approach is about two orders of magnitude greater than this new method.

This representation of the likelihood function is also interesting from an analytic viewpoint. The leading term in the a element summations corresponds to beamforming. The other terms associated with the off-diagonal b elements represent interference that is accounted for in the more complete ML approach. Another practical benefit of this representation is the ability to analytically calculate derivatives of the likelihood function. These are used in maximization searches to locate the best bearing estimates.

References

[1] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 1553–1560, 1988.

[2] P. Stoica and K. Sharman, "Maximum likelihood methods for direction-of-arrival estimation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 1132–1143, 1990.

[3] Q. Wu and D. Fuhrmann, "A parametric method for determining the number of signals in narrow-band direction finding," *IEEE Trans. Signal Processing*, vol. 39, pp. 1848–1857, 1991.

[4] S. Orfanidis, *Optimal Signal Processing*. New York: Macmillan, 1988, pp. 369–370.

[5] J. Burg, D. Luenberger, and D. Wenger, "Estimation of structured covariance matrices," *Proc. IEEE*, vol. 70, pp. 963–974, 1982.

[6] D. Fuhrmann, "Application of Toeplitz covariance estimation to adaptive beamforming and detection," *IEEE Trans. Signal Processing*, vol. 39, pp. 2194–2198, 1991.

[7] D. Tufts and A. Shah, "Estimation of a signal waveform from noisy data using low-rank approximation to a data matrix," *IEEE Trans. Signal Processing*, vol. 41, pp. 1716–1720, 1993.

Comments on "A Further Analysis of Decorrelation Performance of Spatial Smoothing Technique for Real Multipath Sources"

K. C. Indukumar and V. U. Reddy

I. Introduction

In [1], the authors have analyzed and discussed the decorrelation performance of spatial smoothing based on first-order prediction approximation to time-delay narrowband signals when nonzero bandwidth signals impinge on the array. Based on the approximation model, they have shown that the spatial smoothing progressively reduces the correlation between signals until certain smoothing is reached and then builds up the correlation to that of the original on further smoothing.

In this note, we first show that the prediction model used in [1] is valid only for short time delays. In large arrays, the time delay (with respect to the reference sensor) becomes very large and the approximation fails. This failure leads to erroneous results, such as decreasing of correlation initially and then increasing beyond so-called optimum smoothing. We show that this result is due to modeling error and not due to nonzero bandwidth signals. Further, using the true signal model for nonzero bandwidth signals, we show that the spatial smoothing progressively decorrelates the signals. A detailed discussion on the decorrelation effect of smoothing in the case of nonzero bandwidth signals using a broadband array is given in [2].

Manuscript received December 15, 1992; revised July 16, 1993. The associate editor coordinating the review of this paper and approving it for publication was Prof. John A. Stutler.

The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, India.

IEEE Log Number 9214185.

Let $s(t)$ be a sample function from a finite-bandwidth random process and $s(t+\tau)$ be a delayed version of the same signal. Based on the prediction model, $s(t+\tau)$ is given by [1]

$$s(t+\tau) = a(\tau)s(t) + b(\tau)s'(t) \tag{1}$$

where $s(t)$ and $s'(t)$ are the uncorrelated signals of same power and $a(\tau)$ and $b(\tau)$ are the coefficients given by

$$a(\tau) = R^1(\tau) \tag{2}$$

and

$$b(\tau) = (1 - |R(\tau)|^2)^{1/2} \tag{3}$$

with $R(\tau)$ denoting the auto-correlation function of the signal for lag τ. Assuming the random processes to be of flat power spectrum having bandwidth B and centered at f_o, the power spectral density is

$$P(f) = \begin{cases} \frac{1}{B}, & f_o - \frac{B}{2} \le f \le f_o + \frac{B}{2} \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

The auto-correlation function is then given by

$$R(\tau) = \text{sinc}\,(B\tau)e^{j2\pi f_o \tau} \tag{5}$$

where $\text{sinc}\ (\alpha) = (\sin(\alpha)/\alpha)$.

