodesics numerically in a general two-parameter form makes the problem computationally intractable.

In this paper, we present a method for treating the mutual coupling between individual elements of a conformal antenna array on a general paraboloid of revolution in the one-parameter form. Actual mutual coupling results have been presented using the Geodesic Constant Method (GCM) [3].

Formulation

A general paraboloid of revolution incorporates a shaping parameter in its parametric equation set

$$x = au \cos \phi \qquad y = au \sin \phi \qquad z = -u^2 \tag{1}$$

which determines the "sharpness/flatness" of the GPOR. We identify the parabolic coordinates (u,ϕ) with the geodesic coordinates (u,v). The equation of the general right m-th order geodesic on the GPOR is given as [4]

$$\frac{d\phi}{du} = \frac{\pm h_{rm}(4u^2+a^2)^{1/2}}{au(a^2u^2-h_{rm}^2)^{1/2}} \tag{2}$$

where the `+/-` sign of h_{rm} depends on whether ϕ is increasing/decreasing with respect to u. Although several surface ray geometric parameters can be readily

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computed using the differential geometric form of the parametric equation, the main difficulty lies in the evaluation of the integrals for (i) the relation between the geodesic parameters u and ϕ, (ii) the arc length and (iii) the generalized Fock parameter required in the UTD formulation. Integrating eq (2), we obtain the relation between ϕ and u as

$$\phi = \frac{h_{rm}}{a^2} \ln \frac{a[4u^2+a^2]^{1/2}+2[a^2u^2-h_{rm}^2]^{1/2}}{a[4u^2+a^2]^{1/2}-2[a^2u^2-h_{rm}^2]^{1/2}} + \sin^{-1}\left|\frac{a[a^2u^2-h_{rm}^2]^{1/2}}{u[a^4+4h_{rm}^2]^{1/2}}\right| + h'_{rm} \qquad (3)$$

The constants h_{rm} and h'_{rm} are the constants of integration which appear due to the two successive integrations of the geodesic equations and are readily determined since the source and observation points are known a priori. The arc length s_{rm} for the m-th order right geodesic is found to be

$$s_{rm} = \frac{[(4u^2+a^2)(a^2u^2-h_{rm}^2)]^{1/2}}{2a} + \frac{a^4+4h_{rm}^2}{8a^2} \ln \frac{a[4u^2+a^2]^{1/2}+2[a^2u^2-h_{rm}^2]^{1/2}}{a[4u^2+a^2]^{1/2}-2[a^2u^2-h_{rm}^2]^{1/2}} \qquad (4)$$

while the generalized Fock parameter has been derived in [4]. We have derived these ray parameters in the one-parameter form, i.e., in terms of the First Geodesic Constant h_{rm}. This is the reason for calling this method as Geodesic Constant Method (GCM).

Discussion

It has been observed that the ray geometric parameters like the arc length and the generalized Fock parameters are extremely sensitive to the value of h and any truncation in these leads to large errors in the determinations of the ray parameters. For most practical applications, the value of h must be determined accurately upto eight decimal places.

In the case of the GPOR, we have further observed the phenomena of ray splitting. For instance there could be more than one primary geodesic in the anti-clockwise direction itself. In effect, it means a doubling of the rays which must be accounted for in the EM computations. Since our method is one of the univariate search, this phenomenon can be dealt with within the realm of computation tractability.

Using the ray parameters derived by this method, it has been possible to compute the mutual coupling between pairs of antenna elements located arbitrarily on the surface of a general paraboloid of revolution (GPOR). The mutual coupling results presented here are for the variation of the observation slot centroid along the geodesic directions u and v.

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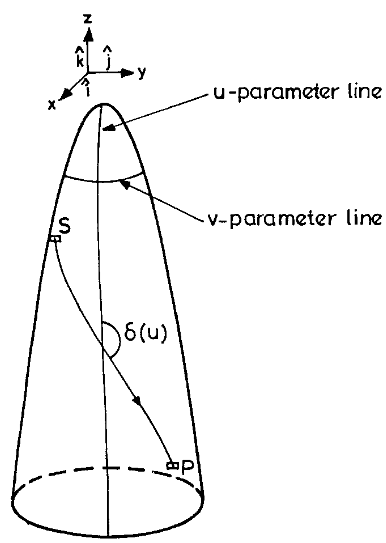

Fig. 1 Geodesic path simulation on a general paraboloid of revolution.

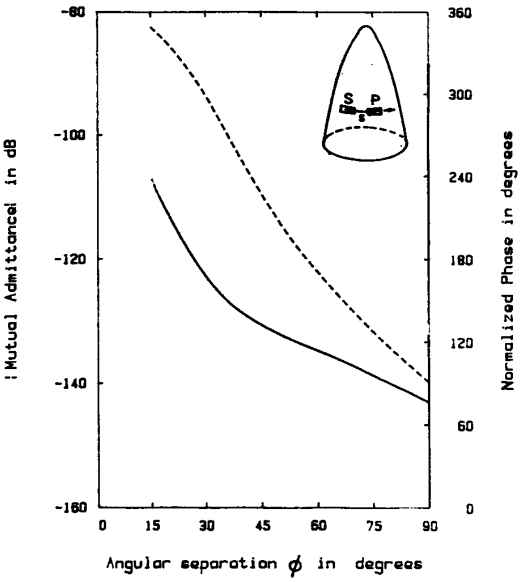

Fig. 2 Mutual admittance vs. angular separation between two rectangular slots $(0.5\lambda \times 0.2\lambda)$ on a GPOR of shaping parameter $a = 5.0$. $u_s = u_f = 2.0$. Magnitude (——), Phase (----).

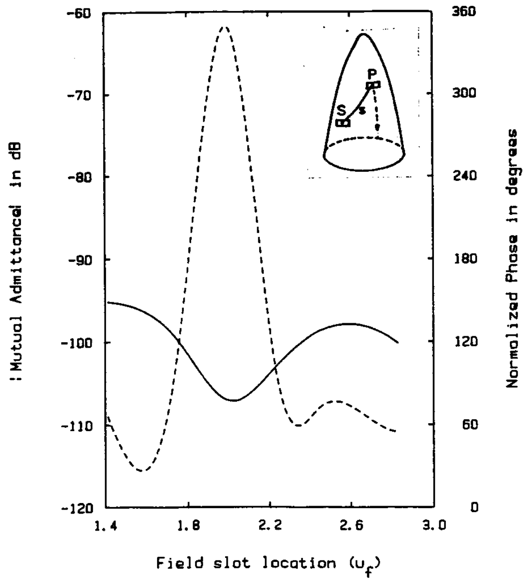

Fig. 3 Mutual admittance vs. the u_f coordinate between two rectangular slots $(0.5\lambda \times 0.2\lambda)$ on a GPOR of shaping parameter $a = 5.0$. $u_s = 2.0$. Angular separation = 15° (constant). Magnitude (——), Phase (----).