Now, consider two delayed versions of the signal, $s(t+\tau_1)$ and $s(t+\tau_2)$. The correlation between them is given by

$$R(\tau_2 - \tau_1) = E[s(t+\tau_1)s^*(t+\tau_2)]. \tag{6}$$

Assuming the signals to be of unit power, the correlation based on the prediction model (1), $R_m(\tau_2-\tau_1)$,[2] is given by

$$R_m(\tau_2 - \tau_1) = a(\tau_1)a^*(\tau_2) + b(\tau_1)b^*(\tau_2) \tag{7}$$

which can be rewritten as

$$R_m(\tau_2 - \tau_1) = R^*(\tau_1)R(\tau_2) + (1 - |R(\tau_1)|^2)^{1/2} \cdot (1 - |R(\tau_2)|^2)^{1/2}. \tag{8}$$

From (5) and (8), we get

$$R_m(\tau_2 - \tau_1) = \text{sinc}\,(B\tau_1)\,\text{sinc}\,(B\tau_2)e^{j2\pi f_o(\tau_2-\tau_1)} + (1 - \text{sinc}\,(B\tau_1)^2)^{1/2} \cdot (1 - \text{sinc}\,(B\tau_2)^2)^{1/2}. \tag{9}$$

But from (5), the true correlation for lag $(\tau_2 - \tau_1)$ is given by

$$R(\tau_2 - \tau_1) = \text{sinc}\,(B(\tau_2 - \tau_1))e^{j2\pi f_o(\tau_2-\tau_1)}. \tag{10}$$

Consider (9) and (10). For small values of $B\tau$ (i.e., $B\tau_1 \ll 1$ and $B\tau_2 \ll 1$), sinc $(B\tau) \approx 1$ and we get

$$R_m(\tau_2 - \tau_1) \approx R(\tau_2 - \tau_1) \tag{11}$$

[1] denotes conjugate operator

[2] The subscript m in $R_m(\cdot)$ indicates correlation computed with the prediction model.

1053-587X/94$04.00 © 1994 IEEE

implying that the prediction model is valid for very small values of time-bandwidth product. In the case of large arrays, the signals received at faraway elements with respect to the reference sensor experience large delays. We now compute the modeling error in the covariance matrix obtained with the prediction model.

II. Modeling Error

Assume that nonzero bandwidth signals, $s_1(t), \cdots, s_D(t)$, are impinging on a linear array grouped into overlapping subarrays of M elements each with K subarrays. Assuming the signals from zero-mean, bandlimited complex Gaussian random processes and noise from a zero-mean white complex Gaussian random process are uncorrelated from sensor to sensor, the true data covariance matrix of kth subarray is given by [2]

$$\mathbf{R}^{(k)} = \int_{f_o-B/2}^{f_o+B/2} \mathbf{A}(\theta, f)(\mathbf{G}(\theta, f))^{(k-1)} \cdot \mathbf{S}(f)(\mathbf{G}(\theta, f))^{-(k-1)}\mathbf{A}(\theta, f)^H \, df + \sigma_n^2 \mathbf{I} \quad (12)$$

where $\mathbf{S}(f)$ is the source power spectral density matrix, and

$$\mathbf{A}(\theta, f) = [\mathbf{a}(\theta_1, f), \cdots, \mathbf{a}(\theta_D, f)]$$

is the direction matrix at frequency f with

$$\mathbf{a}(\theta_i, f) = \left[1, \exp\left(-j2\pi f \frac{d}{c} \sin\theta_1\right), \cdots, \cdot \exp\left(-j2\pi f(M-1)\frac{d}{c}\sin\theta_i\right)\right]$$

and

$$\mathbf{G}(\theta, f) = \mathrm{diag}\left(\exp\left(-j2\pi f\frac{d}{c}\sin\theta_1\right), \cdots, \cdot \exp\left(-j2\pi f\frac{d}{c}\sin\theta_D\right)\right).$$

The smoothed data covariance matrix is given by

$$\overline{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R}^{(k)}. \quad (13)$$

Combining (12) and (13), and taking the summation operator inside the integral, we get

$$\overline{\mathbf{R}} = \int_{f_o-B/2}^{f_o+B/2} \mathbf{A}(\theta, f)\left\{\frac{1}{K}\sum_{k=1}^{K}(\mathbf{G}(\theta, f))^{(k-1)} \cdot \mathbf{S}(f)(\mathbf{G}(\theta, f))^{-(k-1)}\right\}\mathbf{A}(\theta, f)^H \, df + \sigma_n^2\mathbf{I}. \quad (14)$$

The source power spectral density matrix can be separated into two matrices as

$$\mathbf{S}(f) = \mathbf{S}_{\mathrm{diag}}(f) + \mathbf{S}_{\text{non-diag}}(f) \quad (15)$$

where $\mathbf{S}_{\mathrm{diag}}(f)$ is a diagonal matrix that represents the auto power spectrum and $\mathbf{S}_{\text{non-diag}}(f)$ is the nondiagonal part of $\mathbf{S}(f)$ that gives the cross spectrum (which would be zero when the signals are uncorrelated) between the signals. Substituting (15) in (14), we get

$$\overline{\mathbf{R}} = \left\{\int_{f_o-B/2}^{f_0+B/2}\mathbf{A}(\theta, f)\mathbf{S}_{\mathrm{diag}}(f)\mathbf{A}(\theta, f)^H \, df + \sigma_n^2\mathbf{I}\right\} + \left\{\int_{f_0-B/2}^{f_o+B/2}\mathbf{A}(\theta, f)\left\{\frac{1}{K}\sum_{k=1}^{K}(\mathbf{G}(\theta, f))^{(k-1)} \cdot \mathbf{S}_{\text{non-diag}}(f)(\mathbf{G}(\theta, f))^{-(k-1)}\right\}\mathbf{A}(\theta, f)^H \, df\right\}. \quad (16)$$

The first part (auto-correlation part) represents the data covariance matrix that we would obtain when the sources are uncorrelated, while the second part (cross-correlation part) arises due to correlation between the signals.

The modeling error in the auto- and crosscorrelation parts of data covariance matrix because of prediction model can be computed as follows. In the two-source case, the mnth element of $\overline{\mathbf{R}}$ (from (16)) and $\overline{\mathbf{R}}_m$ (from [1]) are given by

$$[\overline{\mathbf{R}}]_{mn} = [(\overline{\mathbf{R}})_{\mathrm{auto}}]_{mn} + [(\overline{\mathbf{R}})_{\mathrm{cross}}]_{mn} \quad (17)$$

and

$$[\overline{\mathbf{R}}_m]_{mn} = [(\overline{\mathbf{R}}_m)_{\mathrm{auto}}]_{mn} + [(\overline{\mathbf{R}}_m)_{\mathrm{cross}}]_{mn} \quad (18)$$

where

$$[(\overline{\mathbf{R}})_{\mathrm{auto}}]_{mn} = e^{-j\pi(m-n)\sin\theta_1}\,\mathrm{sinc} \cdot \left(\frac{FBW}{2}\pi(m-n)\sin\theta_1\right) + e^{-j\pi(m-n)\sin\theta_2}\,\mathrm{sinc} \cdot \left(\frac{FBW}{2}\pi(m-n)\sin\theta_2\right) + \sigma_n^2\delta(m-n) \quad (19)$$

$$[(\overline{\mathbf{R}}_m)_{\mathrm{auto}}]_{mn} = e^{-j\pi(m-n)\sin\theta_1} + e^{-j\pi(m-n)\sin\theta_2} + \sigma_n^2\delta(m-n) \quad (20)$$

$$[(\overline{\mathbf{R}})_{\mathrm{cross}}]_{mn} = \left\{\frac{1}{K}\sum_{k=1}^{K}\left(e^{-j\pi[(m+k-2)\sin\theta_1-(n+k-2)\sin\theta_2]} \cdot \mathrm{sinc}\left(\frac{FBW}{2}\pi[(m+k-2) \cdot \sin\theta_1 - (n+k-2)\sin\theta_2]\right) + e^{-j\pi[(m+k-2)\sin\theta_2-(n+k-2)\sin\theta_1]} \cdot \mathrm{sinc}\left(\frac{FBW}{2}\pi[(m+k-2) \cdot \sin\theta_2 - (n+k-2)\sin\theta_1]\right)\right)\right\} \quad (21)$$

$$\begin{aligned}[(\overline{\mathbf{R}}_m)_{\mathrm{cross}}]_{mn} = \frac{1}{K}\sum_{k=1}^{K}\Bigg(&e^{-j\pi[(m-1)\sin\theta_1-(n-1)\sin\theta_2]} \cdot \left[\mathrm{sinc}\left(\frac{FBW}{2}(k-1)\sin\theta_1\right) \cdot \mathrm{sinc}\left(\frac{FBW}{2}(k-1)\sin\theta_2\right) \cdot e^{-j\pi(k-1)(\sin\theta_1-\sin\theta_2)}\right. \\ &\left.+\left(1-\mathrm{sinc}^2\left(\frac{FBW}{2}(k-1)\sin\theta_1\right)\right)^{1/2} \cdot \left(1-\mathrm{sinc}^2\left(\frac{FBW}{2}(k-1)\sin\theta_2\right)\right)^{1/2}\right]\Bigg) \\ &+\Bigg(e^{-j\pi[(m-1)\sin\theta_2-(n-1)\sin\theta_1]} \cdot \left[\mathrm{sinc}\left(\frac{FBW}{2}(k-1)\sin\theta_2\right) \cdot \mathrm{sinc}\left(\frac{FBW}{2}(k-1)\sin\theta_1\right) \cdot e^{-j\pi(k-1)(\sin\theta_2-\sin\theta_1)}\right. \\ &\left.+\left(1-\mathrm{sinc}^2\left(\frac{FBW}{2}(k-1)\sin\theta_2\right)\right)^{1/2} \cdot \left(1-\mathrm{sinc}^2\left(\frac{FBW}{2}(k-1)\sin\theta_1\right)\right)^{1/2}\right]\Bigg).\end{aligned} \quad (22)$$

Here FBW is the fractional bandwidth defined as B/f_o.

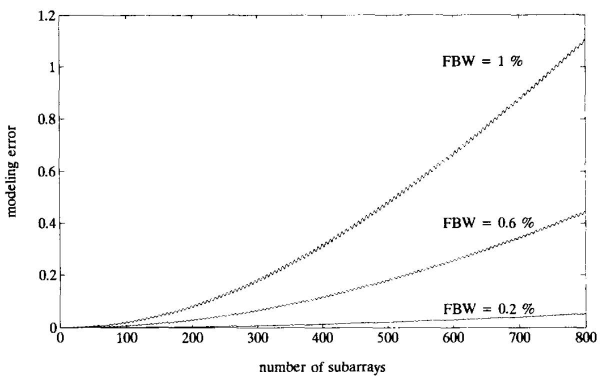

Fig. 1. Modeling error as a function of smoothing for different fractional bandwidths of signals (DOA's: 10° and 30° with zero multipath delay, powers: 0 dB, subarray size: 6).

We now define the modeling error as the Frobenius norm of the difference of true and prediction-based data covariance matrices. Let the modeling error in the auto-correlation and cross-correlation parts of the data covariance matrix be denoted as

$$\zeta_A = \left(\sum_{m=1}^{M} \sum_{n=1}^{M} |[(\overline{\mathbf{R}})_{\text{auto}}]_{mn} - [(\overline{\mathbf{R}}_m)_{\text{auto}}]_{mn}|^2 \right)^{1/2} \tag{23}$$

and

$$\zeta_C = \left(\sum_{m=1}^{M} \sum_{n=1}^{M} |[(\overline{\mathbf{R}})_{\text{cross}}]_{mn} - [(\overline{\mathbf{R}}_m)_{\text{cross}}]_{mn}|^2 \right)^{1/2}. \tag{24}$$

We now show how this modeling error depends on the smoothing. As in [1], we considered a scenario with nonzero bandwidth signal arriving from 10° and its multipath from 30° (for simplicity we assume zero multipath delay) and a uniform linear array with six sensors per subarray. Noise and signal powers were set to unity and the modeling errors were computed for different fractional bandwidths: 1%, 0.6%, and 0.2%, and the results are shown in Fig. 1. Fig. 1 shows the plots of modeling error ζ_C as a function of the number of subarrays. The plots show that the modeling error in the cross-correlation part increases with the number of subarrays. On the other hand, the modeling error in the auto-correlation part is fixed e.g., $\zeta_A = 5.4298 \times 10^{-4}$ for $FBW = 1\%$. Comparing these plots with those of Fig. 1 in [1], it is evident that the increase in the correlation coefficient with smoothing (beyond certain value) is because of the modeling error.

III. Decorrelation Performance of Smoothing with True Model

We now show that the spatial smoothing progressively decorrelates the signals. Note from (16) that only the cross-correlation part is affected by smoothing and also that the cross-correlation part vanishes when the sources are uncorrelated. Hence, we study the effect of spatial smoothing on the cross-correlation part only. The mnth element of the cross-correlation matrix is given as

$$\begin{aligned}[\cdot]_{mn} = \frac{1}{B} \int_{f_o - B/2}^{f_o + B/2} \sum_{p=1}^{D} \sum_{q=1}^{D} \exp \\ \cdot \left(-j \frac{f}{f_o} \pi \left[\left(\frac{2m - 3 + K}{2} \right) \sin \theta_p \right.\right. \\ \left.\left. - \left(\frac{2n - 3 + K}{2} \right) \sin \theta_q \right] \right) \\ \cdot \left(\text{sinc} \left[\frac{f\pi}{f_o 2} K (\sin \theta_p - \sin \theta_q) \right] \right) \Big/ \\ \cdot \left(\text{sinc} \left[\frac{f\pi}{f_o 2} (\sin \theta_p - \sin \theta_q) \right] \right) df. \end{aligned} \tag{25}$$

In the limit, as $K \to \infty$, the term $(\text{sinc}[(f\pi/f_o 2) K(\sin \theta_p - \sin \theta_q)]) \to 0$ for all frequencies in the band giving

$$\lim_{K \to \infty} [\cdot]_{mn} = 0, \qquad \forall \quad m, n. \tag{26}$$

Thus, all the elements in the cross-correlation matrix tend to zero as the number of subarrays tends to infinity, implying that the signals get decorrelated asymptotically with the number of subarrays.

References

[1] J. F. Yang and C. J. Tsai, "A further analysis of decorrelation performance of spatial smoothing technique for real multipath sources," *IEEE Trans. Signal Processing*, pp. 2109–2112, Aug. 1992.
[2] K. C. Indukumar and V. U. Reddy, "Broad-band DOA estimation and beam-forming in multipath environment," in *Proc. IEEE Int. Radar Conf.*, May 1990, pp. 532–537.

An Algorithm for Efficient, Unbiased, Equation-Error Infinite Impulse Response Adaptive Filtering

Carlos E. Davila

Abstract— **An algorithm for efficiently adjusting the coefficients of equation-error infinite impluse response (IIR) adaptive filters is described. Unlike the RLS algorithm, the proposed algorithm yields unbiased filter coefficients. Simulations involving the identification of unknown pole-zero systems demonstrate the algorithm's improved performance over the equation-error RLS algorithm.**

I. Introduction

Adaptive infinite impulse response (IIR) filters offer certain advantages over adaptive finite impulse response (FIR) filters, including the need for lower filter orders and the ability to effectively model a wider variety of systems than FIR filters. Correspondingly, adaptive IIR filters have received considerable interest in the signal processing literature over the past several years [1]–[3]. The equation-error adaptive IIR filter output is given by

$$y(t) = \sum_{m=1}^{N-1} a_m(t) d(t - m) + \sum_{m=0}^{M-1} b_m(t) x(t - m) \tag{1}$$

where $d(t)$ is the desired signal. In vector notation, (1) may be written as $y(t) = \theta_t^T \phi_t$, where

$$\theta_t = [a_1(t) a_2(t) \cdots a_{N-1}(t) b_0(t) b_1(t) \cdots b_{M-1}(t)]^T \tag{2}$$

Manuscript received December 10, 1990; revised March 3, 1993. The associate editor coordinating the review of this paper and approving it for publication was Prof. V. John Mathews.

The author is with the Electrical Engineering Department, Southern Methodist University, Dallas, TX 75275-0335.

IEEE Log Number 9214179.

1053-587X/94$04.00 © 1994 IEEE